Original Article

Nonlinear Vibration of Porous Functionally Graded Cylindrical Panel Using Reddy’s High Order Shear Deformation

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Abstract: The nonlinear dynamic response and vibration of porous functionally graded cylindrical panel (PFGCP) subjected to the thermal load, mechanical load and resting on elastic foundations are determined by an analytical approach as the Reddy’s third order shear deformation theory, Ahry’s function... The results for dynamic response of PFGCP present the effect of geometrical ratio, elastic foundations: Winkler foundation and Paskernak foundation, loads: mechanical load and thermal load, the material properties and distribution type of porous. The results are shown as numerical results, figures and are determined by using Galerkin methods and Fourth-order Runge-Kutta method.

Keywords: Nonlinear dynamic response, porous functionally graded cylindrical panel, the high order shear deformation theory, mechanical load, thermal load, nonlinear vibration.

1. Introduction

With the requirements of working ability such as bearing, high temperature in the harsh environment of some key industries such as defense industries, aircraft, space vehicles, reactor vessels and other engineering structures, in the world, many advanced materials have appeared, including Functionally Graded Materials (FGMs). FGMs is a composites material and is made by a combination...
of two main materials: metal and ceramic. Therefore, the material properties of FGMs will have all the outstanding properties of the two-component materials and vary with the thickness of the structure. With these outstanding properties, FGMs have attracted the attention of scientists in the world. In recent years, a lot of research carried out on FGMs, dynamic FGMs.


In fact, porous materials appear around us and play in many areas of life such as fluid filtration, insulation, vibration dampening, and sound absorption. In addition, porous materials have high rigidity leading to the ability to work well in harsh environments. In recent years, porous materials have been researched and applied along with other materials to create new materials with the preeminent properties of component materials. Porous functionally graded (PFG) is one of the outstanding materials among them and has research works such as: Gao et al. [17] researched about dynamic characteristics of functionally graded porous beams with interval material properties. Dual-functional porous copper films modulated via dynamic hydrogen bubble template for in situ SERS monitoring electrocatalytic reaction are proposed by Yang et al. [18]. Foroutan et al. [19] investigated nonlinear static and dynamic hygrothermal buckling analysis of imperfect functionally graded porous cylindrical shells. Li [20] presented nonlinear vibration and dynamic buckling analyses of sandwich functionally graded porous plate with graphene platelet reinforcement resting on Winkler–Pasternak elastic foundation. Li [21] carried out experimental research on dynamic mechanical properties of metal tailings porous concrete. Transient [22] published response of porous FG nanoplates subjected to various pulse loads based on nonlocal stress-strain gradient theory. Esmaeilzadeh and Kadkhodayan [23] researched about dynamic analysis of stiffened bi-directional functionally graded plates with porosities under a moving load by dynamic relaxation method with kinetic damping. Vibration analysis of magneto-electro-elastic heterogeneous porous material plates resting on elastic foundations are studied by Ebrahimi et al [24]. Arshid and Khorshidvand [25] presented free vibration analysis of

The proposed method used in this study is the third-order shear deformation theory and the effect of the thermal load has been applied in a number of case studies such as Zhang [28] published nonlinear bending analysis of FGM beams based on physical neutral surface and high order shear deformation theory. Reddy and Chin [29] investigated thermo-mechanical analysis of functionally graded cylinders and plates. Modeling and analysis of FGM rectangular plates based on physical neutral surface and high order shear deformation theory is presented by Zhang [30]. In [31], Thon and Bélanger presented EMAT design for minimum remnant thickness gauging using high order shear horizontal modes. Dynamic analysis of composite sandwich plates with damping modelled using high order shear deformation theory are carried out by Meunier and Shenoi [32]. Cong et al. [33] investigated nonlinear dynamic response of eccentrically stiffened FG plate using Reddy’s TSDT in thermal environment. Stability of variable thickness shear deformable plates—first order and high order analyses are gave by Shufrin and M. Eisenberger [34]. In [35], Stojanović et al showed exact closed-form solutions for the natural frequencies and stability of elastically connected multiple beam system using Timoshenko and high order shear deformation theory. A high order shear element for nonlinear vibration analysis of composite layered plates and shells are proposal by Attia and El-Zafrany [36]. Khoa et al. [37] observed nonlinear dynamic response and vibration of functionally graded nanocomposite cylindrical panel reinforced by carbon nanotubes in thermal environment. In [38], Allahyari and Asgari found that thermo-mechanical vibration of double-layer graphene nanosheets in elastic medium considering surface effects; developing a nonlocal third order shear deformation theory. Wang and Shi [39] presented a refined laminated plate theory accounting for the third order shear deformation and interlaminar transverse stress continuity.

From the literature review, the authors often used the high order shear deformation theory to investigate the nonlinear static, nonlinear dynamic or nonlinear vibration of FGM or plate porous functionally graded (PFG). For the nonlinear dynamic and vibration of PFGCP has not carried out. Therefore, in order to observe the nonlinear dynamic and vibration of PFGCP under mechanical load and thermal load, using the Reddy’s high order shear deformation theory and Ahry’ function are proposed in this paper. The natural frequency of PFGCP is obtained by using cylindrical panel fourth-order Runge-Kutta method. Besides, the effect of geometrical ratio, elastic foundations: Winkler foundation and Pasternak foundation, the material properties and distribution type of porous on the modeling will be shown.

2. Theoretical formulation

Fig. 1 show a PFGCP resting on elastic foundations included Winkler foundation and Pasternak foundation in a Cartesian coordinate system \( x, y, z \), with

\( x y \) - the midplane of the panel
\( z \) - the thickness coordinator, \( -h/2 \leq z \leq h/2 \).
\( a \) - the length
\( b \) - the width
\( h \) - the thickness of the panel.
\( R \) - the radius of the cylindrical panel.
The volume fractions of metal and ceramic, $V_m$ and $V_c$, are assumed as [25] with using a simple power-law distribution:

$$V_c(z) = \left(\frac{2z + h}{2h}\right)^N; V_m(z) = 1 - V_c(z),$$

(1)

in which the volume fraction index $N$, subscripts $m$ and $c$ stand for the metal and ceramic constituents, respectively.

