Simulation Research on Hollow Cavities in the Body of Dikes and Dams by Geophysical Methods

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Abstract: This paper presents the results of simulation study of hollow caves in the body of dikes, dams through theoretical modeling and calculation based on the knowledge of Mathematics, Informatics, Physics in general and Geophysics in particular and also the information processed by the software of EarthImage 2D and Reflex 2D on the methods of multi-electrode electrical sounding and ground penetrating radar. From there, it is possible to draw conclusions on the applicability of both methods for finding, identifying hollow cave, and also looking at ways to conduct field work appropriate to improve the efficiency of the methods mentioned above. These results will be applied as the basis for the discovery of the hollow cave of dikes and dams in Vietnam.

Keywords: Simulation, hazard, hollow cave, dike, dam.

1. Rationale

Hollow cave in the body of dikes, dams is causing subsidence hazards leading to the risk of broken dikes and dams. There are many reasons for this generation of hollow caves such as earth for dam embankment is not handled carefully initially leading after the prolonged time to uneven subsidence, living organisms for their nest in body of dams, culverts or other underground works from the past which are not used and forgotten, etc. We can only discover and estimate those hollow caves expressed on the surface or by water seepage from inside to outside and vice versa, then the safety of the dike, dam is threatened.

The regular inspection and monitoring to ensure the safety of dikes, dams system is a mandatory requirement, particularly through the survey results by the scientific methods, including the methods of geophysics. In order to well-do that, it should be studied in order to have preliminary information on relative precision based on scientific basis for the selection and application of conducting search
methods, determining the hollow caves to increase their efficiency (faster, less expensive, able to identify the type, location and size of the hollow cave itself in body of dikes, dams). One of the issues that need to be set is to make simulation studies of hollow cave using geophysical methods as the basis for the discovery of the hollow cave of dikes and dams in Vietnam.

In this paper we present some new results obtained by simulation study of the hollow cave by multi-electrode electrical sounding method (MES) [1-3] or improved multi-electrode electrical sounding method (IMES) [4] (both are called MES method) and ground penetrating radar (GPR) [3, 5], and assess the ability and effectiveness of the methods in detecting objects.

2. Theoretical basic

2.1. Method of Multi-electrode electrical sounding

Theory of single-line electricity wire is a part of theory of electrodynamics, basic equations of physics problems about the relationship between the current density distribution in the environment due to the development of direct current electrical power with capacity of I on the environment with the conductivity of the environment. It is described by the following basic equation:

\[- \text{div} (\sigma(x, y, z) \text{grad}U(x, y, z)) = I \delta(x-x_s) \delta(y-y_s) \delta(z-z_s)\]

Among them: \text{div}, \text{grad} - the operators;

\[\sigma\] - function describing the distribution of electrical conductivity in the environment according to the coordinates \(x, y, z\) (the actual parameters used for resistivity \(\rho = 1/\sigma\) measured in \(\Omega\text{m}\));

\[U\] - voltage function describing the voltage distribution in the environment according to the coordinates \(x, y, z\);

\[I\] - current intensity development on the environment;

\[\delta\] - is the Dirac function characterizing the distribution of power environment coordinates \(x_s, y_s, z_s\);

Equation for the two-dimensional distribution (2D) can be solved by two main methods which are methods of integral equations and differential equations. However, up till now widely used and most effective method is finite difference one. With constant current, the equation is written as:

\[- \nabla \left[ \sigma(x, y, z) \nabla \phi(x, y, z) \right] = 0\]

Among them:

\[\phi(x, y, z)\] - a function that satisfies the conditions of continuous functions with conductivity \(\sigma(x, y, z)\), constant current density \(J\) and Neumann, Dirichlet boundary conditions;

\((x_s, y_s, z_s)\) are the coordinates of the source located in space \((x, y, z)\).

