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Assessment of the influence of interpolation techniques on the accuracy of digital elevation model

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Abstract. Digital Elevation Model (DEM) is an important component of GIS applications in many socio-economic areas. Especially, DEM has a very important role in monitoring and managing natural resources, preventing natural hazards, and supporting spatial decision making.

Usually, DEM is built by interpolation from a limited set of sample points. Thus, the accuracy of the DEM is depended on the used interpolation method. By analyzing the data of experimental DEM creation using three popular interpolation techniques (inverse distance weighted - IDW, spline, and kriging) in four different survey projects (Thai Nguyen, Go Cong Tay, Co Loa, and Duong Lam), the paper has made an assessment of influence of interpolation technique on the DEM accuracy. Based on that, some recommendations on choosing interpolation technique has been made: for mountainous areas the spline regularized is the most suitable, for hilly and flat areas, the IDW or kriging ordinary with exponential model of variogram are recommended.

Keywords: Digital elevation model (DEM); DEM accuracy; Interpolation technique.

1. Introduction

Digital elevation model (DEM) is an important part of the spatial data infrastructure (SDI). DEMs are widely used in natural resource management, natural hazard prevention, land-related decision making, etc.

Usually, the DEMs are produced by interpolating the elevations of a set of sample points for predicting the elevations at all positions inside the DEM area [4]. Consequently, interpolation technique will contribute to the error budget of DEM.

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Several researches were conducted on the relation between DEM accuracy and interpolation technique. Fencík and Vajsáblová [3] investigated the DEM accuracy of Morda-Harmonia territory (Hungary) created by using kriging interpolation with various variogram models. The author concluded that the linear model of variogram is the most suitable for the study area.

Research of El Hassan [2] on the accuracy comparison of some spline interpolation algorithms for the test areas in Cairo (Egypt) and Riyadh (Saudi Arabia) shown that the pseudo-quintic spline algorithm gives the best accuracy of DEM.

Chaplot et al. [1] used some interpolation techniques (kriging, inverse distance weighted, multiquadratic radial basis function, and spline) for creating DEM in various regions of Laos and France. The author has concluded that for a high density of sample points, all of the interpolation techniques perform similarly; and for a low density of sample points, kriging and inverse distance weighted interpolation techniques are better than the others. However, the research carried out by Peralvo [8] in the two watersheds of Eastern Andean Cordillera of Ecuador shows other results: the inverse distance weighted interpolation produced the most inaccurate DEM.

Our review of conducted researches shows that they usually were carried out in small areas (less than 100 ha). Due to the differences in types of topography, surveying methods, and levels of technology application in various countries, the results of these research sometimes are contrary each to others.

This research investigates the influence of interpolation techniques on the accuracy of DEM in the examples of four projects in Vietnam. The projects have various areas, and are belonging to typical types of topography of Vietnam. The research is limited to two surveying methods: digital photogrammetry, and total station / GPS. The LIDAR and contour digitizing methods are out of scope.

2. Research method

2.1. The tested interpolation techniques

This research uses three popular interpolation methods for experimental creation of DEMs: inverse distance weighted, spline, and kriging.

- *The inverse distance weighted (IDW) interpolation* determines the elevation of a specific point using a linearly weighted combination of the elevations of nearby located sample (known) points [5]. The weight w_i of a sample point i is a function of inverse distance as follows:

$$w_i = 1/d_i^p, \quad (1)$$

where d_i is the distance from point of interest to the sample point i; and the power p controls the significance of sample points to the interpolated values, based on their distance to the output point. The higher the power, the more emphasis can be put on the nearest points. Thus, nearby data will have the most influence, and the surface will have more detail (less smooth).

