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# Radion Production in $\gamma\mu^-$ Collisions

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**Abstract:** We have calculated the cross – section of the $\gamma\mu^- \to \phi\mu^-$ process in the Randall – Sundrum model, which address the Higgs hierarchy problem in particle physics. Based on the results we have showed that the radion can give observable values at the high energy if the radion mass is in order of GeV.

*Keywords:* Radion, electron, cross – section.

## 1. Introduction

In 1999, the Randall – Sundrum (RS) model was conceived to solve the Higgs hierarchy problem, which the gravity scale and the weak scale can be naturally generated. The RS setup involves two three – branes bounding a slice of 5D compact anti-de Sitter space taken to be on an $S^1/Z_2$ orbifold. Gravity is localized UV brane, while the Standard Model (SM) fields are supposed to be localized IR brane [1]. The Golberger –Wise mechanism is presented to stabilize the radius of the extra dimension without reintroducing a fine tuning. Fluctuations about the stabilized RS model include both tensor and scalar modes. The fluctuations of the size of the extra dimension, characterized by the scalar component of the metric otherwise known as the radion [1].

The radion may turn out to be the lightest new particle in the RS model. The phenomenological similarity and potential mixing of the radion and Higgs boson warrant detailed to distinguish between the radion and Higgs signals at colliders. The radion can be produced in the $e^+e^-$ and $\gamma\gamma$ colliders which we have calculated in last paper. In this paper, we study the production of radion in $\gamma\mu^-$ colliders, our results can compare with results of [2]. This paper is organized as follows. In Sec.2, we give a review of the RS model. Section III is devoted to the creation of radion in high energy $\gamma\mu^-$ colliders. Finally, we summarize our results and make conclusions in Sec. IV.

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## 2. A review of Randall – Sundrum model

The RS model is based on a 5D spacetime with non – factorizable geometry [3]. The single extradimension is compactified on an $S^1/Z_2$ orbifold of which two fixed points accommodate two three – branes (4D hyper – surfaces): the Planck brane and TeV brane.

The fundamental action is the sum of the Hilbert – Einstein action $S_H$ and a matter part $S_M$:

$$S = S_H + S_M = \int d^4x \int_{-L}^{L} dy \sqrt{-g}\left(M^3 R - \Lambda\right) , \tag{1}$$

where M is the fundamental 5D mass scale, R is the 5D Ricci scalar and g is the determinant of the metric, $\Lambda$ is the 5D cosmological constant [4].

The background metric reads:

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 , \tag{2}$$

where $x^\mu$ $(\mu = 0, 1, 2, 3)$ denote the coordinates on the 4D hyper – surfaces of constant y with metric $\eta_{\mu\nu} = diag(-1, 1, 1, 1)$.

To determine the function σ(y), we calculate the 5D Einstein equations:

$$G_{MN} = R_{MN} - \frac{1}{2} g_{MN} R , \tag{3}$$

where

$$R = 8\sigma''(y) - 20\sigma'^2(y). \tag{4}$$

The 55 component of the Einstein equation gives:

$$G_{55} = 6\sigma'^2 = \frac{-\Lambda}{2M^3} . \tag{5}$$

From that equation, we show that $\sigma'^2$ is equal to a constant called $k^2$:

$$\sigma'^2 = \frac{-\Lambda}{12M^3} \equiv k^2 . \tag{6}$$

With respect to orbifold symmetry, we choose: $\sigma(y) = k\,|\,y\,|$. (7)

Therefore, the background metric in the Randall – Sundrum model is parameterized by:

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 , \tag{8}$$

with $-L \le y \le L$.

The Higgs action can be shown as

$$S_H = \int d^4x \left[ \eta^{\mu\nu} D_\mu \tilde{H}^+ D_\nu \tilde{H} - \lambda\left(\tilde{H}^+\tilde{H} - \left(e^{-kL}v_0\right)^2\right)^2 \right] , \tag{9}$$

where $v_0$ is a mass parameter, the Higgs field $H = e^{kL}\tilde{H}$.

The vacuum expectation value is an exponential function:

$$v \equiv e^{-kL} v_0 . \tag{10}$$

The physical Higgs mass can be written:

$$m \equiv e^{-kL} m_0 , \tag{11}$$

where $m_0$ is of Planck scale. If the value of $m_0$ is of the order of $10^{19}$ GeV, $m \simeq 1$ TeV.

