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Original Article

Influence of Confined Phonons on the Hall Coefficient in a Cylindrical Quantum Wire with an Infinite Potential (for Electron–acoustic Optical Phonon Scattering)

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Abstract: This paper studies the influence of confined acoustic phonons on the Hall coefficient (HC) in a cylindrical quantum wire (CQW) with an infinite potential (for electron – confined acoustic phonons scattering). The paper considers the case where CQW is placed in a perpendicular magnetic field $\vec{B}$, a constant electric field $\overrightarrow{E_1}$ and an intense electromagnetic wave $\vec{E} = \overrightarrow{E_0} \sin \Omega t$. By using the quantum kinetic equation for electrons interacting with confined optical phonons (COP), analytical expressions for HC are obtained. The application of numerical calculations to GaAs/GaAsAl cylindrical quantum wire shows that the HC depends on magnetic field B, temperature T, frequency Ω and amplitude E_0 of laser radiation and on quantum indices m_1 and m_2 characterizing the phonon confinement. This influence is due to the quantum indices m1 and m_2 which increase the Hall coefficient by 2.3 times in comparison with the case of unconfined phonons. When the quantum numbers m_1 and m_2 go to zero, the result is the same as in the case of unconfined phonons.

Keywords: Hall coefficient, quantum kinetic equation, cylindrical quantum wire, confined acoustic phonon.

1. Introduction

In recent years, the study of low – dimensional semiconductor systems has been increasingly interested, include the electrical, the magnetic and the optical properties. In these systems, the motion of carriers is restricted, thus leading to their new properties under the action of external fields for example: the absorption coefficient of an electromagnetic wave, the Hall effect, the Radioelectric

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effect, the Acoustoelectric effect. The Hall effect — the effect of drag of charge carriers caused by the external magnetic field has been studied extensively [1–3]. There have been study of the Hall effect in bulk semiconductor in the presence of electromagnetic waves, in which classical theory of Hall effect in bulk semiconductor when placed in electricity, the magnetic field is perpendicular to the presence of an electromagnetic wave is built on the basis of Boltzman's classical kinetic equation, while quantum theory is based on quantum-kinetic equation [4]. In two-dimensional semiconductor systems, there have been studies on the Hall effect with the electronic – confined phonon scattering [5-9]. In one-dimensional semiconductor system, there have been studies on the Hall effect with the confined electronics – unconfined phonon [10]. But the influence of the confined phonons on the HC in one-dimensional semiconductor system is not studied. In this work, we study new properties of the HC under the effect of COP. Considering an infinite potential quantum wire subjected to a dc electric field $\vec{E}$, a magnetic field $\vec{B}$and a laser radiation $\vec{E} = \overrightarrow{E_0}\, sin\, \Omega t$. The article is organized as follows: in section 2 we present the confinement of electron and optical phonons in a CQW. Thus, by using the quantum kinetic equation method, we obtained analytical expressions for the Hall coefficient. Numerical results and discussions for the GaAs/GaAsAlcylindrycal quantum wire are given in section 3. Finally, section 4 shows remarks and conclusions.

2. The influence of confined phonons on the hall coefficient in a cylindrycal quantum wire with an infinite potential

Consider a cylindrycal quantum wire with an infinite potential V= $\pi R^2 L$ subjected is placed in a perpendicular magnetic field $\vec{B}$, a constant - electric field $\overrightarrow{E_1}$ and an intense electromagnetic wave $\vec{E} = \overrightarrow{E_0}\, sin\, \Omega t$. Under the influence of the material confinement potential, the motion of carriers is restricted in x, y direction and free in the z one. So, the wave function of an electron and its discrete energy now becomes:

$$\Psi_{\mathrm{n,l,\vec{k}}}(\vec{r}, \Phi, z) = \frac{1}{\sqrt{V_0}} e^{im\Phi} e^{i\vec{k}z} \varphi_{n,l}(r) \text{ , where } \varphi_{n,l}(r) = \frac{1}{J_{n+1}(B_{n,l})} J_n(B_{n,l}\frac{r}{R}) \quad (1)$$

