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Calculation the Irreducible water saturation S_{wi} and determination Capillary pressure curve P_C from Well Log data

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Abstract. Calculation of irreducible water saturation S_{wi} and determination Capillary pressure curve P_C is very important in the Oil and gas exploration and production. The complex reservoirs always represent a quite challenge to geologist and engineers to calculate S_{wi} [1]. Capillary pressure curvers are usually determined in the laboratory in core analysis and only can perform when we known the irreducible water saturation S_{wi}. It is very difficult in the fact.

This research gives a method: Calculate S_{wi} and determine of P_c. curve from obtain's data set $(S_w;\ P_c)_j$ with: $j = 1 \ldots p$ (which is easyly to collect .for every reservoirs). The method of this study can use for both the carbonate reservoirs and the Sandstone reservoirs. These reservoirs consict of 90% oil and gas in the world

The declared actual testing result from data in variety of diefferent huge oil fields around the world which are found on website and PVEP data has affirmed the appropriateness of this method.

1. Introduction

Calculation of irreducible water saturation S_{wi} is an important step in 3 D reservoir modeling studies. The irreducible water saturation S_{wi} distribution will dictate the original oil in place (STOIP) estimation and influence to the subsequent steps in establishment of dynamic modeling. The complex reservoirs always represent a quite challenge to geologist and engineers to calculate S_{wi} [1].

The capillary phenomemna occurs in porous media when more immiscible fluids are present in the pore space [2]. In the interface between the two phases, Capillary pressureis defined as the difference in pressure betwwen the wetting and nonwetting phases [2]:

$$P_c = P_{nw} - P_w \qquad (1)$$

where P_w is wetting phase capillary and P_{nw} is non wetting phasecapillary.

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Because the gravity forces are balanced by the Capillary foces, so that Capillary pressure at a point in the reservoir can be estimated from the hight above the oil-water contact and the diffrrence in fluid densities. For the oil-water media, we have:

$$P_c = (\rho_w - \rho_0)gh \qquad (2)$$

Capillary forces are reflected by Capillary pressure curvers affect the recovery efficiency of oil displaced by water, gas or different chemicals. thus, Capillary pressure functions are need for performing reservoir simulation studies of the different oil recovery processes. In [3] and more other studies suggest methods to plot P_c curvers by empiricalism and analise the relationships between it and orther parameters.

Interpretation of Capillary pressure curvers may yield useful informations regarding the petrophysical properties of rocks and the fluid rock interaction.. Relative permeability, absolute permeability and pore distribution to the nonwetting and wetting phases can obtain from the Capillary pressure curvers.

This research gives a method: From obtain's data set $(S_w; P_c)_j$ with: $j = 1...p$, we calculate S_{wi} and determine function of P_c.curve , determine three constants: $S_{wi}, a; b$.

The object of this study is both the carbonate reservoirs and the Sandstone reservoirs. These reservoirs consist 90% reserver oil and gas in the world. Verification for both these reservoirs

The most important result of this study is calculation of the irreducible water saturation S_{wi} and plot P_c curve as the graph of a continuous function from $(S_w; P_c)_j$. data, which is easyly to collect in the fact. On the other hand, the measurement of it in the laboratory by core analysis is very difficult, espensive and time-consuming.

The declared actual testing result from data in variety of diefferent huge oil fields around the world which are found on website and PVEP data has affirmed the appropriateness of this method

Nomenclature:

S_{wi}: irreducible water saturation

Well log data :

P_c: Capillary pressure

2. The theoretical basic of the method

2.1. The empirical method:

Capillary pressure curvers are usually determined in the laboratory in core analysis by the mercury injection method. The determination of Capillary pressure using reservoir fluids is usually done by the restored – state method or using a centrifuge, and according to the correclation coefficient to obtain the reservoir pressure [4].

2.2.The method of this study:

Capillary pressure curvers are presented by the equation [5]:

$$P_c = \frac{a}{(S_w - S_{wi})^b} \qquad (3)$$

Where : $a, b, S_{wi\blacklozenge}$ are three constants with $0 < a < 1$ and $0 < b < 2$

Formula (3) is established empirically and used for more studies. But the empirical establishment of this function is vry difficunt .

It requires to know the value S_{wi}, which is hard to implement and also sufficial measurement data is not simple to obtaint in practice. Nevertheless, the measurement to gather data set from 8 to 10 values is feasible in every reservoir.

This research offers a method: From obtain's data set $(S_w;\ P_c)_j$ with: $j = 1...p$, we calculate S_{wi} and determine function of P_c.curve , determine three constants: $S_{wi}, a; b$, and plot the graph of (3).