The material properties of PFGCP with porosity distribution are given as

**Porosity – I:**

$$[E(z), \alpha(z), \rho(z)] = [E_m, \alpha_m, \rho_m] + [E_{cm}, \alpha_{cm}, \rho_{cm}] \left[\left(\frac{2z + h}{2h}\right)^N - \frac{N}{2}\left[E_m + E_c, \alpha_m + \alpha_c, \rho_m + \rho_c\right]\right].$$

(2)

**Porosity – II:**

$$[E(z), \alpha(z), \rho(z)] = [E_m, \alpha_m, \rho_m] + [E_{cm}, \alpha_{cm}, \rho_{cm}] \left[\left(\frac{2z + h}{2h}\right)^N - \frac{N}{2}\left(1 - \frac{1}{h}z\right)\left[E_m + E_c, \alpha_m + \alpha_c, \rho_m + \rho_c\right]\right].$$

(3)

with

$$E_{cm} = E_c - E_m, \alpha_{cm} = \alpha_c - \alpha_m, \rho_{cm} = \rho_c - \rho_m, K_{cm} = K_c - K_m,$$

(4)

and in this paper, the Poisson ratio $\nu(z)$ can be considered constant $\nu(z) = \nu$.

![Fig. 1. Geometry of the PFGCP on elastic foundation.](image)

![Fig. 2. The porosity distribution type.](image)

**Porosity - I**

**Porosity - II**
2.1. Governing equations

By using the Reddy’s high order shear deformation theory (HSDT), the governing equations and the dynamic analysis of PFGCP are determined.

The train-displacement relations taking into account the Von Karman nonlinear terms as [10,29,33]:

\[
\begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{pmatrix} = \begin{pmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{pmatrix} + z \begin{pmatrix}
k^1_x \\
k^1_y \\
k^1_{xy}
\end{pmatrix},
\begin{pmatrix}
\gamma_{xz} \\
\gamma_{yz}
\end{pmatrix} = \begin{pmatrix}
\gamma_{xz}^0 \\
\gamma_{yz}^0
\end{pmatrix} + z^2 \begin{pmatrix}
k^2_{xz} \\
k^2_{yz}
\end{pmatrix},
\]

(5)

With

\[
\begin{pmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{pmatrix} = \begin{pmatrix}
\frac{\partial u}{\partial x} + \frac{w}{R} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 \\
\frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 \\
\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}
\end{pmatrix};
\begin{pmatrix}
k^1_x \\
k^1_y \\
k^1_{xy}
\end{pmatrix} = \begin{pmatrix}
\frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2} \\
\frac{\partial \phi_y}{\partial y} + \frac{\partial^2 w}{\partial y^2} \\
\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} + 2 \frac{\partial^2 w}{\partial x \partial y}
\end{pmatrix};
\begin{pmatrix}
\gamma_{xz}^0 \\
\gamma_{yz}^0
\end{pmatrix} = \begin{pmatrix}
\phi_x + \frac{\partial w}{\partial x} \\
\phi_y + \frac{\partial w}{\partial y}
\end{pmatrix} + \begin{pmatrix}
k^2_{xz} \\
k^2_{yz}
\end{pmatrix} = \begin{pmatrix}
\phi_x + \frac{\partial w}{\partial x} \\
\phi_y + \frac{\partial w}{\partial y}
\end{pmatrix};
\]

(6)

in which

\[e_1 = 4/3h^2, \varepsilon_x, \varepsilon_y\]

\[\varepsilon_x, \varepsilon_y\] - normal strains,

\[\gamma_{xy}\] - the in-plane shear strain,

\[\gamma_{xz}, \gamma_{yz}\] - the transverse shear deformations

\[u, v, w\] - the displacement components along the \(x, y, z\) directions, respectively

\[\phi_x, \phi_y\] - the slope rotations in the \((x, z)\) and \((y, z)\) planes, respectively.

The strains are related in the compatibility equation [27, 28]:
\[
\frac{\partial^2 \varepsilon_0}{\partial y^2} + \frac{\partial^2 \varepsilon_0}{\partial x^2} - \frac{\partial^2 \gamma_0}{\partial x \partial y} = \left( \frac{\partial^2 w}{\partial x^2} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \frac{1}{R} \frac{\partial^2 w}{\partial x^2}.
\] (7)

The effect of temperature will be described in Hooke’s law for a PFGCP as:

\[
\left( \sigma_x, \sigma_y \right) = \frac{E}{1-\nu^2} \left[ (\varepsilon_x, \varepsilon_y) + \nu (\varepsilon_x, \varepsilon_y) - (1+\nu)\alpha \Delta T \left( 1, 1 \right) \right],
\]

\[
\left( \sigma_{xy}, \sigma_{yz}, \sigma_{zx} \right) = \frac{E}{2(1+\nu)} \left[ \gamma_{xy}, \gamma_{yz}, \gamma_{zx} \right],
\]

in which \( \Delta T \) is temperature rise.

The force and moment resultants of the PFGCP can be obtained with equations of stress components along with thickness of PFGCP as:

\[
(N_x, M_x, P_x) = \int_{-h/2}^{h/2} \sigma_i \left( 1, z, z^3 \right) dz; i = x, y, xy,
\]

\[
(Q, R) = \int_{-h/2}^{h/2} \sigma_j \left( 1, z^2 \right) dz; i = x, y; j = xz, yz.
\]

Replacing Eqs. (3), (5) and (8) into Eq. (9) gives:

\[
(N_x, M_x, P_x) = \frac{1}{1-\nu^2} \left[ \left( E_1, E_2, E_4 \right) \left( \varepsilon_0^0 + v \varepsilon_x^0 \right) + \left( E_2, E_3, E_5 \right) \left( k_3^1 + v k_3^1 \right) \right],
\]

\[
+ \left( E_4, E_5, E_6 \right) \left( k_3^1 + v k_3^1 \right) - (1+\nu) \left( \Phi_{\alpha}, \Phi_{\beta}, \Phi_{\gamma} \right) \right],
\]

\[
(N_y, M_y, P_y) = \frac{1}{1-\nu^2} \left[ \left( E_1, E_2, E_4 \right) \left( \varepsilon_0^0 + v \varepsilon_y^0 \right) + \left( E_2, E_3, E_5 \right) \left( k_3^1 + v k_3^1 \right) \right],
\]

\[
+ \left( E_4, E_5, E_6 \right) \left( k_3^1 + v k_3^1 \right) - (1+\nu) \left( \Phi_{\alpha}, \Phi_{\beta}, \Phi_{\gamma} \right) \right],
\]

\[
(N_{xy}, M_{xy}, P_{xy}) = \frac{1}{2(1+\nu)} \left[ \left( E_1, E_2, E_4 \right) \gamma_{xy}^0 + \left( E_2, E_3, E_5 \right) k_{xy}^1 + \left( E_4, E_5, E_6 \right) k_{xy}^3 \right],
\]

\[
(Q, R_x) = \frac{1}{2(1+\nu)} \left[ \left( E_1, E_2 \right) \gamma_{xy}^0 + \left( E_2, E_3 \right) k_{xy}^1 \right],
\]

\[
(Q, R_y) = \frac{1}{2(1+\nu)} \left[ \left( E_1, E_2 \right) \gamma_{xy}^0 + \left( E_2, E_3 \right) k_{xy}^1 \right],
\]

with

\[
\left( E_1, E_2, E_4, E_5, E_6 \right) = \int_{-h/2}^{h/2} \left( 1, z, z^2, z^3, z^4, z^5 \right) E(z) dz,
\]

\[
\left( \Phi_{\alpha}, \Phi_{\beta}, \Phi_{\gamma} \right) = \int_{-h/2}^{h/2} \left( 1, z, z^3 \right) E(z) \alpha(z) \Delta T(z) dz,
\]

and the coefficients \( E_i (i = 1+5, 7) \) are given in Appendix.