To facilitate the calculation, one solves equations in one variable Fourie space by converting space index \((y)\) into the frequency domain \(K_y\) by transformation forward and reverse Fourie equation we have:
-\nabla [\sigma(x,z)\nabla \phi(x,K_z,z)] + K_y^2 \sigma(x,y)\phi(x,K_y,z) = Q \delta(x) \delta(y)

General form of the above equation can be expressed as differential self-adjoint, strongly connected and nonseparable elliptic equation:

\[-\frac{\partial}{\partial x} (P(x,y) \frac{\partial \phi}{\partial x}(x,K_y,z)) - \frac{\partial}{\partial z} (P(x,z) \frac{\partial \phi}{\partial z}(x,K_y,z)) + \sigma(x,z)\phi(x,K_y,z) = f(x,z)\]

\((x,z)\) defined in the domain \(R\) - contained, fully and continuously enclosed by boundary \(\Gamma\) continuous with normal vectors outside; function \(P\) and the function \(f\) is continuous function in the domain by \(R\).

To determine the lower boundary of the half-infinite space with any conductivity distribution, \(R\) is set by the domain boundaries instead of assuming away the infinite horizontal (\(x\)-axis) and vertical (\(z\)-axis). Not limited to the bottom half of this is represented as a discrete grid.

Grid was selected as the rectangle with the edges of any size. The \(x\)-axis grid nodes are denoted by \(i = 1,2,3,\ldots, N\) and the \(z\) axis grid nodes are denoted by \(j = 1,2,3,\ldots, M\). Left side and the right edge of the road network is denoted by \(i = 1\) and \(i = N\). Bottom edges of the mesh are denoted by \(j = M\). The primary source of cause, as well as the secondary due to the heterogeneity induced conductivity in the lower half space is inversely proportional to the distance \((x, y, z)\) and \(K_0(K_y,r)\) in transformation space (here \(K_0\) is the modified Bessel function and \(r\) is the distance). Thus, by choosing large enough values for \(M\) and \(N\) with appropriate discrete grid \((i \to 1 \text{ and } i \to N, \text{ and } j \to M)\) and applying the appropriate boundary conditions, the boundary edge can be modeled infinite interview by choosing the limited number of \(M\) and \(N\). From any node \((i, j)\) which is the characteristic equation developed will be approximately in a region corresponding to an area of \(\Delta A_{ij}\).

As such, always amperage determined; voltage value \(U\) is determined by field measurement devices. As the conductivity distribution function of the environment is what we need to know \(\sigma\) can be determined by solving the above equation.

In the example results obtained by simulation studies of the hollow cave by MES method presented in Section 3 below, we use the Wenner-Schlumberger array (Figure 1). Coefficient devices: \(K = \pi an\), with \(n = 1, 2, 3,\ldots, N\). The first measure point, the first measurements with \(n = 1\) (\(C1P1 = P1P2 = P2C2 = a\)), the second period with \(n = 2\) (\(P1P2 = a; C1P1 = P2C2 = 2a\)); Tuesday measurements with \(n = 3\) (\(P1P2 = a; C1P1 = P2C2 = 3a\)),..., \(n^{th}\) measurements with \(n = N\) (\(P1P2 = a; C1P1 = P2C2 = na\)).

Then we only measured the first translation of a walk in a measure the second point. Measurement process is repeated until the end of the measuring line. If the value obtained in the expansion \(C1C2\) \(P1P2\) small, we can expand \(P1P2 = 2a\).

\[
\begin{array}{cccc}
\downarrow & \downarrow & \downarrow & \downarrow \\
C1 & P1 & P2 & C2 \\
\end{array}
\]

Fig 1. Wenner-Schlumberger array.
2.2. Method of ground penetrating radar

GPR method is an agricultural geophysical method based on the principle of electromagnetic wave transceiver at ultrasonic frequencies (from 15 ÷ 2400 MHz). When the electromagnetic wave propagating in the beams emitted from the transmitter and receiver antennas are created from reflective objects are the boundaries of the geological environment.

The depth of penetration of the method depends on transmitting and receiver antenna frequency and the physical properties of the geological environment in which the value of relative dielectric constant and relatively high power authority ($\varepsilon_r$) and electrical conductivity ($\sigma$) are dominant. The higher the frequency, the conductivity and dielectric constant greater the smaller the depth of the survey.