$$\varepsilon_{n,l}(\vec{k}_z) = \left(N + \frac{n}{2} + \frac{l}{2} + \frac{1}{2}\right)\hbar\omega_c + \frac{\hbar^2 k^2}{2m} - \frac{1}{2m}\left(\frac{eE1}{\omega_c}\right)^2 \quad (2)$$

where k, m is the wave vector and the effective mass of an electron, R being the radius of the CQW, $n = 1,2,3,...$ and $l = 0, \pm 1, \pm 2, ...$ being the quantum numbers charactering the electron confinement, $\hbar$ is the Planck constant, $\omega_c = \frac{eB}{m}$is the cyclotron frequency.

When phonons are confined in CQW, the wave vector and frequency of them are given by [11,12]:

$$\vec{q} = \left(\vec{q}_{m_1 m_2}, q_z\right),\ \omega_{m_1,m_2,\vec{q}_\perp} = \sqrt{\omega_0^2 - \beta^2(q_{m_1 m_2}^2 + q_z^2)} \quad (3)$$

Where β is the velocity parameter, m_1, m_2 = 1,2,3,…being the quantum numbers charactering phonon confinement. Also, matrix element for confined electron – confined optical phonon interaction in the CQW now becomes [11]

$$D_{n_1,l_1,n_2,l_2,q_z}^{m_1,m_2} = C_{\vec{q}}^{m_1,m_2} * I_{n_1,l_1,n_2,l_2}^{m_1,m_2} \text{where } \left|C_{\vec{q}}^{m_1,m_2}\right|^2 = \frac{e^2\omega_0}{2\varepsilon_0 V}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)\frac{1}{q_z^2 + q_{m_1,m_2}^2} \quad (4)$$

$$I_{n_1,l_1,n_2,l_2}^{m_1,m_2} = \frac{2}{R^2}\int_0^R J_{|n_1 - n_2|}(q, R)\varphi_{n_2,l_2}^*(r)\varphi_{n_1,l_1}(r) r\, dr. \quad (5)$$

Though equations (1-5), it has been seen that the CQW with new material confinement potential gives the different electron wave function and energy spectrum. In addition, the contribution of confined phonon could enhance the probability of electron scattering. As a result, the Hall Coefficient in a CQW under influence of confined optical phonon and laser radiation should be studied carefully to find out the new properties. The effect of confined optical phonons and the laser radiation modify the Hamitonian of the confined electron – confined optical phonons system in the CQW. This leads the quantum kinetic equation for electron distribution. Using Hamiltonian of the confined electrons — confined optical phonons in a CQW, we establish the quantum kinetic equation for electron distribution function. After some manipulation, the expression for the conductivity tensor is obtained:

$$\sigma_{ie} = \frac{\tau}{1+\omega_c^2\tau^2}\left\{\delta_{ik} - \omega_c\tau\varepsilon_{ijk}h_k + \omega_c^2\tau^2 h_i h_j\right\}\left\{a\delta_{el} + b\left(\delta_{je} - \omega_c\tau\varepsilon_{jef}h_f + \omega_c^2\tau^2 h_e h_f\right)\right\} \quad (6)$$

here δ_{ik} is the Kronecker delta; ε_{ijk} being the antisymmetric Levi-Civita tensor; symbols i, j, k, l, p corresponding the components x, y, z of the Cartesian coordinates. From this we obtain the expression for the hall coefficient

$$R_H = -\frac{1}{B}\frac{\sigma_{yx}}{\sigma_{xx}^2 + \sigma_{yx}^2} \quad (7)$$