- *The spline interpolation* estimates the elevation of a specific point using a mathematical function that minimizes the overall surface curvature, resulting in a smooth surface that passes exactly through the input points [5]. Conceptually, the sample points are extruded to the height of their magnitude; spline bends a sheet of rubber that passes through the input points while minimizing the total curvature of the surface. It fits a mathematical function to a specified number of nearest input points while passing through the sample points. There are two spline methods: regularized and tension. The regularized method creates a smooth, gradually changing surface with values that may lie outside the sample data range. The tension method controls the stiffness of the surface according to the character of the modeled phenomenon. It creates a less smooth surface with values more closely constrained by the sample data range. The main parameters of the spline interpolation are the number of sampled points used for interpolation, and the weight. For the regularized spline, the higher the weight, the smoother the output surface. For the tension spline, the higher the weight, the coarser the output surface. More detailed information about the spline interpolation can be found in [6].

- *The kriging interpolation* assumes that the distance or direction between sample points reflects a spatial correlation that can be used to explain the variation in the surface [5]. Kriging fits a mathematical function to a specified number of points, or all points within a specified radius, to determine the output value for each location. It is a multistep process including: exploratory statistical analysis of the data, variogram modeling, creating the surface. Kriging is most appropriate when there is a spatially correlated distance or directional bias in the data. Kriging is similar to IDW in that it weights the surrounding measured values to derive a prediction for an unmeasured location. However, in kriging, the weights are based not only on the distance between the measured points and the prediction location but also on the overall spatial arrangement of the measured points. To use the spatial arrangement in the weights, the spatial autocorrelation must be quantified through empirical semivariograms. The semivariogram can have one of the following models: circular, spherical, exponential, gaussian, and linear. There are two kriging methods: ordinary and universal. The ordinary kriging assumes that the constant mean is unknown, while the universal kriging assumes that there is an overriding trend in the data and this trend is modeled by a polynomial. Detailed information about the kriging interpolation can be found in [7].

Among the three tested interpolation techniques, IDW is the fastest and kriging is the slowest technique. Spline gives the smoothest DEM surface.

2.2. The workflow

The assessment of influence of interpolation technique on the accuracy of DEM is carried out according to the workflow presented in Fig. 1. The computation is done by using ArcGIS software developed by ESRI [5].

The input data consists of two point sets: the set of source (sample) points, and the set of control (check) points. The control points are evenly distributed and accurately measured. The number of control points is about 0.5-1.0% of the number of source points, but not less than 50.

Both point sets are imported into a geodatabase as point feature classes having an attribute field *Elevation*. The source point set is then interpolated to create a raster DEM with a relatively high resolution. The high resolution is defined in order to eliminate the influence of the output resolution on the accuracy of DEM. The three described above interpolation techniques are applied with varying parameters.

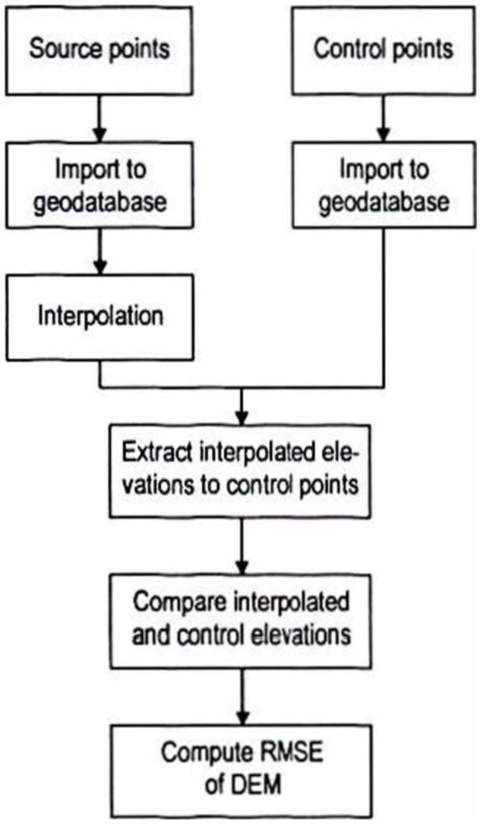

Fig. 1. The workflow for assessing the influence of interpolation technique on the accuracy of DEM by using ArcGIS software.