The physical quantities characteristic of electromagnetic waves are used in GPR method are: wave propagation velocity ($v$), wavelength ($\lambda$), reduction coefficient ($\alpha$), evaluation of the relative power ($\varepsilon$), magnetic permeability ($\mu$), electrical conductivity ($\sigma$)...

The reason GPR method can detect objects above is that in the geological environment in general there always exists the potential dangers such as karst caves, heterogeneous blocks, tropical caves and hollow porous. These are the areas with different physical properties than the surrounding geological environment. At the boundary between the potential dangers and the surrounding environment electromagnetic wave reflection occurs, the reflection coefficient is calculated using the formula:

$$R = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

with: $\varepsilon_1$ - the relative dielectric constant of the first medium and $\varepsilon_2$ - relative dielectric constant of the second medium.

Geological environment is considered as a mixture of three components, namely: earth, water and air. Depending on water levels and air entrapment in the porous layer of the geological environment which more or less affect the electrical conductivity of the medium. Electrical conductivity of a geological environment is difficult to be determined accurately. It depends on the conductivity of the water contained in the pores of the earth. It also depends on the characteristics of components, the architecture, the structure of the earth environment.

Archie’s law describes this relationship as follows:

$$\sigma = a \cdot \Phi^m \cdot S^n \cdot \sigma_w \cdot \sigma_c$$

with: $\Phi$ - porosity of the earth environment (%) $S$ - water saturation, $m$ - empirical constant value from 1.3 to 2.5, $a$ - empirical constant value from 0.4 to 2.0; $n$ - constant with a value of 2; $\sigma_w$ - the water in the hole; $\sigma_c$ - surface conductivity of the earth particle surface geology.

The determination of the electromagnetic parameters to standardize the results measured in GPR method will increase the accuracy of data processing. The parameters needed to be determined are the dielectric constant $\varepsilon$, conductivity $\sigma$, the wave velocity and attenuation $\alpha$.

* Dielectric constant $\varepsilon$

Dielectric constant is a scalar, it indicates the ability of positively charged earth environment when an electromagnetic wave field propagates through. Dielectric constant depends mainly on the amount
of water contained in the geological environment and it takes the value from 1 to 81. The formula for calculating $\varepsilon$ in the earth environment and unsaturated sediments is as follows [6]:

$$\sqrt{\varepsilon} = \Phi (1 - S) \sqrt{\varepsilon_a} + \Phi \cdot S \cdot \sqrt{\varepsilon_w} + (1 - \Phi) \sqrt{\varepsilon_s}$$

with: $\Phi$ - porosity of the earth environment (%); $S$ - water saturation, $\varepsilon$ - dielectric constant of rocks; $\varepsilon_a$ - dielectric constant of air; $\varepsilon_w$ - dielectric constant of water; $\varepsilon_s$ - dielectric constant of particle composition.

*Electrical conductivity $\sigma$

Electrical conductivity is the ability of specific electrical conductivity of the physical environment, it is inversely proportional to the resistivity $\rho$ and depends mainly on the amount of water and clay content in the environment. With most earths, electrical conductivity usually get value from $4 \times 10^{-9}$ S/m. The formula for the electrical conductivity of the not saturated geological environment is as follows:

$$\sqrt{\sigma} = \Phi (1 - S) \sqrt{\sigma_a} + \Phi \cdot S \cdot \sqrt{\sigma_w} + (1 - \Phi) \sqrt{\sigma_s}$$

with: $\Phi$ - porosity of the earth environment (%); $S$ - water saturation, $\sigma$ - conductivity of the sediments; $\sigma_a$ - conductivity of air; $\sigma_w$ - electrical conductivity of the water; $\sigma_s$ - conductivity of particle composition.