With $\sigma_{xx} = \frac{\tau}{1+\omega_c^2\tau^2}\{a + b[1 - \omega_c^2\tau^2]\}$; $\sigma_{yx} = \frac{-\tau}{1+\omega_c^2\tau^2}(a+b)\omega_c\tau$ (8)

$$a = \frac{L_z}{2\pi}\frac{e\beta\hbar}{m^2}\frac{\tau_0}{1+\omega_c^2\tau_0^2}\exp\left\{\beta\left[\varepsilon_F - \hbar\omega_c\left(N + \frac{n}{2} + \frac{l}{2} + \frac{1}{2}\right) + \frac{e^2E_1^2}{2m\omega_c^2}\right]\right\}\left(\frac{2m}{\beta\hbar^2}\right)^{3/2}\frac{\sqrt{\pi}}{2} \quad (9)$$

$$b = \frac{e}{m}\frac{\tau}{1+\omega_c^2\tau^2}b_0; \qquad b_0 = \frac{2\pi ie}{m}\sum_{\gamma_1,\gamma_2,m_1,m_2}\left(I_{\gamma_1,\gamma_2,}^{m_1,m_2}\right)^2\left(A_1 - A_2 + A_3 + A_4 - A_5 + A_6 + A_7 - A_8\right) \quad (10)$$

$$A_1 = \frac{L_z}{2\pi}\frac{me^2\omega_0}{2\varepsilon_0 V}\left(\frac{1}{X_\infty} - \frac{1}{X_0}\right)e^{\beta\left[\varepsilon_F - \omega_c\left(N_1 + \frac{n}{2} + \frac{l}{2} + \frac{1}{2}\right) + \frac{1}{2}\left(\frac{eE_1}{\omega_c}\right)^2\right]}\frac{1}{2\pi}\frac{k_BT}{\omega}$$

$$\left[2q_{m_1,m_2}^2 m^2 A_{11}^2\exp\left(-\frac{\beta A_{11}}{2}\right)K_{-2}\left(\frac{\beta A_{11}}{2}\right) + 8A_{11}^4 m^4 q_{m_1,m_2}^2\exp\left(-\frac{\beta A_{11}}{2}\right)K_{-3}\left(\frac{\beta A_{11}}{2}\right)\right. \quad (11)$$

$$\left. - mA_{11}\exp\left(-\frac{\beta A_{11}}{2}\right)K_{-1}\left(\frac{\beta A_{11}}{2}\right) - 4m^3A_{11}^3\exp\left(-\frac{\beta A_{11}}{2}\right)K_{-2}\left(\frac{\beta A_{11}}{2}\right)\right]$$

$$A_2 = \frac{L_z\beta}{4\pi^2}\frac{e^2\omega_0}{2\varepsilon_0 V}\frac{e^2E_0^2}{2\Omega^2}\left(\frac{1}{X_\infty} - \frac{1}{X_0}\right)e^{\beta\left[\varepsilon_F - \omega_c\left(N + \frac{n}{2} + \frac{l}{2} + \frac{1}{2}\right) + \frac{1}{2}\left(\frac{eE_1}{\omega_c}\right)^2\right]}\left[\frac{q_{m_1,m_2}^2}{2m}K_{-2}\left(\frac{\beta A_{11}}{2}\right)\right.$$

$$\left. + \frac{q_{m_1,m_2}^2}{2m}K_0\left(\frac{\beta A_{11}}{2}\right) - A_{11}\left(\frac{\beta A_{11}}{2}\right) - A_{11}\left(\frac{\beta A_{11}}{2}\right)\right] \quad (12)$$

$$A_3 = \frac{L_z}{4\pi}\frac{e^4E_0^2k_BT}{8\varepsilon_0 V}\left(\frac{1}{X_\infty} - \frac{1}{X_0}\right)\exp\left\{\beta\left[\varepsilon_F - \omega_{\gamma_1} - \frac{\hbar^2 A_{11}}{4} + \frac{\hbar^2\Omega}{2}\right]\right\}\left[\Omega K_0\left(\frac{\beta\hbar^2|A_{11} - \Omega|}{2}\right)\right.$$