3. Calculate the Irreducible water saturation S_{wI} and determination Capillary pressure curve P_C from Well Log data :

The capillary pressure curvers are represented by the equation

$$P_c = \frac{a}{(S_w - S_{wi})^b}. \quad (3)$$

The problem is that: From the collection data: $(S_w;\ P_c)_j$ with : $j = 1...p$ we calculate S_{wi}, P_c curver, determine three constants: S_{wi} and $a; b$, plot the graph of function by (3).

Taking common logarithms of both sides of (3) gives:

$$\lg P_c = \lg a - b\lg\left(S_w - S_{wi}\right).$$

Denote: $y = \lg P_c$; $A = -b$; $B = \lg a$ we have: $y = Ax + B$.

Consider S_{wi} is constant, perform the linear regression analysis to determine A and B (to infer $a; b$).

In order to minimize the mean squared error : $F = \sum_{j=1}^{p}\left[y_j - \left(Ax_j + B\right)\right]^2 \to \min$.

Differentiate function F with respect to arguments A and B we have:

$$\begin{cases} \dfrac{\partial F}{\partial A} = -2\sum_{j=1}^{p}\left[y_j - \left(Ax_j + B\right)\right].x_j = 0 \\ \dfrac{\partial F}{\partial B} = -2\sum_{j=1}^{p}\left[y_j - \left(Ax_j + B\right)\right] = 0 \end{cases}$$

$$\text{or: } \begin{cases} (\sum_{j=1}^{p} 1).B + (\sum_{j=1}^{p} x_j).A = \sum_{j=1}^{p} y_j. \\ (\sum_{j=1}^{p} x_j).B + (\sum_{j=1}^{p} x_j^2).A = \sum_{j=1}^{p} x_j y_j. \end{cases}$$

in the matrix form:

$$\begin{pmatrix} \sum_{j=1}^{p} 1 & \sum_{j=1}^{p} x_j \\ \sum_{j=1}^{p} x_j & \sum_{j=1}^{p} x_j^2 \end{pmatrix} . \begin{pmatrix} B \\ A \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^{p} y_j \\ \sum_{j=1}^{p} x_j y_j \end{pmatrix} \ldots (4)$$

Using the liner regression method represents in [5] to solve equation (4), we find out A and B. The constants a, b are calculated as following:

$$a = 10^B \ ; b = -A$$

Obviously that: $S_{wi} \leq \min(VtS_w)$, we give consecutively: $S_{wi} = 0; \lambda, 2\lambda.......(n-1)\lambda$ with $\lambda = 0.0025$ and: $n\lambda = \min(VtS_w)$

For every value : $S_{wi} = 0; \lambda, 2\lambda.......(n-1)\lambda$ we calculate F, then choose $F\min$ is the smallest value in the series n values of F, we determine immediately three constants: S_{wi} and $a; b$ in (3). With 3 parameters S_{wi} and $a; b$, we plot the graph of (3)

Programming by the MATLAB language .

Application's conditions

Differentiate (3) we have :

$P_c' = \dfrac{-a.b}{(S_w - S_{wi})^{b+1}} < 0$ so function P_c is degenerated and non uniform continuous on $(S_{wi};\ 1]$; thus application's conditions is the collection data: $(S_w;\ P_c)_j$ with: $j = 1...p$ must be unit value and monotono degreeing. It is mean that : Data $(S_w;\ P_c)_j$ must satisfy condition:

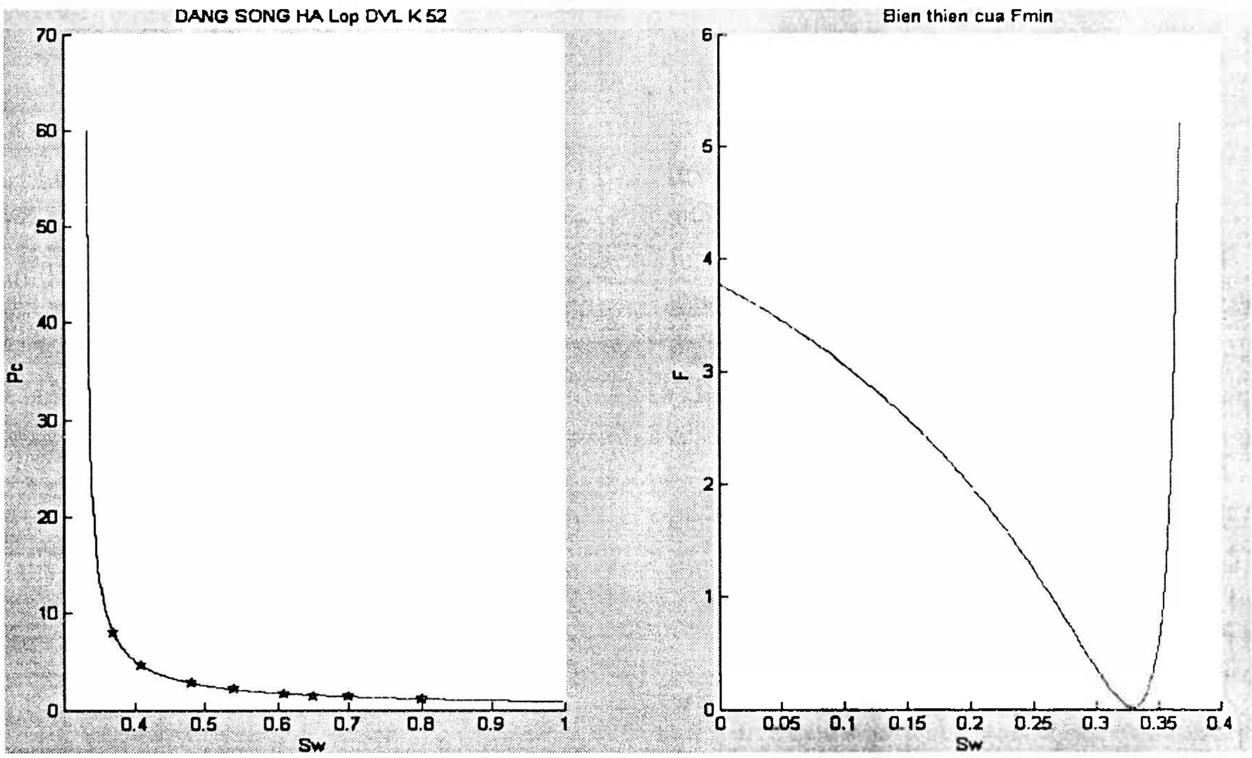
If $(S_w)_k < (S_w)_j$. then $(P_c)_k > (P_c)_j$.

This condition is satisefied easily.

Notice:

1) The value S_{wi} calculates by this study usually smaller than the measured value a little, becausee S_{wi} calculates while $P_c \to +\infty$ and S_{wi} measured value with P_c only big enough.

2) The accuracy of the calculation result can be evaluated by analising and interpretation the constants $a; b$ and S_{wi} (exemple: satisfy condition: $0 < a < 1$; $0 < b < 2$) and the variable behaviour of the function F . In the MATLAB Programming we plot the grapth F-S_w. Consider the theory and testing on the practical data, we see that: The F curve reflects the accuracy of calculation result. The result is good if the minimum of function is reflected clearly., It is mean that function F decreases quickly to the minimum and increases quickly as the following figure (on the right):



Verification:

1) On the Internet: Consist of 6 `Samples` from the big oil and gas fields in the world. Reader can find them in [1,4,6] on the Internet.

<u>Sample 1:</u>

```
VtPc=[8.00  4.56  2.78  2.15  1.64  1.40  1.30  1.15];
```

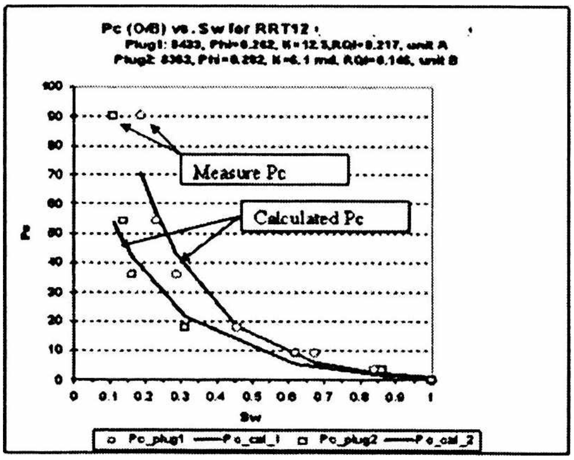
```
VtSw=[0.37  0.41  0.48  0.54  0.61  0.65  0.70  0.80];
Sample 2:
VtPc=[0.867  1.16  1.45  1.73  2.02  2.31  2.89  7.8  21.7];
VtSw=[0.90   0.80  0.70  0.60  0.50  0.45  0.40  0.35  0.30];
Sample 3:
VtPc=[0.15  0.30   0.50   1      2      4      7      10   ];
VtSw=[1     0.982  0.883  0.771  0.698  0.664  0.648  0.639];
Sample 4:
VtPc=[0.15  0.30   0.50   1      2      4      7      10   ];
VtSw=[1     0.917  0.815  0.699  0.616  0.581  0.566  0.560];
Sample 5:
VtPc=[0.15   0.30   0.50   1      2     4      7      10   ];
VtSw=[0.984  0.942  0.868  0.786  0.72  0.687  0.673  0.665];
Sample 6:
VtPc=[0.15   0.30   0.50   1      2      4      7      10   ];
VtSw=[0.983  0.929  0.823  0.708  0.648  0.617  0.603  0.596];
```