From Eq. (10), the inverse expression are obtained:
\[ e_s^0 = \frac{1}{E_i} \left[ N_x - v N_y - E_2 k_2^1 - E_4 k_4^1 + \Phi_a \right];
\]
\[ e_y^0 = \frac{1}{E_i} \left[ N_y - v N_x - E_2 k_2^1 - E_4 k_4^1 + \Phi_a \right]. \tag{12} \]

The equations of motion are [26] based on HSDT:

\[
\begin{align*}
\frac{\partial N_x}{\partial x} + \frac{\partial N_{sy}}{\partial y} & = I_0 \frac{\partial^2 u}{\partial t^2} + J_1 \frac{\partial^2 \phi_y}{\partial t^2} - c_i I_6 \frac{\partial^3 w}{\partial x \partial t^2}, \\
\frac{\partial N_{sx}}{\partial x} + \frac{\partial N_y}{\partial y} & = I_0 \frac{\partial^2 v}{\partial t^2} + J_1 \frac{\partial^2 \phi_x}{\partial t^2} - c_i I_6 \frac{\partial^3 w}{\partial x \partial t^2}, \\
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} & = c_i \left( \frac{\partial R_x}{\partial x} + \frac{\partial R_y}{\partial y} \right) + N_x \frac{\partial^2 w}{\partial x^2} + 2 N_{sy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \\
& + c_i \left( \frac{\partial^2 P}{\partial x^2} + 2 \frac{\partial^2 P_{xy}}{\partial x \partial y} + \frac{\partial^2 P_y}{\partial y^2} \right) - k_w + k_z \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + \frac{N_x + q}{R} \\
& = I_0 \frac{\partial^2 w}{\partial t^2} + 2 \epsilon I_0 \frac{\partial w}{\partial t} - c_i I_6 \left( \frac{\partial^3 w}{\partial x^2 \partial t} + \frac{\partial^3 w}{\partial y^2 \partial t} \right) \\
& + c_i \left[ I_3 \left( \frac{\partial^3 u}{\partial x \partial t^2} + \frac{\partial^3 v}{\partial y \partial t^2} \right) + J_4 \left( \frac{\partial^3 \phi_x}{\partial x \partial t^2} + \frac{\partial^3 \phi_y}{\partial y \partial t^2} \right) \right], \tag{13} \\
\frac{\partial M_x}{\partial x} + \frac{\partial M_{sy}}{\partial y} & = -c_i \left( \frac{\partial P_x}{\partial x} + \frac{\partial P_{sy}}{\partial y} \right) - Q_x - c_i R_x = J_1 \frac{\partial^2 u}{\partial t^2} + K_2 \frac{\partial^2 \phi_y}{\partial t^2} - c_i J_4 \frac{\partial^3 w}{\partial x \partial t^2}, \\
\frac{\partial M_{sx}}{\partial x} + \frac{\partial M_y}{\partial y} & = -c_i \left( \frac{\partial P_{sx}}{\partial x} + \frac{\partial P_y}{\partial y} \right) - Q_y - c_i R_y = J_1 \frac{\partial^2 v}{\partial t^2} + K_2 \frac{\partial^2 \phi_x}{\partial t^2} - c_i J_4 \frac{\partial^3 w}{\partial y \partial t^2},
\end{align*}
\]

where

\[
\begin{align*}
I_i &= \int_{h_i/2}^{h_i/2} \rho(z) z' \, dz, \quad (i = 0,1,2,3,4,6), \\
J_i &= I_i - c_i I_{n+2}, K_2 = I_2 - 2 c_i I_4 + c_i^2 I_6, c_2 = 3 c_1,
\end{align*}
\tag{14}
\]

and the coefficients \( I_i (i = 0 \div 4,6) \) are noted in Appendix, and

- \( k_1 \) - Winkler foundation modulus
- \( k_2 \) - Pasternak foundation model
- \( q \) - an external pressure uniformly distributed on the surface of the plate
- \( \epsilon \) - damping coefficient.

The stress function \( f(x,y,t) \) is introduced as
\begin{equation}
N_x = \frac{\partial^2 f}{\partial y^2}, \quad N_y = \frac{\partial^2 f}{\partial x^2}, \quad N_{xy} = -\frac{\partial^2 f}{\partial x \partial y}.
\end{equation}

Replacing Eq. (15) into the two first Eqs. (13) yields
\begin{align}
\frac{\partial^2 u}{\partial t^2} &= -\frac{J_i}{I_0} \frac{\partial^2 \phi}{\partial t^2} + c_i I_3 \frac{\partial^3 w}{\partial x \partial t^2}, \\
\frac{\partial^2 v}{\partial t^2} &= -\frac{J_i}{I_0} \frac{\partial^2 \phi}{\partial t^2} + c_i I_3 \frac{\partial^3 w}{\partial y \partial t^2}.
\end{align}

By substituting Eqs. (16) into the Eqs. (13) and Eqs. (13) can be rewritten
\begin{align}
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - c_1 \left( \frac{\partial R_x}{\partial x} + \frac{\partial R_y}{\partial y} \right) &= N_x \frac{\partial^2 w}{\partial x^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \\
&+ c_1 \left( \frac{\partial^2 P_x}{\partial x^2} + 2 \frac{\partial^2 P_y}{\partial x \partial y} + \frac{\partial^2 P_y}{\partial y^2} \right) - k_I + k_2 \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + N_o + q \\
= I_0 \frac{\partial^2 w}{\partial t^2} + 2 e I_0 \frac{\partial w}{\partial t} + \left( c_i I_3 - c_i I_6 \right) \left( \frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^4 w}{\partial y^2 \partial t^2} \right) + \\
\left( J_4 c_1 - \frac{J_4 c_1}{I_0} \right) \frac{\partial^3 \phi}{\partial x \partial t^2} + \left( J_4 c_1 - \frac{J_4 c_1}{I_0} \right) \frac{\partial^3 \phi}{\partial y \partial t^2},
\end{align}

Substituting Eqs. (12) and (15) into the equation (7), we have:
\begin{equation}
\frac{1}{E} \left( \frac{\partial^2 f}{\partial x^2} + 2 \frac{\partial^2 f}{\partial x \partial y^2} + \frac{\partial^2 f}{\partial y^2} \right) = \left( \frac{\partial^2 w}{\partial x^2} \right)^2 - \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial y^2}.
\end{equation}