$$+ \left(\frac{1}{|2mA_{11} - 2m\Omega|}\right)^{3/2}K_{-3/2}\left(\frac{\beta\hbar^2|A_{11} - \Omega|}{2}\right)\Omega q_{m_1,m_2}^2 + \frac{q_{m_1,m_2}^2}{2m}K_0\left(\frac{\beta\hbar^2|A_{11} - \Omega|}{2}\right) \quad (13)$$

$$-\frac{1}{2}|2mA_{11} - 2m\Omega|K_1\left(\frac{\beta\hbar^2|A_{11} - \Omega|}{2}\right) - A_{11}K_0\left(\frac{\beta\hbar^2|A_{11} - \Omega|}{2}\right) - \left(\frac{1}{|2mA_{11} - 2m\Omega|}\right)^{3/2}$$

$$\left.\left(\Omega q_{m_1,m_2}^2 + A_{11}q_{m_1,m_2}^2\right)K_{-3/2}\left(\frac{\beta\hbar^2|A_{11} - \Omega|}{2}\right) + \frac{1}{2}|2mA_{11} - 2m\Omega|K_1\left(\frac{\beta\hbar^2|A_{11} - \Omega|}{2}\right)\right]$$

$$A_4 = \frac{m^2 e^2 k_B T}{2\varepsilon_0 V}\beta \frac{e^2 E_0^2}{4m^2\Omega^3}\frac{L_z}{4\pi^2}\left(\frac{1}{X_\infty}-\frac{1}{X_0}\right)\exp\left[\beta\left(\varepsilon_F-\omega_{\gamma_1}-\frac{\hbar^2 A_{11}}{4}-\frac{\hbar^2\Omega}{2}\right)\right]\left[\left(q_{m_1,m_2}^2-A_{11}-\Omega\right)K_0\right. \quad (14)$$

$$\left.\left(\frac{\beta\hbar^2\left|A_{11}-\Omega\right|}{2}\right)-2m\left|A_{11}-\Omega\right|K_1\left(\frac{\beta\hbar^2\left|A_{11}-\Omega\right|}{2}\right)+\left(\Omega q_{m_1,m_2}^2+A_{11}q_{m_1,m_2}^2\right)K_{-1}\left(\frac{\beta\hbar^2\left|A_{11}-\Omega\right|}{2}\right)\right]$$

$$A_5 = \frac{m^2 e^2 k_B T}{2\varepsilon_0 V}\beta \frac{e^2 E_0^2}{4m^2\Omega^3}\frac{L_z}{4\pi^2}\left(\frac{1}{X_\infty}-\frac{1}{X_0}\right)\exp\left[\beta\left(\varepsilon_F-\omega_{\gamma_1}-\frac{\hbar^2 B_1}{2}\right)\right]\left[\frac{\hbar^2}{2m}K_0\left(\frac{\beta\hbar^2}{2}\right)\right.$$

$$\left.+\left(\frac{1}{2}-\frac{\hbar^2 q_{m_1,m_2}^2}{4mB_1}\right)K_1\left(\frac{\beta\hbar^2}{2}\right)-\frac{q_{m_1,m_2}^2}{4B_1}K_{-2}\left(\frac{\beta\hbar^2}{2}\right)\right] \qquad \text{K is Bessel fouction type 2}$$

(15)

$$A_6 = \frac{m^2 e^2 k_B T}{2\varepsilon_0 V}\beta \frac{e^2 E_0^2}{4m^2\Omega^4}\frac{L_z}{4\pi^2}\left(\frac{1}{X_\infty}-\frac{1}{X_0}\right)\exp\left[\beta\left(\varepsilon_F-\omega_{\gamma_1}-\frac{\hbar^2 B_1}{2}\right)\right] \quad (16)$$

$$\left[\frac{\hbar^2 B_1}{2m}K_1\left(\frac{\beta B_1}{2}\right)\frac{\hbar^2 q_{m_1,m_2}^2}{2m}K_0\left(\frac{\beta B_1}{2}\right)-B_1 q_{m_1,m_2}^2\left(\frac{1}{2B_1}\right)^{3/2}K_{-3/2}\left(\frac{\beta B_1}{2}\right)\right]$$