In the next step, the elevations of interpolated DEM are extracted to the control points by using the ArcGIS's tool *Extract Values to Points*. Thus, the output points will have two attributes: the original *Elevation*, and the extracted from DEM *Int_Elevation*. These attributes are compared each with other to derive the elevation difference Δ_i for each point *i*:

$$\Delta_i = Int\_Elevation - Elevation \qquad (2)$$

The calculated differences are stored in a newly created attribute field *Elev_Diff*.

In the final step, the RMSE (root mean square error) of the interpolated DEM is calculated by using the following formula:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\Delta_i^2}, \quad (3)$$

where N is the number of control points.

For automated execution of the workflow, we have developed a model in the *Model Builder* extension of ArcGIS software. For each project, the user only has to change the interpolation method and define its parameters in order to re-run the entire process. The model for IDW interpolation is presented in Fig. 2.

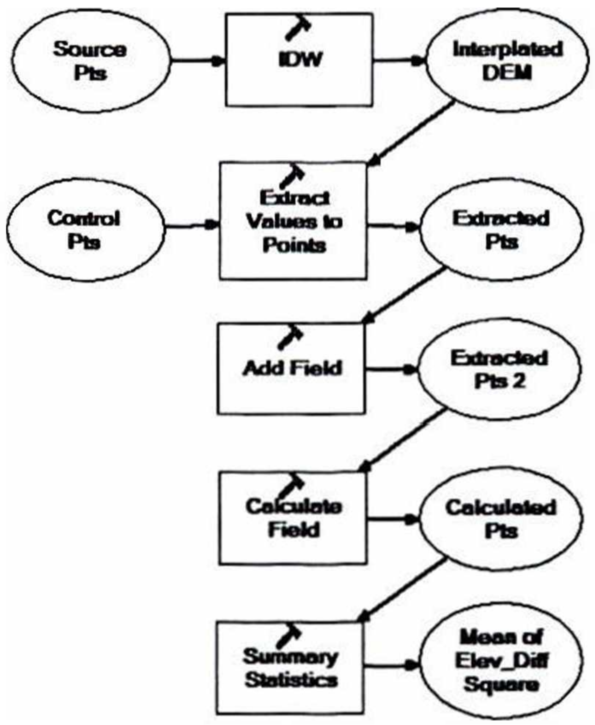

Fig. 2. Automated workflow execution by using ArcGIS's *Model Builder*.

In the model in Fig. 2, the tools (denoted by rectangles) are used as follows:

- IDW: interpolate source points into raster DEM (it can be substituted by spline or kriging for other interpolation techniques).

- Extract Values to Points: extract interpolated elevations from the created DEM into the control point feature class, and create a new feature class (Extracted Pts).

- Add Field: add the *Elev_Diff* field to the feature class Extracted Pts.

- Calculate Field: calculates the elevation difference Δ_i by using Eq. 2 and takes its square value.

- Summary Statistics: calculates RMSE of the interpolated DEM by using Eq. 3.

2.3. The study areas

This research is based on the survey data of four topographic mapping projects: Thai Nguyen, Go Cong Tay, Co Loa, and Duong Lam. The projects are located in areas belonging to different topography types. Table 1 lists the short description of these projects. Since the Thai Nguyen project is relatively large and covers three types of topography, it was divided into three subprojects: Plain Thai Nguyen, Hilly Thai Nguyen, and Mountainous Thai Nguyen.

3. Results and discussion

The results of testing the influence of interpolation technique on the accuracy of DEM is presented in figures 3÷6 as combined graphs. The horizontal axes represent interpolation techniques with varying parameters, and the vertical axes represent the root mean square errors (RMSE) of DEMs in the unit of meter. Fig. 3 uses the following notation:

- Plain, Hill, Mountain: the subprojects of Thai Nguyen project that are located in plain, hilly and mountainous areas respectively.