Since the scale of weak interactions $M_W \simeq 10^{-16} M_{Pl}$, the applicable value for size of the extra dimension is assessed by

$$kL \simeq \ln 10^{16} \simeq 35 . \tag{12}$$

Consequently, the hierarchy problem is addressed.

## 3. Radion production in γμ⁻ collisions

In this section, we consider the process collision in which the initial state contains a photon and a muon, the final state contains a pair of muon and radion.

### *3.1. Radion production in $\gamma\mu^-$ collisions as unpolarized $\mu^-$ beams*

The Feynman diagram of the process collision is:

$$\mu^- (p_1) + \gamma(p_2) \to \mu^- (k_1) + \phi(k_2) , \tag{13}$$

here $p_i, k_i$ $(i=1,2)$ stand for the momentum.

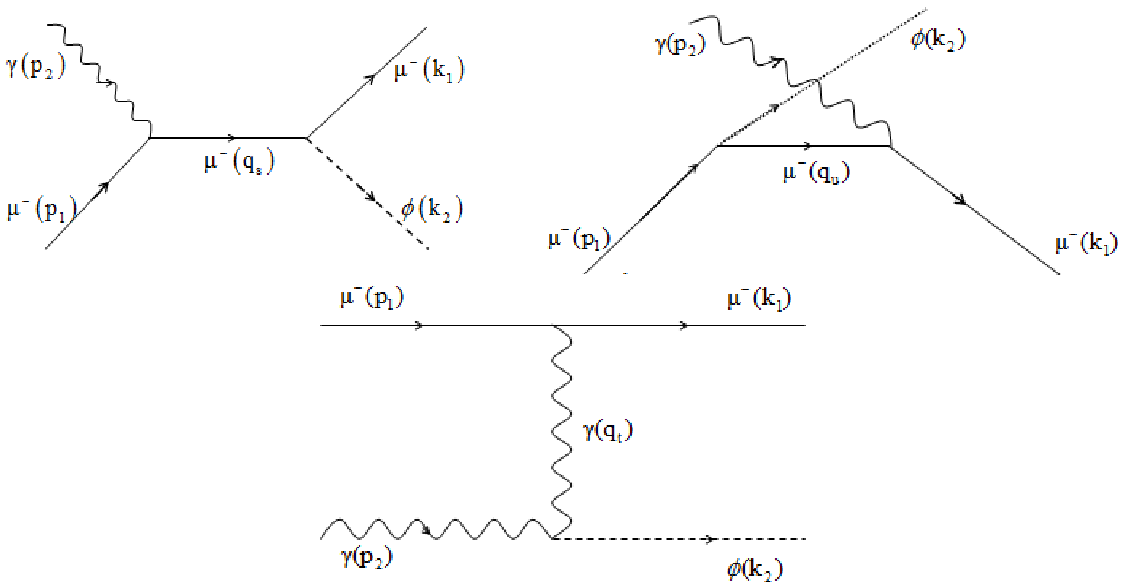


Figure 1. Feynman diagrams for γμ⁻ collisions.

We have amplitude squared of this collisions

$$M^2 = M_s^2 + M_u^2 + M_t^2 + 2\mathrm{Re}(M_s^+ M_u + M_s^+ M_t + M_u^+ M_t), \qquad (14)$$

where

$$M_s = -\frac{ie}{\Lambda'_\phi\left(q_s^2 - m_\mu^2\right)} \varepsilon_\mu(p_2)\bar{u}(k_1)\left(\hat{q}_s + m_\mu\right)\gamma^\mu u(p_1) \;, \qquad (15)$$

$$M_u = -\frac{ie}{\Lambda'_\phi(q_u^2 - m_\mu^2)} \varepsilon_\mu(p_2)\bar{u}(k_1)\gamma^\mu\left(\hat{q}_u + m_\mu\right)u(p_1), \qquad (16)$$

$$M_t = \frac{4e}{\Lambda_\gamma q_t^2}\left[(p_2 q_t)g_\alpha^\nu - p_2^\nu q_{t\alpha}\right]\bar{u}(k_1)\gamma_\nu u(p_1)\varepsilon^\alpha(p_2). \qquad (17)$$