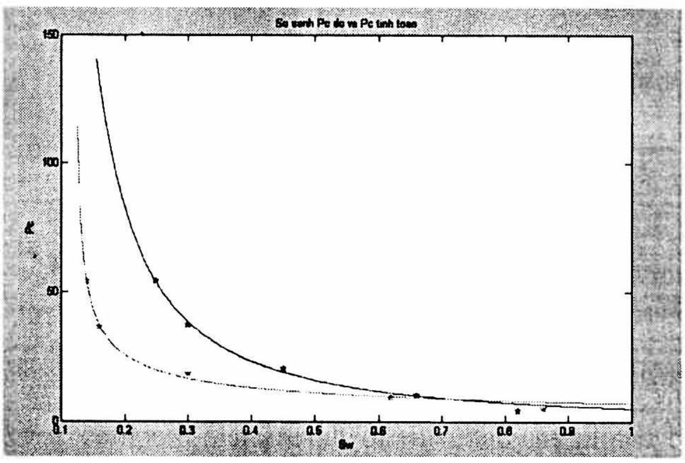
Calculation results from the samples:

Sample	Results in the Internet S_{wi}	Results of this study		
		S_{wi}	a	B
1	0.330	0.3300	0.5925	0.8085
2	0.300	0.2850	0.6012	0.8476
3	0.607	0.6050	0.0555	1.5359
4	0.539	0.5375	0.0803	1.2703
5	0.633	0.6325	0.0401	1.6138
6	0.575	0.5725	0.0631	1.3509

Comparision:

According to the data from "Calculations of Fluid Saturations from Log-Derived J-Functions in Giant Complex Middle-East Carbonate Reservoir"[1] We calcalate and compare: The result of this stady is ploted by MATLAB on the right , the result of the auther [4]on the left in the following figure:





4. Results and Discussions

1) The fact that objective testing data in range of the reservoirs in the world which is found on website and data from PVEP, the appropriateness of parameter a, b and variation of F curve is good has shown that: This research method is used for determining S_{wi} and P_c curve establishment. Good variation of F curve proves that mathematical basic about minimum condition of fuction is it's derivation equals zeros is entrusted theorical foundation

2) The most important result of this study is the calculation of the irreducible water saturation S_{wi} from $(S_w;\ P_c)_j$.data. The influence of S_{wi} on P_c is mentioned detailly in [3] and other documents, but none of them has mentioned about the calculation of S_{wi} from P_c. The calculation of S_{wi} from $(S_w;\ P_c)_j$ data is reasonable. Value S_{wi} determines $(S_w;\ P_c)$ then from $(S_w;\ P_c)$, value S_{wi} can be found out by solving reverse mathematical problem which is used frequently in geology

3) As well as other problem in Petrol and geology, the calculation for S_{wi} and establishment for curve P_c in this research should not be rewiewd separately but analytical comparation S_{wi} of this study with other paramerters as Permeability K, porosity φ of the reseavoir. Thus, this study is an approaching way together with other ones make a solution for sifnificant as well as difficult problem in petrol exploration and geology investigation

5. Conclusions

1. The empirical method only can plot the empirical capillary pressure curvers but can not calculate irreducible water saturation S_{wi} and two parameters a,b

2. The method of this study can calculate S_{wi} , two parameters a,b and plot the graph of (3). Not only S_{wi} but olso two parameters : $a; b$ have their petrophysical meaning. In the reservoirs we usually have more data sets $(S_w;\ P_c)_j$, so we have more data sets $\{S_{wi}\ ; a\ ; b\ \}$ respectively. Analysis, comparision data sets $\{S_{wi}\ ; a\ ; b\ \}$ may yield useful informations regarding the reservoior

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References

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[3] Michael Holmes, *Capillary pressure & Relative Permability Petrophysical Reservoir Models, Denves*, Colorado USA May 2002

[4] Tawfic A.Obedia, Yousef S.Al-Mehin, Karri Suryanarayana: Calculations of Fluid Saturations from Log - Derived J-Functions in Giant Complex Middle-East Carbonate Reservoir.