- S, C, E, G, L: spherical, circular, exponential, gaussian, and linear models of experimental variogram for the ordinary kriging interpolation method.

- LD, QD: linear with linear drift and linear with quadratic drift for the universal kriging interpolation method.

Table 1. Characteristics of the DEM projects

Project	Location	Type of topography	Survey method	Project's area
Thai Nguyen	South of Thai Nguyen Province. 21°18'÷22°00' N, 105°26'÷106°25' E	Combined plain, hills, and mountains	Digital photogrammetry by using aerial photos at 1:30,000 scale. Source point sampling interval ~25m	14,000 ha
Go Cong Tay	South of Go Cong Tay Dist., Tien Giang Prov., Cuu Long River Delta. 10°12'÷10°18' N, 106°32'÷106°40' E	Plain	Digital photogrammetry by using aerial photos at 1:22,000 scale. Source point sampling interval ~30m	1,295 ha
Co Loa	South-East of Dong Anh Dist., Hanoi. 21°06'÷21°08' N, 105°51'÷105°53' E	Plain	Digital photogrammetry by using aerial photos at 1:7,000 scale. Source point sampling interval ~20m	245 ha
Duong Lam	North-West of Son Tay Town, Hanoi. 21°08'÷21°10' N, 105°27'÷105°29' E	Midland, hills, mounds.	Total station in combination with GPS. Source point sampling interval 2÷30m	211 ha

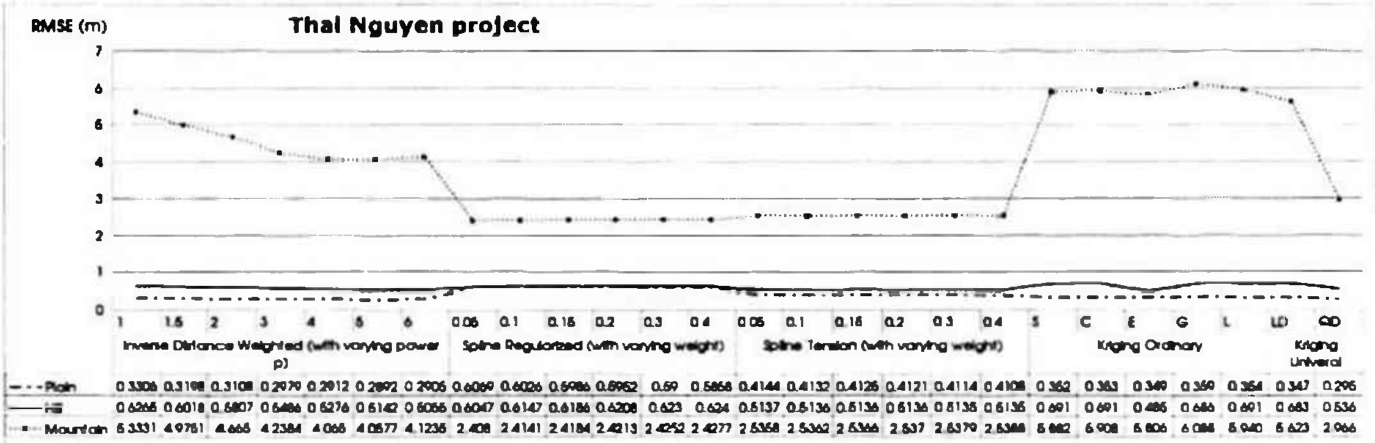

Fig. 3. Results of testing DEM accuracy in the Thai Nguyen project.