*Electromagnetic wave propagation velocity $v$

Electromagnetic wave propagation velocity depends mainly on the value of the dielectric constant of the wave medium and calculated by the following formula:

$$v = \frac{c}{\sqrt{\varepsilon}} = \frac{3 \times 10^8}{\sqrt{\varepsilon}} \text{ (m/s)}$$

*Attenuation of electromagnetic waves $\alpha$

Attenuation of electromagnetic wave is proportional to the quantity of medium conductivity and inversely proportional to the wave and dielectric constant of the medium. Attenuation of electromagnetic waves is calculated by the formula:

$$\alpha = 1.69 \frac{\sigma}{\sqrt{\varepsilon}} \text{ (dB/m)}$$

There are many methods of determining the electromagnetic parameters: geometrical ratio method; common method of deep...

GPR method is based on studies of electromagnetic wave propagation in the ground, especially the reflected waves generated from the surface of the object boundary in the geological environment. With the hazards in the dike, dam can be divided into two basic object types: point format (termite nests, hollow caves, tropical porous, heterogeneous blocks...) and the boundary layer (saturated absorption line, the boundary layer...).

With model, hollow cave is considered as one of the object. When moving the antenna to any position on the survey line, where the line of sight between the transmitting antenna and the object form an angle of approximately $45^\circ$ located in areas with waves reflected from the object, the object to
be probed will be detected. The object’s reflectance probe at a point corresponding to a certain time $t_z$ ($t_z$ - the time that the wave propagation from the antenna to the object). When the antenna closer to the object than the time that the waves propagate from the transmitting antenna to the object and decreasing the time of the wave at the top position of the object is the smallest. Thus the reflected wave signal obtained from the object has the Hyperbole form. Radar waves on schema objects are represented in Figure 2.

![Radar wave diagram on objects.](image)

Based on the above theory, we have calculated and presented some findings in theoretical hollow cave models obtained by GPR method in Section 3 below.

### 3. Some calculation results

#### 3.1. Simulation of the hollow cave by multi-electrode electrical sounding method

Study of physics and geology of dikes and dams reveals that earth resistivity of dikes, dams have greatly fluctuated. The dikes in the Red River Delta with resistivity of 14-40Ωm, the dam resistance from 70-300Ωm.

Hollow cave in body of dikes, dams in practice often have very large resistivity, very small width of several tens of centimeters to several meters, the depth does not look great from the dikes and dams to 3 - 4m depth compared to surface dikes and dams.

**3.1.1. The model of a hollow cave**

From the practic, we choose to conduct calculations and theoretical models in which hollow cave is one-size objects located in a radius of 1m various depths 1m, 2m, 3m and 4m. In the process of calculation we impose environmental noise of 0% (considered as absolute environment) and of 3% (considered as real environment).
Here is an example of the results calculated for the theoretical hollow cave model above with absolute environment and noisy environments.

Results calculated on a theoretical hollow cave model with a depth of objects from the size of the object to 4 times the depth of objects in the environment without interference standards (Figures 3, 5, 7, 9) show that anomalies of hollow cave are always identified. However, when calculating with the environmental assumptions of the actual noise of 3% (Figures 4, 6, 8, 10) with depth is 2 times of the object, the anomaly of the objects is still visible, while increasing depth of 3 times of the object, there is abnormal expression but anomalies were difficult to be identified. As the depth increased to 4 times the size of the object, there is no abnormal expression.

The location of anomalies compared to the position of the object in the theoretical model is completely accurate, but the size of anomaly is much larger than that of the model object even in case where there is no noise environment. So for the small hollow cave object it is very difficult to use MES method to assess their size.
3.1.2. The many hollow caves model

- The model has many hollow caves with the same size located at different depths

In order to study and evaluate the mutual influence of multiple objects with the same size but located at different depths in the same environment, we chose to conduct calculations using the theoretical model with 5 hollow caves in the same size 1m and located at a depth respectively from left to right is 1.5m, 2m, 1m, 4m, and 3m on the 1 measurement line (Figure 11c).

The results of theoretical model with noise environment of 3% with 5 objects the same size (1m) but located in different depths are shown in Figure 11.
Looking at the results we see when there are objects they interact and influence each other, so the object located at a depth of 1m and 1.5m show obvious anomalies, while the objects located in the greater depths exhibit no abnormal expression.