*3.2. Radion production in $\gamma\mu^-$ collisions as polarized $\mu$ beams*

In this section, we calculate the cross – section in $\gamma\mu^-$ collision when $\mu^-$ beams are polarized. Let us consider the following cases:

a) In s – channel, we consider the process collision in which the initial state contains the left – handed $\mu^-$, photon and the final state contains the right – handed $\mu^-$, radion and vice versa. The transition amplitude for this process can be written as:

$$M_{sLR} = -\frac{ie}{\Lambda'_\phi\left(q_s^2 + m_\mu^2\right)} \varepsilon_\mu(p_2)\bar{u}(k_1)\frac{1-\gamma_5}{2}\hat{q}_s\gamma^\mu u(p_1), \qquad (18)$$

$$M_{sRL} = -\frac{ie}{\Lambda'_\phi\left(q_s^2 + m_\mu^2\right)} \varepsilon_\mu(p_2)\bar{u}(k_1)\frac{1+\gamma_5}{2}\hat{q}_s\gamma^\mu u(p_1). \qquad (19)$$

b) In a similar way, we consider the process in u – channel. The transition amplitude for this process can be written as:

$$M_{uLR} = -\frac{ie}{\Lambda'_\phi\left(q_u^2 + m_\mu^2\right)} \varepsilon_\mu(p_2)\bar{u}(k_1)\frac{1-\gamma_5}{2}\gamma^\mu\hat{q}_u u(p_1) \;, \qquad (20)$$

$$M_{uRL} = -\frac{ie}{\Lambda'_\phi\left(q_u^2 + m_\mu^2\right)} \varepsilon_\mu(p_2)\bar{u}(k_1)\frac{1+\gamma_5}{2}\gamma^\mu\hat{q}_u u(p_1) \;. \qquad (21)$$

c) In t – channel, the initial left – handed $\mu^-$ beams produce the photon with the momentum $q_t$ and the final left – handed $\mu^-$ beams. The photon in the intial state collide the photon with the momentum $q_t$ produce the radion in the final state. The transition amplitude for this process is given by:

$$M_{tLL} = \frac{-4ie}{\Lambda_\gamma q_t^2}[(p_2 q_t)g_\alpha^\nu - p_2^\nu q_{t\alpha}]\varepsilon^\alpha(p_2)\bar{u}(k_1)\frac{1+\gamma_5}{2}\gamma_\nu u(p_1). \qquad (22)$$

The initial right – handed $\mu^-$ beams produce the photon with the momentum $q_t$ and the final right – handed $\mu^-$ beams. The photon in the intial state collide the photon with the momentum $q_t$ produce the radion in the final state. The transition amplitude for this process is given by:

$$M_{tRR} = \frac{-4ie}{\Lambda_\gamma q_t^2}[(p_2q_t)g_\alpha^\nu - p_2^\nu q_{t\alpha}]\varepsilon^\alpha(p_2)\bar{u}(k_1)\frac{1-\gamma_5}{2}\gamma_\nu u(p_1) . \quad (23)$$

d) In the s, u channel interference, the transition amplitude of the process in which the initial state contains the left – handed $\mu^-$ and the final state contains the right – handed $\mu^-$ can be given by:

$$M_{sLR}^+ M_{uLR} = \frac{4e}{\Lambda_\phi^{'2}\left(q_s^2 + m_\mu^2\right)\left(q_u^2 + m_\mu^2\right)}(k_1q_s)(p_1q_u) . \quad (24)$$

The transition amplitude for the process in which the initial state contains the right –handed $\mu^-$ and the final state contains the left – handed $\mu^-$ can be written as:

$$M_{sRL}^+ M_{uRL} = \frac{4e}{\Lambda_\phi^{'2}\left(q_s^2 + m_\mu^2\right)\left(q_u^2 - m_\mu^2\right)}(k_1q_s)(p_1q_u). \quad (25)$$

*3.3. The cross-section of the $\gamma\mu^- \to \phi\mu^-$ process*

From the expressions of the differential cross – section and the total cross – section:

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{32\pi s}\frac{|\vec{k}|}{|\vec{p}|}|M|^2 ,$$

where M is the scattering amplitude, we assess the number and make the identification, evaluation of the results obtained from the dependence of the differential cross – section by $\cos\theta$ , the total cross – section fully follows $\sqrt{s}$ and the polarization factors of $\mu^-$ beams $(P_1, P_2)$ .