$$A_7 = \frac{e^2 k_B T}{2\varepsilon_0 V}\beta \frac{e^2 E_0^2}{4\Omega^4}\frac{L_z}{4\pi^2}\left(\frac{1}{X_\infty}-\frac{1}{X_0}\right)\exp\left\{\beta\left[\varepsilon_F-\omega_{\gamma_1}+\frac{\beta}{2}\left(B_1+\Omega\right)\right]\right\}\left[\frac{\hbar^2}{2m}m\left|B_1-\Omega\right|K_1\right. \quad (17)$$

$$\left(\beta\left|B_1-\Omega\right|\right)+\left(\Omega-B_1-\frac{\hbar^2 q_{m_1,m_2}^2}{2m}\right)K_0\left(\beta\left|B_1-\Omega\right|\right)+q_{m_1,m_2}^2\left(\Omega-B_1\right)*\frac{1}{m\left|B_1-\Omega\right|}K_{-1}\left(\beta\left|B_1-\Omega\right|\right)$$

$$A_8 = \frac{e^4 k_B T}{2\varepsilon_0 V}\beta \frac{E_0^2}{4\Omega^3}\frac{L_z}{4\pi^2}\left(\frac{1}{X_\infty}-\frac{1}{X_0}\right)\exp\left\{\beta\left[\varepsilon_F-\omega_{\gamma_1}+\frac{\beta}{2}\left(B_1-\Omega\right)\right]\right\}\left\{-\hbar^2\left(B_1+\Omega\right)\right. \quad (18)$$

$$\left.K_1\left[\beta\left(B_1+\Omega\right)\right]-\left(\Omega+B_1+\frac{\hbar^2 q_{m_1,m_2}^2}{2m}\right)K_0\left[\beta\left(B_1+\Omega\right)\right]-\frac{q_{m_1,m_2}^2}{m}K_{-1}\left[\beta\left|B_1-\Omega\right|\right]\right\}$$

$$A_{11}=\varepsilon_{\gamma_2}-\varepsilon_{\gamma_1}+\omega_{m_1,m_2,q_z}; \qquad B=\frac{\hbar^2 q_z^2}{2m}+\varepsilon_{\gamma_2}-\varepsilon_{\gamma_1}-\omega_{m_1,m_2,q_z}; \qquad B_1=\varepsilon_{\gamma_2}-\varepsilon_{\gamma_1}+\sqrt{\omega_0-\beta q_{m_1,m_2}^2}$$

(19)

The expression (7) is analytics expression of the Hall coefficient in CQW with an infinite potential (for electron – confined optical phonons scattering). From this expression we see, the HC dependent on the magnetic field B,frequency Ω and amplitude E_0 of laser radiation, temperature T of system and specially the quantum numbers m_1, m_2 characterizing the phonon confinement effect. Where m_1, m_2 goes to zero, we obtain results as case of unconfined phonons [10].

3. Numerical results and discussions

In this section, we present the numerical evaluation of the Hall conductivity and the HC for the GaAs/GaAsAl quantum wire. Parameters used in this according to the result in Ref. [11,12]: $m_e = 0.067m_0$, (m_0 is the free mass of an electron),$\chi_\infty = 10.9$, $\chi_0 = 12.9$, $\varepsilon_F = 8.10^{-21}J$, $\tau = 10^{-12}s$,

$\nu = 8.73 \times 10^4 ms^{-1}$, $\hbar\omega_0 = 36.25 meV$, V = 1, $E_0 = 10^5 V/m$, $E_1 = 5.10^5 V/m$ $c = 3.10^8 \frac{m}{s}$, , $k_B = 1.38.10^{-23} J/K$