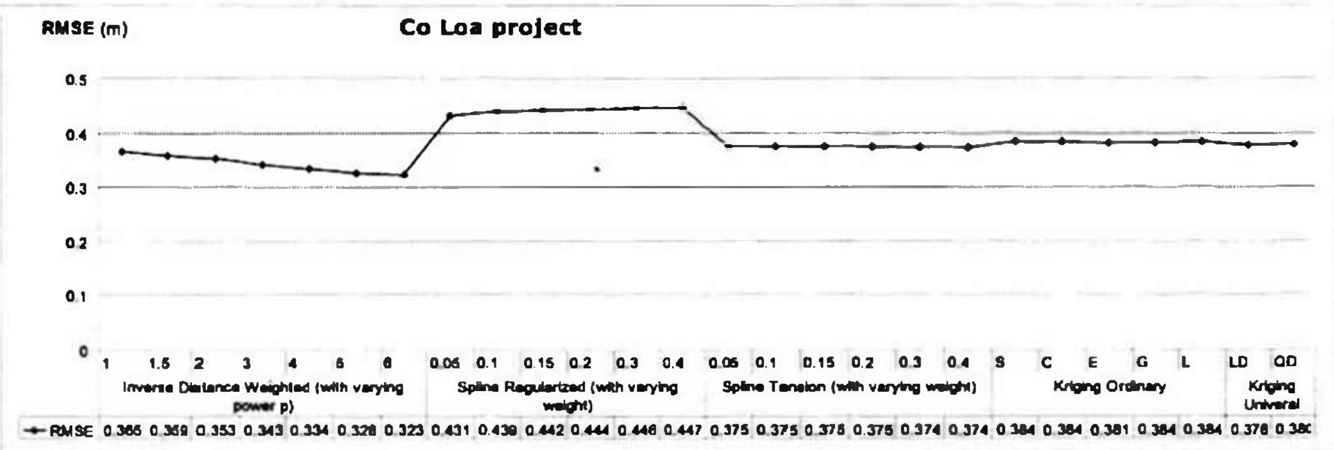

Fig. 4. Results of testing DEM accuracy in the Co Loa project.

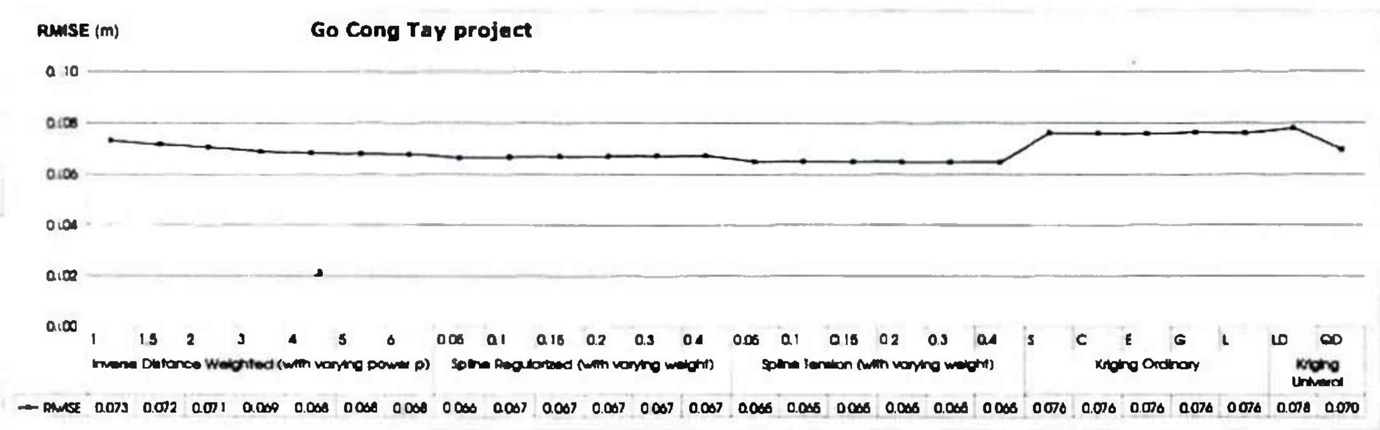

Fig. 5. Results of testing DEM accuracy in the Go Cong Tay project.