In the SI unit, we choose $m_\mu = 0.1058\,\text{GeV}$, $\Lambda_\phi = 5.10^3\,\text{GeV}$, $\Lambda_\gamma = 308250\pi\,\text{GeV}$ to estimate for the cross – section as follows:

i) In Fig.2, we plot the differential cross – section as a function of the $\cos\theta$ . We have chosen a relatively low value of the radion mass $m_\phi = 10\,\text{GeV}$ and the collision energy $\sqrt{s} = 3\,\text{TeV}$ [5]. Typical polarization coefficients $P_1 = P_2 = 1, 0.5, 0$ are shown by the first, second, third line, respectively. The figure shows that the differential cross – section increases when the $\cos\theta$ increases from – 1 to 1. When $\cos\theta \simeq 1$ , the differential cross – section reaches to the maximum value. This is the advantage to collect radion from experiment.

ii) In Fig.3, we plot the differential cross – section as a function of the $\cos\theta$ with typical polarization coefficients $P_1 = -1, P_2 = 1, m_\phi = 10\,\text{GeV}$, $\sqrt{s} = 3\,\text{TeV}$. The figure shows that the differential cross – section decreases as $-1 < \cos\theta < 1$ .

iii) When the $\mu^-$ beams in the initial and final state are polarized, the total cross – section which depends on typical polarization coefficients $P_1, P_2$ is shown in Fig.4. The total cross – section achieves the maximum value in case of $P_1 = P_2 = -1$ or $P_1 = P_2 = 1$ and the minimum value in case of $P_1 = -1,\ P_2 = 1$ or $P_1 = 1,\ P_2 = -1$.

iv) In Fig.5, we plot the total cross – section as a function of the collision energy $\sqrt{s}$ in the cases $P_1, P_2$ similar to Fig.2. We show that the total cross – section is approximately independent on $\sqrt{s}$ when $\sqrt{s} > 500$ GeV. Therefore, it is difficult to collect radion at very high energies.

v) We plot the total cross – section as a function of the collision energy $\sqrt{s}$ with $P_1 = -1,\ P_2 = 1$ in Fig.6. The figure shows that the total cross – section decreases as $1\ \text{TeV} < \sqrt{s} < 5\ \text{TeV}$.

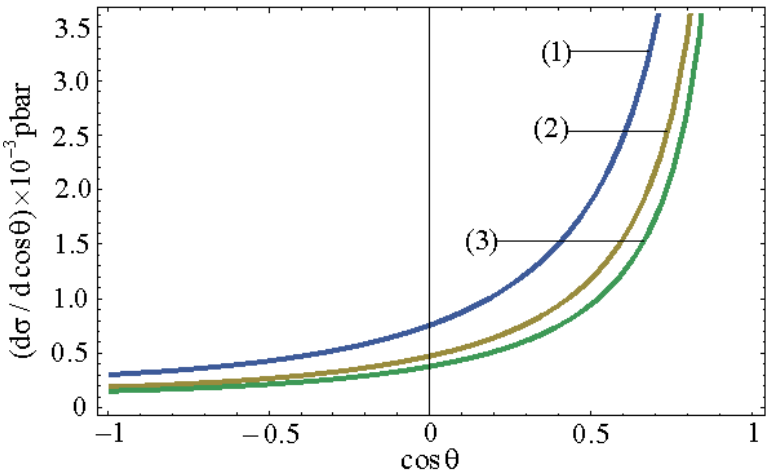


Figure 2. Cross – section as a function of $\cos\theta$. Typical polarization coefficients are chosen as $P_1 = P_2 = 1,\ 0.5,\ 0$ respectively and $m_\phi = 10$ GeV.