The system of motion Eqs. (17) is rewritten as Eq. (19) by replacing Eq. (6) into Eq. (10) and then into Eqs. (17):
\begin{align}
L_1(w) + L_2(\phi_x) + L_3(\phi_y) + P(w, f) + q &= I_0 \frac{\partial^2 w}{\partial t^2} + 2 e I_0 \frac{\partial w}{\partial t} \\
&+ \left( c_i I_3 - c_i I_6 \right) \left( \frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^4 w}{\partial y^2 \partial t^2} \right) + \\
&\left( J_4 c_1 - \frac{J_4 c_1}{I_0} \right) \frac{\partial^3 \phi}{\partial x \partial t^2} + \left( J_4 c_1 - \frac{J_4 c_1}{I_0} \right) \frac{\partial^3 \phi}{\partial y \partial t^2},
\end{align}

\begin{align}
L_2(w) + L_2(\phi_x) + L_3(\phi_y) &= \left( \frac{J_4}{I_0} \right) \frac{\partial^3 \phi}{\partial t^2} + \left( c_i I_1 - c_i J_4 \right) \frac{\partial^3 w}{\partial x \partial t^2}.
\end{align}
\[ L_{11}(\phi) + L_{12}(\phi_x) + L_{13}(\phi_y) = \left( K_2 - \frac{J_1}{I_0} \right) \frac{\partial^2 \phi}{\partial t^2} + \left( c_1I_1 - c_1J_4 \right) \frac{\partial^4 \phi}{\partial y^4}, \tag{19c} \]

and the \( L_{ij}(i = 1-3, j = 1-3) \) and \( P \) are noted in Appendix.

In next section, in order to determined nonlinear dynamical analysis of PFGCP using the third order shear deformation theory, the Eqs. (18)-(19) are used along with boundary conditions and initial conditions

2.2. Boundary conditions

In order to determined nonlinear dynamical analysis of PFGCP, the edges of the PFGCP are simply supported and immovable (IM) and boundary conditions are [27, 28]:

\[ w = u = \phi_x = M_x = P_x = 0, \quad N_x = N_{y0} \text{ at } x = 0, a, \]
\[ w = v = \phi_y = M_y = P_y = 0, \quad N_y = N_{x0} \text{ at } y = 0, b, \tag{20} \]

in which \( N_{x0}, N_{y0} \) are forces along the \( x \) and \( y \) axis.

With the boundary condition (20) and In order to solve Eqs. (18) and (19), the approximate solutions can be written as [26]:

\[ w(x, y, t) = W(t) \sin \alpha x \sin \beta y, \]
\[ \phi_x(x, y, t) = \Phi_x(t) \cos \alpha x \sin \beta y, \]
\[ \phi_y(x, y, t) = \Phi_y(t) \sin \alpha \cos \beta y, \tag{21} \]

with \( \alpha = \frac{m\pi}{a}, \beta = \frac{n\pi}{b} \) and \( m, n = 1, 2, \ldots \) are the numbers of half waves in the direction \( x, y \), \( W, \Phi_x, \Phi_y \) - the amplitudes which are functions dependent on time.

The stress function is defined as:

\[ f(x, y, t) = A_1(t) \cos 2\alpha x + A_2(t) \cos 2\beta y + \frac{1}{2} N_{x0} y^2 + \frac{1}{2} N_{y0} x^2, \tag{22a} \]

with \( A_1 = \frac{E_x \beta^2}{32 \alpha^2} W^2; A_2 = \frac{E_y \alpha^2}{32 \beta^2} W^2. \tag{22b} \)

2.3. Nonlinear dynamical analysis

In order to obtain Eqs. (23), Eqs. (21) and (22) are replaced into the Eq. (19) after that applying Galerkin method:

\[ \left[ l_{11} - \left( N_{x0} \alpha^2 + N_{y0} \beta^2 \right) \right] W + l_{12} \Phi_x + l_{13} \Phi_y + nW^3 + \frac{16}{mnp^2} q = n_2 \frac{\partial^2 W}{\partial t^2} + 2c_1 \frac{\partial W}{\partial t} \]

\[ + \rho_2 \frac{m \pi \partial^2 \Phi_x}{a \partial t^2} + \rho_2 \frac{n \pi \partial^2 \Phi_y}{b \partial t^2}, \tag{23a} \]

\[ l_2W + l_{22} \Phi_x + l_{23} \Phi_y = \rho_1 \frac{\partial^2 \Phi_x}{\partial t^2} + \rho_2 \frac{m \pi \partial^2 W}{a \partial t^2}, \tag{23b} \]
\begin{equation}
l_{31} W + l_{32} \Phi_x + l_{33} \Phi_y = \rho_1 \frac{\partial^2 \Phi_x}{\partial t^2} + \rho_2 \frac{n \pi}{b} \frac{\partial^2 W}{\partial t^2},
\end{equation}

(23e)

Where \( \rho_1 = K_2 - \frac{J^2}{l_0 L_0} ; \rho_2 = \frac{c_1 I J_1}{l_0 L_0} - c_4 J_4 \),

\begin{align*}
l_{11} &= -\frac{1}{2(1 + \nu)} \left[ E_1 - 3c_i E_i - c_2 \left(E_3 - 3c_i E_i \right) \right] \left( \frac{m \pi}{a} \right)^4 + \left( \frac{n \pi}{b} \right)^4 \\
&- \frac{c_i^2}{1 - \nu^2} \left( \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 \right) \left( E_1 - E_i^2 \right) \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 + \frac{m \pi}{a} \frac{n \pi}{b} \frac{1}{R},
\end{align*}

\begin{align*}
l_{12} &= -\frac{1}{2(1 + \nu)} \frac{m \pi}{a} \left[ E_1 - 3c_i E_i - c_2 \left(E_3 - 3c_i E_i \right) \right] + \frac{c_i}{1 - \nu^2} \left( \frac{m \pi}{a} \right)^2 \left[ E_3 - E_i \frac{E_i}{E_i} - c_1 \left(E_3 - E_i \right) \right] \\
&+ c_i \left[ \left( \frac{1}{1 + \nu} + \frac{\nu}{1 - \nu^2} \right) \left( E_3 - E_i \frac{E_i}{E_i} \right) - c_i \left( \frac{\nu}{1 - \nu^2} + \frac{1}{1 + \nu} \right) \left( E_i \frac{E_i}{E_i} \right) \right] \\
&+ \frac{n \pi}{b} \frac{1}{R} \frac{m \pi}{a} \frac{n \pi}{b} \frac{1}{R},
\end{align*}