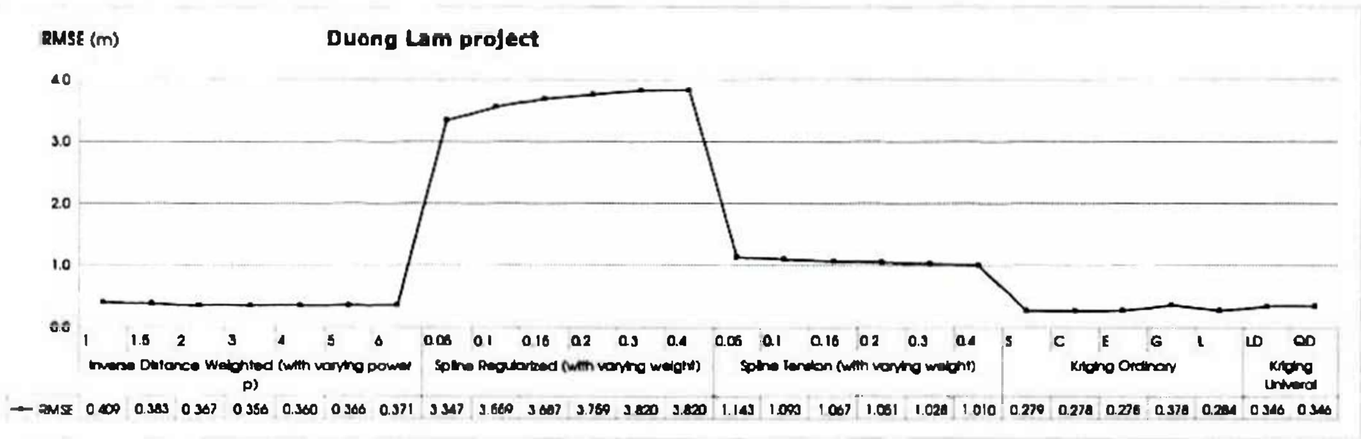

Fig. 6. Results of testing DEM accuracy in the Duong Lam project.

3.1. The Thai Nguyen project

The results of testing DEM accuracy in the Thai Nguyen project is presented in Fig. 3. For this project, some remarks can be made as follows:

- The error of DEM in the mountainous subproject is much higher than those in the plain and hilly subprojects. The reason is that the elevation in mountainous areas strongly varies, while the interpolation techniques can account only for gradual changes over space.

- Among the three tested interpolation techniques, the spline one (regularized or tension) produces a much lower level of error in the mountainous area.

- In the plain and hilly areas, all three interpolation techniques give roughly comparable results. The IDW is slightly better than others in the plain area, while the kriging with exponential model of semivariogram gives the smallest RMSE (0.485m) in the hilly area.

- For the IDW interpolation, when the power *p* increases, the error of DEM decreases, but only by a small amount. Thus, for improving the computational speed, one can choose a relatively small value of *p*.

- For the spline interpolation, the tension method has some advantages over the regularized one in the plain and hilly areas. Conversely, the regularized method is better in the mountainous area.

- For the kriging interpolation, the ordinary method using exponential model and the universal method using linear model with quadratic drift (QD) gives slightly smaller RMSEs than other methods.

3.2. The Co Loa project

The results of testing DEM accuracy in the Co Loa project are presented in Fig. 4. It can be readily seen that the graph for Co Loa is very similar to the one for the plain area of Thai Nguyen project. The IDW with a high value of power *p* produces the best results, while the spline regularized produces the worst. However, due to the relatively flat characters of topography in Co Loa, the interpolation techniques do not have a strong effect on the accuracy of DEM: the errors are within the range from 0.32m to 0.38m except for the cases of using the spline regularized method.