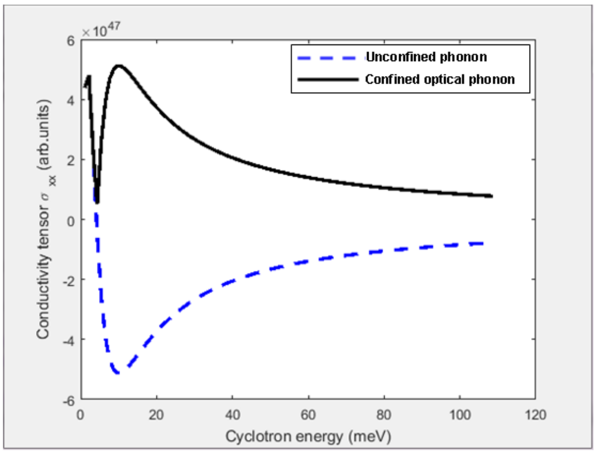

Figure 1.The dependence of the conductivity tensor σ_{xx}on the cyclotron energy for confined phonon (solid curve) and unconfined phonon (dashed curve), here $E_1 = 5.10^5 V/m$ and $L = 30\ nm$

In figure 1, we can see clearly the appearance of oscillations and oscillations are controlled by the ratio of the Fermi energy and energy of cyclotron. First, phonons are confined in 2 dimensions x, y, only motion free in the z one (quantum wires), therefore, The power spectrum of the external phonon depends on the normal effects of free movement, depending on the confined index of phonon m_1, m_2 corresponding to the x and y directions. In case confined phonon get more two resonance peaks comparing with that in case of unconfined phonons. When phonons are confined, specially the confined optical phonons frequency is now modified to $\omega_{m_1,m_2,\vec{q}} = \sqrt{\omega_0^2 - \beta^2(q_z^2 + q_{m_1,m_2}^2)}$. Hence, confined optical phonons make remarkable contribution on the resonance condition.

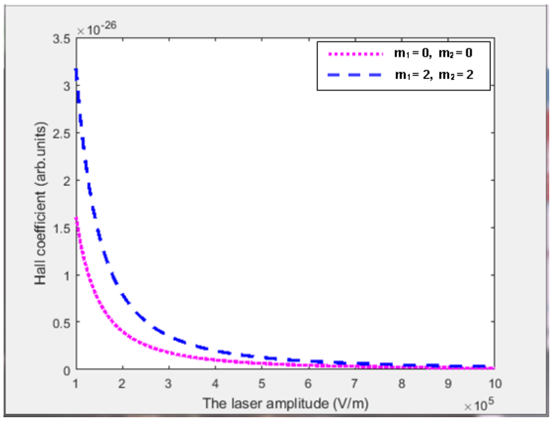

Figure 2. The dependence of the Hall coefficient on the laser amplitude for unconfined phonon $m_1 = 0$, $m_2 = 0$ (dotted curve) and confined phonon $m_1 = 2$, $m_2 = 2$ (dashed curve)

Figure 2 shows the nonlinear dependence of the Hall coefficient on the laser amplitude at different values of number m_1, $m_2$characterizing the phonon confinement. When the laser amplitude has been valid small, which makes an increase of Hall coefficient by 2,3 times in comparison with the case of unconfined phonons. It has been seen that the HC decreases as the increasing of the laser amplitude and the HC reaches saturation when this amplitude is large. When the quantum number m_1 and m_2 goes to zero, the result is the same as in the case of unconfined phonons [10].

4. Conclusions

In this article, the influence of confined optical phonons on the Hall coefficient in a quantum wires with infinite potential (for electron – confined optical phonons scattering) has been theoretically studied base on quantum kinetic equation method. We obtained the analytical expression of the Hall coefficient in the CQW under the influence of COP. Numerical calculations are also applied for GaAs/GaAsAlcylindrycal quantum wire, we see theHCdepends on magnetic field B, temperature T, frequency Ω and amplitude E_0 of laser radiation and especially quantum index m_1 and m_2 characterizing the phonon confinement. This influence is due to the quantum index m_1 and m_2, which makes an increase of Hall coefficient by 2,3 times in comparison with the case of unconfined phonons.

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