\begin{align*}
l_{13} &= -\frac{1}{2(1 + \nu)} \frac{n \pi}{b} \left[ E_1 - 3c_i E_i - c_2 \left(E_3 - 3c_i E_i \right) \right] + \frac{c_i}{1 - \nu^2} \left( \frac{n \pi}{b} \right)^4 \left[ E_3 - E_i \frac{E_i}{E_i} - c_1 \left(E_3 - E_i \right) \right] \\
&+ c_i \left[ \left( \frac{1}{1 + \nu} + \frac{\nu}{1 - \nu^2} \right) \left( E_3 - E_i \frac{E_i}{E_i} \right) - c_i \left( \frac{\nu}{1 - \nu^2} + \frac{1}{1 + \nu} \right) \left( E_i \frac{E_i}{E_i} \right) \right] \\
&+ \frac{m \pi}{a} \frac{n \pi}{b} \frac{1}{R},
\end{align*}

\begin{align*}
n_1 &= \frac{E_1}{16} \left( \frac{m \pi}{a} \right)^4 + \left( \frac{n \pi}{b} \right)^4 \\
n_2 &= I_0 + c_i^2 \left( I_0 - \frac{I_1}{I_0} \right) \left( \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 \right),
\end{align*}

\begin{align*}
l_{21} &= -\frac{c_i}{1 - \nu^2} \frac{m \pi}{a} \left( \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 \right) \left( E_i \frac{E_i}{E_i} - E_i \frac{E_i}{E_i} + E_i \frac{E_i}{E_i} \right) \\
&- \frac{1}{2(1 + \nu)} \frac{m \pi}{a} \left[ E_1 - 3c_i E_i \right] + c_2 \left(E_3 - 3c_i E_i \right),
\end{align*}

\begin{align*}
l_{22} &= -\frac{1}{2(1 + \nu)} \frac{m \pi}{a} + \frac{1}{2(1 + \nu)} \frac{n \pi}{b} \left( \left( \frac{m \pi}{a} \right)^2 + \left( \frac{n \pi}{b} \right)^2 \right) \left[ E_3 - E_i \frac{E_i}{E_i} - 2c_i \left(E_3 - E_i \frac{E_i}{E_i} \right) + c_i^2 \left(E_3 - E_i \right) \right] \\
&- \frac{1}{2(1 + \nu)} \left[ E_3 - 3c_i E_i \right] + c_2 \left(E_3 - 3c_i E_i \right),
\end{align*}

\begin{align*}
l_{23} &= -\frac{1}{2(1 + \nu)} \frac{m \pi}{a} \frac{n \pi}{b} \left[ E_3 - E_i \frac{E_i}{E_i} - 2c_i \left(E_3 - E_i \frac{E_i}{E_i} \right) + c_i^2 \left(E_3 - E_i \right) \right].
\end{align*}
\[
\begin{align*}
I_{31} &= \frac{c_1}{1 - \nu} \left[ \frac{n\pi}{b} \left( c_i E_i - c_i \frac{E_i^2}{E_1} - E_i^2 - E_i c_i \frac{E_i}{E_1} \right) \left( \frac{(m\pi)^2}{b^2} \right) + \frac{n\pi}{b} \left( \frac{(n\pi)^2}{b^2} \right) \right] - \frac{1}{2(1 + \nu)} \left[ E_i (3c_i E_i + c_i (E_i - 3c_i E_i)) \right],
\end{align*}
\]
\[
\begin{align*}
I_{32} &= -\frac{1}{2(1 - \nu)} \left[ E_i^2 - \frac{E_i^2}{E_1} - 2c_i \left( E_i^2 - E_i c_i \frac{E_i}{E_1} \right) + c_i^2 \left( E_i - \frac{E_i^2}{E_1} \right) \right],
\end{align*}
\]
\[
\begin{align*}
I_{33} &= \left( \frac{1}{2(1 - \nu)} \left( \frac{n\pi}{b} \right) + \frac{1}{2(1 + \nu)} \left( \frac{m\pi}{a} \right) \right) \left[ E_i^2 - \frac{E_i^2}{E_1} - 2c_i \left( E_i^2 - E_i c_i \frac{E_i}{E_1} \right) + c_i^2 \left( E_i - \frac{E_i^2}{E_1} \right) \right]
- \frac{1}{2(1 + \nu)} \left[ E_i (3c_i E_i + c_i (E_i - 3c_i E_i)) \right],
\end{align*}
\]

In order to investigate the effect of thermal load on the PFGCP. The modeling is simply supported and immovable in all edges. In other words, boundary conditions of case stud is \( u = 0 \) (on \( x = 0, a \)) and \( v = 0 \) (on \( y = 0, b \)), and and is satisfied equations as \([10, 29]\):
\[
\int_0^b \int_0^b \frac{\partial u}{\partial x} dx dy = 0, \quad \int_0^b \int_0^b \frac{\partial v}{\partial y} dx dy = 0.
\]

With Eqs. (6) and (12) along with (15), Eq (25) is determined as:
\[
\frac{\partial u}{\partial x} = \frac{1}{E_1} \left( f_{xx} - \nu f_{yy} \right) - \frac{E_i}{E_1} \phi_{,x} + \frac{c_i E_i}{E_1} \left( \phi_{,x,x} + w_{,x} \right) - \frac{w}{R} - \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 + \frac{\Phi_f}{E_1},
\]
\[
\frac{\partial v}{\partial y} = \frac{1}{E_1} \left( f_{yy} - \nu f_{xx} \right) - \frac{E_i}{E_1} \phi_{,y} + \frac{c_i E_i}{E_1} \left( \phi_{,y,y} + w_{,y} \right) - \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 + \frac{\Phi_f}{E_1}.
\]

In order to obtain Eqs. (26), Eqs. (21) and (22) are placed into Eq. (25) then the obtained result are replaced into Eq. (24) we have:
\[
N_{x0} = -\frac{\Phi_f}{1 - \nu} - \frac{4}{mn\pi^2 \left( 1 - \nu^2 \right)} \left[ (E_i - c_i E_i) (\alpha \phi_{,x} + \nu \beta \phi_{,y}) - c_i E_i (\alpha^2 + \nu \beta^2) W \right]
+ \frac{E_i}{8 \left( 1 - \nu^2 \right)} (\alpha^2 + \nu \beta^2) W^2;
\]
\[
N_{y0} = -\frac{\Phi_f}{1 - \nu} - \frac{4}{mn\pi^2 \left( 1 - \nu^2 \right)} \left[ (E_i - c_i E_i) (\nu \alpha \phi_{,y} + \beta \phi_{,x}) - c_i E_i (\nu \alpha^2 + \beta^2) W \right]
+ \frac{E_i}{8 \left( 1 - \nu^2 \right)} (\nu \alpha^2 + \beta^2) W^2.
\]