- The model has many hollow caves with different size located at different depths

We chose to conduct calculations using the theoretical model with 5 hollow caves in size respectively from left to right is 1m, 1.5m, 1m, 2.5m and 2m respectively located at a depth of 1.5m, 2m, 1m, 4m and 3m above the 1 measurement line (Figure 12c).

The results of theoretical model with noise environment of 3% with 5 objects of different sizes and located at different depths are shown in Figure 12.
Calculation results show that the objects of model mentioned above show clear relevant anomalies. It suggests that although there are many environmental hollow caves situated near one another, MES method can still identify objects located at a depth greater than 1.5 times corresponding to their size when environmental noise is 3%.

3.2. Simulation of the hollow cave by GPR method

- The model has many hollow caves with the same size located at different depths

We chose to conduct calculations and theoretical models with 5 hollow caves in the same size 1m and located at a depth respectively from left to right is 1.5 m, 2m, 1m, 4m and 3m on the 1 measurement line (like the theoretical model for a MES method - see Figure 13) located in the environment dielectric constant of the medium of 18, resistors 100Ωm environment.

Results calculated using a theoretical model shows that with the GPR method, the objects of the same size of 1 meter in different depth show very clear parabolic anomalies (Figure 14) located in depths coincide with the depths of the corresponding model objects; and treatment by allowing Migration shift (Figure 15) gives us the exact size of the objects. However, the GPR method can not identify the below bottom of the objects that define the peaks and their width.

Fig 13. Theoretical hollow cave model with 5 objects of the same size at different depths used for GPR method

Fig 14. The results of theoretical model with environmental dielectric constant 18 and 5 hollow caves with a dielectric constant of 1, the same size at different depths using 200 MHz antenna.
The model has many hollow caves with different size located at different depths

We chose to conduct calculations and theoretical model with 5 hollow caves in size respectively from left to right is 1m, 1.5m, 1m, 2.5m and 2m respectively located at a depth of 1m, 5m, 2m, 1m, 4m, and 3m on the 1 measurement line (such as theoretical model for MES method - see Figure 16) with a dielectric constant of 18, environmental resistance $100\Omega\text{m}$ and cave is hollow thus we set the air dielectric constant equal to 1.

Fig 16. Theoretical hollow cave model with 5 different size objects at different depths used for GPR method.

Fig 17. The results of theoretical model with environmental dielectric constant 18 and 5 hollow caves with a dielectric constant of 1, different sizes and located at different depths using the 200 MHz antenna.
Fig 18. The results of theoretical model with environmental dielectric constant 18 and 5 hollow caves with a dielectric constant of 1, different sizes and located at different depths using the 200 MHz antenna after treatment by allowing movement of Migration.

Results calculated on a theoretical model shows that GPR method completely determine the location, depth on the surface of the objects of different sizes and located at different depths through the abnormal expression of the parabola (Figure 17). We are able to determine the width of moving objects by allowing Migration (Figure 18). However, this method cannot show the bottom of the object.

4. Conclusion

1. For MES method
   - Multi-electrode sounding method can identify the object of hollow cave at a depth greater than 1.5 times the size of our with natural environment noise of 3% even when hollow caves situated close to each other. And when the environment is only a hollow cave, it is possible to determine the independent hollow cave situated at depths greater than 2 times their size with environmental noise of 3%.
   - Location of anomalies compared to the object's position in the theoretical model is completely accurate, but the size of the anomaly is much larger than the size of the object even in cases where there is no environmental interference. Thus, uncertainty about the size of the object obtained is significant.

2. For GPR method
   - With the same theoretical model, GPR method allows to always detect objects even when MES method does not have the ability.
   - In environments with 100 $\Omega$m resistivity, dielectric constant of 18, GPR method accurately determines earth depth as well as on the size of the object to 4m depth.
   - GPR method cannot determine under the bottom of the object.
3. Thus, the study shows that the application of MES method and GPR method completely effective for the search and detection of objects hidden in body of dikes, dams which are hollow caves. However, the combined application of these methods is needed to in order to be able to accurately determine the depth and size of the object. At the same time, more research is needed to improve methods for more efficient collecting and processing. This is the direction of our future research and results will be published elsewhere.

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