3.3. The Go Cong Tay project

Fig. 5 shows the DEM accuracy obtained in the Go Cong Tay project. Since the project area is very flat with elevation varied only from 0 to 4 m, the interpolation does not have much influence, and the accuracy of DEM is very high. All three interpolation techniques give almost the same results, only the kriging one shows a slightly higher level of error. Thus, for a very flat area like the Go Cong Tay project, the DEM accuracy isn't the main criterion for choosing interpolation technique. The criterion can be the computational speed (choosing IDW) or the smoothness of the DEM (choosing spline).

3.4. The Duong Lam project

The results of testing DEM accuracy in the Duong Lam project are shown in Fig. 6. Since the survey method used in this project (total station and GPS) differs from the one used in other projects (digital photogrammetry), the graph in Fig. 6 has a shape that is dissimilar to those in figures 3÷5. The spline regularized interpolation gives an extreme (abnormal) RMSE of DEM, reaching 3.8 m, what is 13.7 times more than the error given by kriging ordinary interpolation (0.278 m). The spline tension interpolation is much better than the spline regularized one, but still has an error significantly large than other techniques. The phenomenon can be explained as follows:

- In total station / GPS surveying, the number of surveyed (sampled) points is very limited. However, these points are very well distributed, usually along breaklines where the terrain surface sharply changes. The location of each surveyed point is chosen manually by the surveyors based on their interpretation of topography and with some statistical meaning. Meanwhile, the spline interpolation assumes that the surface is smoothly passed through sampled points, and thus it is not suitable for the cases when most of these sample points are allocated along breaklines.
- The abnormal error given by spline regularized method is due to the fact that the elevation peaks in the Duong Lam project were already surveyed in the field by placing sample points on them. The spline regularized tends to interpolate the elevation beyond the surveyed range, i.e. might give a elevation far higher than the surveyed peaks that leads to the abnormal error.
- Since the distribution of sample points in total station (or GPS) surveying has some statistical meaning, kriging interpolation - the most statistically rigid interpolation technique - may have some advantages over others.

As it shows in Fig. 6, among the three tested interpolation techniques, the kriging ordinary with circular or exponential model has the best accuracy (RMSE of 0.278 m). The IDW interpolation is a bit less accurate with RMSE of 0.356 m. However, the IDW is much faster than the kriging, and thus the choice of optimal interpolation technique for the projects similar to Duong Lam is not obvious, especially if they cover a large area.

3.5 Recommendations on choosing interpolation technique

From the above discussions, we have made some recommendations on choosing appropriate interpolation techniques based on the type of topography and surveying method (Table 2).

Table 2. Recommendations on choosing interpolation technique

Type of topography	Survey method	Interpolation technique		
		Recommended	Can be considered	Not recommended
Mountainous	Digital photogrammetry	Spline regularized with any weight	Spline tension	Kriging
Hilly	Digital photogrammetry	IDW with power $p > 3$	Spline tension	
Plain (Flat)	Digital photogrammetry	IDW with power $p=3\div5$	Spline or kriging	
Hilly or flat	Total station / GPS	Kriging ordinary with exponential model for small areas, IDW with $p=2\div3$ for large areas		Spline, especially spline regularized

If there are several topography types available in the project area then the project can be divided into subprojects with relatively homogeneous type of topography. This can be done automatically by analyzing the variation of elevation by using statistical indicators, such as variance or standard deviation.

4. Conclusions

Interpolation technique plays an important role in achieving a high accuracy of DEM. The influence of interpolation technique on the DEM accuracy depends on the type of topography, and the distribution of sample points, what is directly related to the surveying method. This research has examined three interpolation techniques (IDW, spline, and kriging) in four different survey projects. Based on the analysis of obtained results, some recommendations on choosing the optimal interpolation technique has been made: for mountainous areas, the spline regularized is the most suitable; and for hilly and flat areas, the IDW or kriging ordinary with exponential model of variogram are recommended.

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