Putting Eqs. (26) into the Eq. (23). We obtain:
\[
\left[ l_{11} + \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right] W + l_{12} x + l_{13} y + l_{14} \frac{W}{t} + l_{15} \frac{W}{t^2} + l_{16} \frac{W}{t^3} + l_{17} \frac{W}{t^4}
+ \frac{16}{mn} q = \frac{n_2}{t^2} + 2 l_0 \frac{W}{t} + \frac{m}{a} \frac{W}{t^2} + \frac{n}{b} \frac{W}{t^2},
\]
\[
\begin{align*}
l_{21} W + l_{22} \Phi_x + l_{23} \Phi_y = \rho_1 \frac{\partial^2 \Phi_x}{\partial t^2} + \rho_2 \frac{m\pi}{a} \frac{\partial^2 W}{\partial t^2},
\end{align*}
\]
\[ l_{31}w + l_{32}\Phi_x + l_{33}\Phi_y = \rho_1 \frac{\partial^2 \Phi_y}{\partial t^2} + \rho_2 \frac{n\pi}{b} \frac{\partial^3 \Phi_y}{\partial t^3}, \]  
\tag{27c}

with
\[ l_{14} = \frac{4(E_2-c_4E_4)\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}{mn\pi^2(1-v^2)} \frac{m\pi}{a}, \]
\[ l_{15} = \frac{4(E_2-c_4E_4)\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}{mn\pi^2(1-v^2)} \frac{n\pi}{b}, \]
\[ l_{16} = \frac{4c_4E_4(\alpha^4 + 2\nu\alpha^2\beta^2 + \beta^4)}{mn\pi^2(1-v^2)}, \]
\[ l_{17} = n_l - \frac{E_1}{8(1-v^2)} \left[ \left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4 \right]. \]

The Eqs. (27) are used to determine dynamic analysis of PFGCP with supported by elastic foundations along with the effect of temperature with the appearance of the coefficient \( \Phi_u \).

In order to obtain the natural frequencies (\( \omega \)) of PFGCP, solving determinant of matrix (28) with linear coefficients taken from Eqs. (27) along with \( q = 0 \). The obtained results are three natural frequencies, the smallest value will be selected.

\[ \begin{vmatrix} l_{11} + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \Phi \frac{1}{1-v} + n_2\omega^2 & l_{12} + \rho_2 \frac{m\pi}{a} \phi \omega^2 & l_{13} + \rho_2 \frac{n\pi}{b} \omega^2 \\ l_{21} + \rho_2 \frac{m\pi}{a} \phi \omega^2 & l_{22} + \rho_1 \phi \omega^2 & l_{23} \\ l_{31} + \rho_2 \frac{n\pi}{b} \omega^2 & l_{32} & l_{33} + \rho_1 \phi \omega^2 \end{vmatrix} = 0. \]  
\tag{28}

3. Numerical results and discussion

In this paper, consider PFGCP is under the influence of a uniformly distributed load on the surface with equations in the harmonic function \( q = Q\sin \Omega t \) with \( Q \) is the amplitude of uniformly excited load. Besides, the fourth-order Runge-Kutta method is used to solve equations (27) and the material properties of the component material are given as [40]:

\[ E_c = 380 \times 10^9 \text{ N/m}^2, \rho_c = 3800 \text{ kg/m}^3, \alpha_c = 7.4 \times 10^{-6} \text{ C}^{-1}, K_c = 10.4 \text{ W/mK}, \]
\[ E_m = 70 \times 10^9 \text{ N/m}^2, \rho_m = 2702 \text{ kg/m}^3, \alpha_m = 23 \times 10^{-6} \text{ C}^{-1}, K_m = 204 \text{ W/mK}, \]
\[ v = 0.3, \]
3.1. Natural frequency

In order to verify the reliability of the used method, the numerical results of this paper will be compared with the published results of Duc et al [40]. In the Duc author's paper, the used method is high order shear deformation for FGM plate. In order to obtain the highest accuracy of the comparison results, the modeling of this paper was brought to the same format as the modeling in Duc’s paper by \( R = \infty \). The comparison results of the free natural frequency are shown in Table 1. It can be seen that the results of both papers are not much different. It verifies that the used method is reliable.

Table 1. Comparison of natural frequency \( (s^{-1}) \) of PFGCP with other paper with same conditions as \( a/b = 1, a/h = 20, N = 1 \).

<table>
<thead>
<tr>
<th>( (k_1;k_2) ) ((GPa/m;GPa.m))</th>
<th>( \Delta T (^\circ\text{C}) )</th>
<th>Natural frequency ( (s^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duc et al [40]</td>
<td>Present study</td>
<td></td>
</tr>
<tr>
<td>((0;0))</td>
<td>0</td>
<td>1941.73</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>1570.25</td>
</tr>
<tr>
<td></td>
<td>105</td>
<td>1015.19</td>
</tr>
<tr>
<td>((0.25;0))</td>
<td>0</td>
<td>2303.00</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>1999.79</td>
</tr>
<tr>
<td></td>
<td>105</td>
<td>1601.28</td>
</tr>
<tr>
<td>((0.25;0.02))</td>
<td>0</td>
<td>2779.46</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>2533.92</td>
</tr>
<tr>
<td></td>
<td>105</td>
<td>2232.86</td>
</tr>
</tbody>
</table>

The natural frequencies \( (s^{-1}) \) of PFGCP with the influence of index \( N \), modes \( (m,n) \) along with \( a/b = 1, a/h = 20, k_1 = 0, k_2 = 0, \Delta T = 100^\circ\text{C} \) and using type 2 of porosity distribution. We can see that the natural frequencies will raise when modes \( (m,n) \) increase in which index \( N \) is constant. In contrast, when index \( N \) increase in which modes \( (m,n) \) is constant, the natural frequencies increase. Form equation (1), it can be explained that index \( N = 0 \) corresponding to isotropic uniform panel is made from ceramic materials, \( N = 1 \) is the case when the ceramic and metal components are distributed linearly over the thickness of the structure wall and when \( N \) increase, the volume ratio of the metal component in the structure increases.

Table 2. The natural frequencies \( (s^{-1}) \) of PFGCP with the influence of index \( N \), modes \( (m,n) \) along with \( a/b = 1, a/h = 20, k_1 = 0, k_2 = 0, R/h = 400 \) and \( \Delta T = 100^\circ\text{C} \).