[5] Noaman El-Khatib: *Development of a Modified Capillary Pressure J-Function*, KingSaud University

[6] Crain's Petrophysical Handbook–Capillary pressure

Appendix

1. Programm calculation for S_{wi} and establishment for curve P_c

```
VtSw=[0.37 0.41 0.48 0.54 0.61 0.65 0.70 0.80]; %thay 2 dong nay khi can
VtPc=[8.00 4.56 2.78 2.15 1.64 1.40 1.30 1.15]; % thay 2 dong nay khi can
x=min(VtSw); l=round((x/0.0025));
p=length(VtSw);
Kqua =zeros(1,4); for n=1:l
      Swi=0.0025*(n-1);
      Kqua(n,1)=Swi;
      Hso=zeros(2,2);
      Nghiem=zeros(2,1);
      Tudo=zeros(2,1);
      for j=1:p
          Sw=VtSw(j);
          Pc=VtPc(j);
          Hso(1,2)=Hso(1,2)+log10(Sw-Swi);
          Hso(2,2)=Hso(2,2)+(log10(Sw-Swi))^2;
          Tudo(1,1)=Tudo(1,1)+log10(Pc);
          Tudo(2,1)=Tudo(2,1)+log10(Sw-Swi)*log10(Pc);
      end
      Hso(1,1)=p;
      Hso(2,1)=Hso(1,2);
      Nghiem=inv(Hso)*Tudo;
      a1=Nghiem(1,1); a=10^a1;     Kqua(n,2)=a;
      b1=Nghiem(2,1);  b=-b1;      Kqua(n,3)=b;

         Fmin=0;
         for j=1:p
            Sw=VtSw(j);
            Pc=VtPc(j);
            Fmin= Fmin+[Pc-(a/((Sw-Swi)^b))]^2;
         end
        Kqua(n,4)= Fmin;
      end

Fmin=Kqua(1,4);
for n=2:l
      x=Kqua(n,4);
      if (x<Fmin)
         Fmin=x;
      end
end
for n=1:l
     x=Kqua(n,4);
     if (x==Fmin)
         Swi=Kqua(n,1);
         a=Kqua(n,2);
         b=Kqua(n,3);
     end
end
```

```
% Hien thi
disp('Gia tri Swi ='); disp(Swi);
disp('             Bang Ket qua ');
disp('      Swi         a         b         Fmin'); disp(Kqua);
disp('Gia tri  a =');disp(a);
disp('Gia tri  b =');disp(b);
cach= (a/200)^(1/b);
dau= Swi+cach;
u=[dau:0.0025:1]; v=u;
m=length(u);
for j=1:m
       Sw= u(j);
       v(j)=a/((Sw-Swi)^b);
end
figure
subplot(1,2,1);
title('DANG SONG HA Dai hoc Mo Dia chat'); xlabel('Sw'); ylabel(' Pc ');
hold on :
plot(u,v,'r');
hold on :
plot(VtSw,VtPc,'pb');
hold on :

u=Kqua(:,1);v=Kqua(:,4);

subplot(1,2,2);
plot(u,v,'b');
title('Bien thien cua Fmin');
xlabel('Saturation');ylabel('F');
```

2. Some data set

```
VtSw=[  0.90    0.80    0.70    0.60    0.50    0.45    0.40    0.35   0.30];    %
1 VtPc=[  0.867    1.16    1.45    1.73    2.02    2.31    2.89    7.8    21.7];
%0.578

VtSw=[1  0.904    0.782    0.640    0.518    0.478    0.447    0.409];
VtSw=[1    0.982    0.883    0.771    0.698    0.664    0.648    0.639 ];
Swi= 0.6050 a=0.0555 b=1.5359
VtSw=[1    0.917    0.815    0.699    0.616    0.581    0.566    0.560];
Swi=0.5375  a=0.0803 b=1.2703
VtSw=[0.983    0.929    0.823    0.708    0.648    0.617    0.603    0.596];%
Swi=0.5725 a=0.0631 b=1.35.9
VtSw=[1    0.967    0.808    0.706    0.568    0.517    0.483    0.438];
%VtSw=[0.952  0.930    0.865    0.740    0.633    0.594    0.555    0.499];
%VtPc=[ 2      4          8          15         35         70         120        200];

MtSw =[      1    0.982    0.883    0.771    0.698    0.664    0.648    0.639;
             1    0.917    0.815    0.699    0.616    0.581    0.566    0.560;
         0.984    0.942    0.868    0.786    0.72     0.687    0.673    0.665;
         0.983    0.929    0.823    0.708    0.648    0.617    0.603    0.596;];
VtPc =[  0.15     0.30     0.50        1        2        4        7      10];
```