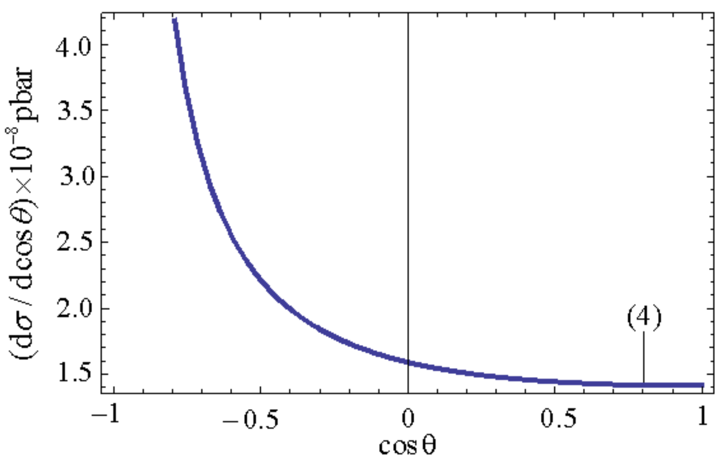


Figure 3. Cross – section as a function of $\cos\theta$ with $P_1 = -1,\ P_2 = 1$.

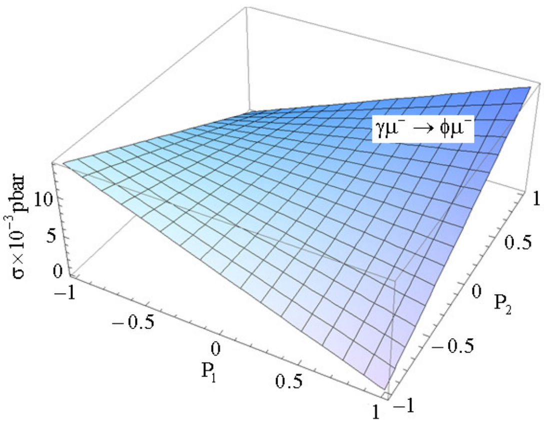


Figure 4. The total cross – section as a function of the polarization coefficients $P_1$, $P_2$.

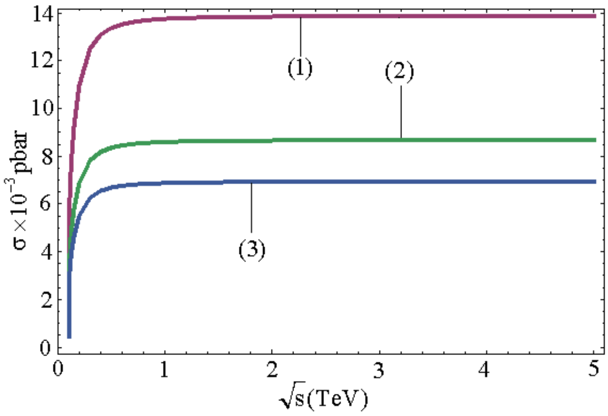


Figure 5. The total cross – section as a function of the collision energy $\sqrt{s}$ . Typical polarization coefficients are chosen as $P_1 = P_2 = 1,\ 0.5,\ 0$ respectively and $m_\phi = 10$ GeV.

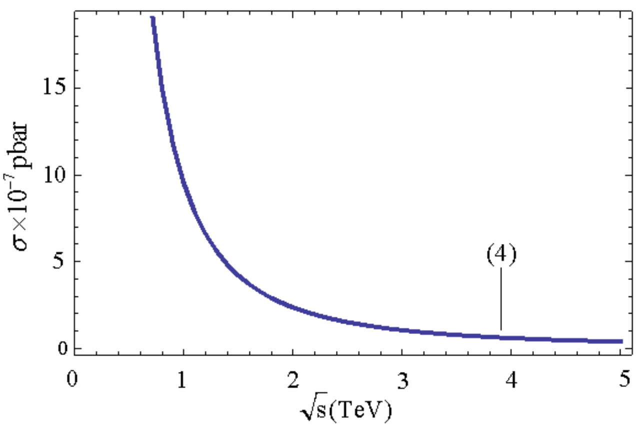


Figure 6. The total cross – section as a function of the collision energy $\sqrt{s}$ with $P_1 = -1, P_2 = 1$ .

## 4. Conclusion

In this work, the radion production in $\gamma\mu^-$ collisions are evaluated in detail. The result has shown that cross sections depend strongly on the polarization factors of $\mu^-$ beams $(P_1, P_2)$. In the region high energy, the total scattering cross section does not depend on the collision energy $\sqrt{s}$. However, the total scattering cross section is very small and is much smaller than that in $\gamma e^-$ collisions [2], (about 3.5 times). Therefore, the possibility to observe radion from laboratory is very difficult.

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