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \omega_1 ) ((m=1,n=1))</th>
<th>( \omega_2 ) ((m=1,n=3))</th>
<th>( \omega_3 ) ((m=1,n=5))</th>
<th>( \omega_4 ) ((m=3,n=5))</th>
<th>( \omega_5 ) ((m=5,n=5))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>2702</td>
<td>11175</td>
<td>27051</td>
<td>34564</td>
<td>48871</td>
</tr>
<tr>
<td>1</td>
<td>2434</td>
<td>9766</td>
<td>24060</td>
<td>30838</td>
<td>43767</td>
</tr>
<tr>
<td>3</td>
<td>2251</td>
<td>8817</td>
<td>21037</td>
<td>26808</td>
<td>37785</td>
</tr>
<tr>
<td>5</td>
<td>2172</td>
<td>8505</td>
<td>20218</td>
<td>25719</td>
<td>36137</td>
</tr>
<tr>
<td>12</td>
<td>2080</td>
<td>8174</td>
<td>19369</td>
<td>24594</td>
<td>34440</td>
</tr>
<tr>
<td>( \infty )</td>
<td>1629</td>
<td>6195</td>
<td>14636</td>
<td>18610</td>
<td>26149</td>
</tr>
</tbody>
</table>
Table 3 shows the natural frequencies \( (s^{-1}) \) of PFGCP with the influence of ratio \( a/b \), elastic foundation, porosity distribution type along with \( a/h = 20, N = 1, \Delta T = 100^\circ C, (m,n) = (1,1), R/h = 400. \) It can easily see that the natural frequencies increase when ratio \( a/b \) raises as well as when modeling is supported by elastic foundations. Furthermore, the comparison of the effect between type 1 of porosity distribution and type 2 of porosity distribution are shown in table 3. The effect of type 2 of porosity distribution on the natural frequencies is smaller than the effect type 1 of porosity distribution.

Table 3. The natural frequencies \( (s^{-1}) \) of PFGCP with the influence of ratio \( a/b \), elastic foundation \( (m,n) = (1,1), R/h = 400, a/h = 20, N = 1, \Delta T = 100^\circ C. \)

<table>
<thead>
<tr>
<th>((k_1;k_2)) ((GPa/m;GPa.m))</th>
<th>(a/b)</th>
<th>Natural frequency ( (s^{-1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0;0))</td>
<td>1</td>
<td>2525</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>3757</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5448</td>
</tr>
<tr>
<td>((0.2;0))</td>
<td>1</td>
<td>2757</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>3917</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5559</td>
</tr>
<tr>
<td>((0.2;0.04))</td>
<td>1</td>
<td>3528</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>4816</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6554</td>
</tr>
</tbody>
</table>

3.2. Nonlinear dynamic response

Figure 2 shows the effect of porosity distribution on nonlinear dynamic response of the PFGCP with \( a/b = 1, a/h = 20, \Delta T = 0, \varepsilon = 0.1. \) It can easily see that the amplitude of deflection for PFGCP in case Porosity I bigger more than the amplitude of deflection for PFGCP in Porosity II. In other words, the porosity distribution Porosity II will enhance the loading carrying capacity of PFGCP more than the
Porosity I. Thus, other results will use the Porosity II to investigate the effect of other factors on the nonlinear dynamic response of the panel.

Figures 3 and 4 give the nonlinear dynamic response of PFGCP with the influence of ratio $a/b$, ratio $a/h$ and supported by elastic foundations. The amplitude of deflection of the PFGCP will raise when increasing ratio $a/h$ or decrease $a/b$. In figure 3, the amplitude of deflection has the highest value with $a/b=1$. When increasing $a/b$ from 1 to 2, the amplitude of deflection decreases about 7 times. In figure 4, the amplitude of deflection has the smallest value with $a/h=40$. When increasing $a/h$ from 20 to 40, the amplitude of deflection raise about 9 times.

![Graph showing nonlinear dynamical response](image)

**Fig. 3.** The nonlinear dynamical response of the PFGCP with influence of effect of ratio $a/b$.

The influence of index N on the nonlinear dynamic response of the PFGCP is investigated in figure 5. From figure 5 can see that the amplitude vibration of the PFGCP will raise when the index N increase and change about 2.3 times. This behavior can be explained that the index N increase results in a volume ratio of ceramic more bigger than a volume ratio of metal. It leads to bigger change in Elastic modulus, in other words, it leads to decrease load carrying capacity of the PFGCP.

![Graph showing influence of N](image)
Fig. 4. The nonlinear dynamical response of the PFGCP with influence of effect of ratio $a/h$

Fig. 5. Effect of power law index $N$ on nonlinear dynamical response of the PFGCP.

The influence of elastic foundations on the nonlinear dynamic response of the PFGCP with $a/b = 1$, $a/h = 20$, $N = 1$, $\Delta T = 100^\circ C$ are shown in figures 6 and 7. It clear that the elastic foundations: Winkler foundation (figure 6) and Pasternak foundations (figure 7) has a positive influence on the amplitude vibrations. When increasing the Winkler foundation, the Pasternak foundations, the amplitude vibrations will decrease. But the influence of the Pasternak foundation better than Winkler foundations, the amplitude vibrations will enhance about 200% with the change of the Winkler foundation in which the amplitude vibrations will enhance about 250%. In order to enhance the load carrying capacity of modeling, the elastic foundations have an important role.

Fig. 6. Effect of the linear Winkler foundation on nonlinear dynamical response of the PFGCP.
Fig. 7. Effect of the Pasternak foundation on nonlinear dynamical response of the PFGCP.

The influence of exciting force amplitude on nonlinear dynamic response of the PFGCP with \( Q = (1500, 1800, 2500 \text{ N/m}^2) \) and \( \Delta T = 100^\circ C \) are proposed in figure 8. The amplitude of vibration will raise when the excited force amplitude increase. In other words, the exciting force amplitude decrease, the amplitude of vibration will decrease.

Fig. 8. Nonlinear dynamic responses of the PFGCP with different loads.

The nonlinear response of the PFGCP with the effect of uniform temperature is presented in figure 9. From figure 9, it can easily seen that the uniform temperature has a negative influence on the amplitude deflection of the cylindrical panel PFG. The amplitude deflection of the PFGCP will increase when the uniform temperature rise. It can be explained that the uniform temperature changes the stiffness matrix.
Fig. 9. Effect of uniform temperature rise on nonlinear response of the PFGCP.

4. Conclusions

This is the first paper uses the high order shear deformation theory and Ahry function to investigate the nonlinear dynamic response and vibration of panel FGM with porosity distribution. Besides, the PFGCP is supported by elastic foundations and under mechanical load, thermal load. By using the Galerkin method as well as the four-order Runge-Kutta method, the numerical results for nonlinear dynamic response of modelings are described by figure and table with the influences of the materials properties, thermal load, mechanical load, elastic foundations, geometrical parameters, the porosity distribution type. This paper obtained some remarkable results as:

- The porosity distribution type 2 will enhance the loading carrying capacity of PFGCP more than the porosity distribution type 1.
- The amplitude of deflection of the PFGCP will raise when increasing ratio \( a/h \) or decrease \( a/b \).
- The amplitude vibration of the PFGCP will raise when the index \( N \) increase and changes about 2.3 times.
- When increasing the Winkler foundation, the Pasternak foundations, the amplitude vibrations will decrease. The influence of the Pasternak foundation better than Winkler foundations.
- The amplitude of vibration will raise when the exciting force amplitude increase.
- The uniform temperature has a negative influence on the amplitude deflection of the cylindrical panel PFG. The amplitude deflection of the PFGCP will increase when the uniform temperature rise.

Acknowledgement

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References


[34] Shufrin, M. Eisenberger, Stability of variable thickness shear deformable plates—first order and high order analyses, Thin-Walled Structures. 43 (2005) 189-207.


Appendix

\[ L_{11}(w) = \frac{1}{2(1+\nu)} \left[ E_{t} - 3 c_{1} E_{s} - c_{2} (E_{s} - 3 c_{1} E_{t}) \right] \left( \frac{\partial^{2} w}{\partial x^{2}} + \frac{\partial^{2} w}{\partial y^{2}} \right) \]
\[- \frac{c_{1}^{2}}{1 - \nu^{2}} \left[ \frac{\partial^{4} w}{\partial x^{2} \partial y^{2}} + 2 \frac{\partial^{4} w}{\partial x^{2} \partial y^{2}} + \frac{\partial^{4} w}{\partial y^{2}} \right] \left( E_{t} - \frac{E_{s}}{E_{t}} \right) - k_{1} w + k_{2} \left( \frac{\partial^{2} w}{\partial x^{2}} + \frac{\partial^{2} w}{\partial y^{2}} \right). \]

\[ L_{12}(\phi_{s}) = \frac{1}{2(1+\nu)} \left[ E_{t} - 3 c_{1} E_{s} - c_{2} (E_{s} - 3 c_{1} E_{t}) \right] \frac{\partial \phi_{s}}{\partial x} + \frac{c_{1}}{1 - \nu^{2}} \left[ E_{t} - \frac{E_{s}}{E_{t}} - c_{1} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) \right] \frac{\partial \phi_{s}}{\partial x^{3}} + c_{1} \left[ \frac{1}{1 + \nu} + \frac{\nu}{1 - \nu^{2}} \right] \left( E_{t} - \frac{E_{s}}{E_{t}} \right) - c_{1} \left[ \frac{\nu}{1 - \nu^{2}} + \frac{1}{1 + \nu} \right] \left( E_{t} - \frac{E_{s}}{E_{t}} \right) \frac{\partial \phi_{s}}{\partial x y^{2}}. \]

\[ L_{13}(\phi_{s}) = \frac{1}{2(1+\nu)} \left[ E_{t} - 3 c_{1} E_{s} - c_{2} (E_{s} - 3 c_{1} E_{t}) \right] \frac{\partial \phi_{s}}{\partial y} + \frac{c_{1}}{1 - \nu^{2}} \left[ E_{t} - \frac{E_{s}}{E_{t}} - c_{1} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) \right] \frac{\partial \phi_{s}}{\partial y^{3}} + c_{1} \left[ \frac{1}{1 + \nu} + \frac{\nu}{1 - \nu^{2}} \right] \left( E_{t} - \frac{E_{s}}{E_{t}} \right) - c_{1} \left[ \frac{\nu}{1 - \nu^{2}} + \frac{1}{1 + \nu} \right] \left( E_{t} - \frac{E_{s}}{E_{t}} \right) \frac{\partial \phi_{s}}{\partial x y^{2}}, \]

\[ P(x, f) = \frac{\partial^{2} f}{\partial y^{2}} \frac{\partial^{2} w}{\partial x^{2}} - 2 \frac{\partial^{2} f}{\partial x \partial y} \frac{\partial^{2} w}{\partial x \partial y} + \frac{\partial^{2} f}{\partial x^{2}} \frac{\partial^{2} w}{\partial y^{2}} + \frac{1}{R} \frac{\partial^{2} f}{\partial x^{2}}, \]

\[ L_{21}(w) = \frac{c_{1}}{1 - \nu^{2}} \left[ c_{1} E_{s} - c_{1} E_{t} - E_{s} + \frac{E_{s} E_{t}}{E_{s}} \right] \left( \frac{\partial^{2} w}{\partial y^{2}} + \frac{\partial^{2} w}{\partial x^{2}} \right) - \frac{1}{2(1+\nu)} \left[ (E_{t} - 3 c_{1} E_{s}) + c_{1} (E_{s} - 3 c_{1} E_{t}) \right] \frac{\partial^{2} w}{\partial x}, \]

\[ L_{22}(\phi_{s}) = \left[ \frac{1}{1 - \nu^{2}} + \frac{1}{2(1+\nu)} \right] \left( E_{t} - \frac{E_{s}}{E_{t}} \right) - 2 c_{1} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) + c_{1} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) \frac{\partial^{2} w}{\partial x y^{2}}, \]

\[ L_{23}(\phi_{s}) = \frac{1}{2(1+\nu)} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) - 2 c_{1} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) + c_{1} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) \frac{\partial^{2} w}{\partial x y^{2}}, \]

\[ L_{31}(w) = \frac{c_{1}}{1 - \nu^{2}} \left[ c_{1} E_{s} - c_{1} E_{t} - E_{s} + \frac{E_{s} E_{t}}{E_{s}} \right] \left( \frac{\partial^{2} w}{\partial y^{2}} + \frac{\partial^{2} w}{\partial x^{2}} \right) - \frac{1}{2(1+\nu)} \left[ (E_{t} - 3 c_{1} E_{s}) + c_{1} (E_{s} - 3 c_{1} E_{t}) \right] \frac{\partial^{2} w}{\partial y}, \]

\[ L_{32}(\phi_{s}) = \frac{1}{2(1+\nu)} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) - 2 c_{1} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) + c_{1} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) \frac{\partial^{2} w}{\partial x y^{2}}, \]

\[ L_{33}(\phi_{s}) = \frac{1}{2(1+\nu)} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) - 2 c_{1} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) + c_{1} \left( E_{t} - \frac{E_{s}}{E_{t}} \right) \frac{\partial^{2} w}{\partial x y^{2}}, \]

\[ - \frac{1}{2(1+\nu)} \left[ (E_{t} - 3 c_{1} E_{s}) + c_{1} (E_{s} - 3 c_{1} E_{t}) \right] \phi_{s}, \]

\[ - \frac{1}{2(1+\nu)} \left[ (E_{t} - 3 c_{1} E_{s}) + c_{1} (E_{s} - 3 c_{1} E_{t}) \right] \phi_{s}. \]