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Effects of elastic foundation and the Poisson's ratio $v = v(z)$ on the nonlinear buckling and postbuckling behaviors of imperfect FGM plates subjected to mechanical loads

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Abstract: This paper presents an analytical approach to investigate effects of elastic foundation and the Poisson's ratio $v = v(z)$ on the nonlinear buckling behavior of imperfect FGM plates, subjected to mechanical loads. Material properties are assumed to be temperature independent, and graded in the thickness direction according to a power law distribution in terms of volume fractions of constituents. Equilibrium and compatibility equations are derived by using classical plate theory taking into account geometrical nonlinearity, initial geometrical imperfection and elastic foundation with Pasternak model. Galerkin method is used to determine explicit expressions of buckling loads and postbuckling paths. Analysis is carried out to assess the effects of material, geometrical, elastic foundation parameters on the stability of FGM plates.

Keywords: Buckling and postbuckling, Functionally graded material, Plate, Elastic foundations, Poisson's ratio $v = v(z)$.

1. Introduction

Due to high performance is heat resistance capacity and excellent characteristics in comparison with conventional composites, Functionally Graded Materials (FGMs) which are microscopically composites and composed from mixture of metal and ceramic constituents have attracted considerable attention recent years. By continuously and gradually varying the volume fraction of constituent materials through a specific direction, FGMs are capable of withstanding ultrahigh temperature environments and extremely large thermal gradients. Therefore, these novel materials are chosen to use in structure components of aircraft, aerospace vehicles, nuclear plants as well as various temperature shielding structures widely used in industries. Buckling and postbuckling behaviors of FGM structures under different types of loading are important for practical applications and have received considerable interest. Eslami and his co-workers used analytical approach, classical and higher order plate theories in conjunction with adjacent equilibrium criterion to investigate the

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buckling of FGM plates with and without imperfection under mechanical and thermal loads [3,8,9]. According to this direction, Lanhe [10] also employed first order shear deformation theory to obtain closed-form relations of critical buckling temperatures for simply supported FGM plates. Zhao et al. [14] analyzed the mechanical and thermal buckling of FGM plates using element-free Ritz method. Liew et al. [15,16] used the higher order shear deformation theory in conjunction with differential quadrature method to investigate the postbuckling of pure and hybrid FGM plates with and without imperfection on the point of view that buckling only occurs for fully clamped FGM plates. The postbuckling behavior of pure and hybrid FGM plates under the combination of various loads were also treated by Shen [17,18] using two-step perturbation technique taking temperature dependence of material properties into consideration. Recently, Lee et al. [19] made of use element-free Ritz method to analyze the postbuckling of FGM plates subjected to compressive and thermal loads.

The components of structures widely used in aircraft, reusable space transportation vehicles and civil engineering are usually supported by an elastic foundation. Therefore, it is necessary to account for effects of elastic foundation for a better understanding of the postbuckling behavior of plates and shells. Librescu and Lin have extended previous works [20] to consider the postbuckling behavior of flat and curved laminated composite panels resting on Winkler elastic foundations [20]. In spite of practical importance and increasing use of FGM structures, investigation on FGM plates and shells supported by elastic media are limited in number. The bending behavior of FGM plates resting on Pasternak type foundation has been studied by Huang et al. [21] using state space method, Zenkour [22] using analytical method and by Shen and Wang [23] making use of asymptotic perturbation technique. To the best of authors' knowledge, there is no analytical studies have been reported in the literature on the postbuckling of thick FGM plates resting on elastic foundations. In [11], the authors Dao Van Dung and Nguyen Thi Nga have studied the stability of the composite FGM plate when $\nu = \nu(z)$ and $E = E(z)$ (without elastic foundation). In [12], the author Do Nam has studied the stability of the FGM plate on the elastic foundation with classical plate theory, in [6] the authors Nguyen Dinh Duc and Hoang Van Tung have studied postbuckling of the high order shear deformable FGM plates on elastic foundation, but these studies assume $E = E(z)$ and $\nu = const$.

The aim of the paper proposed is of studying the nonlinear stability of FGM plate on the elastic foundation under the effect of the load in the case both elastic modules are variable $\nu = \nu(z)$ and $E = E(z)$, the study of the effect of the initial imperfect shape, proportion metal-ceramic, and the elastic foundation parameters and the geometric parameters on the nonlinear stability of the FGM plate

2. FGM plates on elastic foundations

Consider a rectangular functionally graded plate of length a, width b and thickness h. An orthogonal coordinate system $Oxyz$ is choose so that the plane coincides Oxy with the middle surface of the plate and the axis Oz is in the thickness direction $(-h/2 \le z \le h/2)$ as shown in Fig. 1.

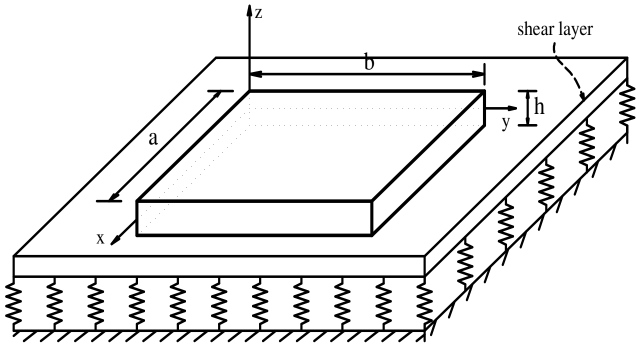

Fig.1.Geometry and coordinatr system of an FGM plate on elastic foundation

The volume fractions of constituents are assumed to vary through the thickness according to the following power law distribution

$$V_m(z) = \left(\frac{2z+h}{2h}\right)^N \ , \ V_c(z) = 1 - V_m(z) \qquad (1)$$

where N is volume fraction index ($0 \le N < \infty$). Effective properties $\Pr_{eff}$ of FGM panel are determined by linear rule of mixture as

$$\Pr_{eff}(z) = \Pr_m V_m(z) + \Pr_c V_c(z) \quad (2)$$

where $\Pr$ denotes a temperature independent material property, and subscripts m and c stand for the metal and ceramic constituents, respectively.

Specific expressions of modulus of elasticity E, Poisson ratio ν the coefficient of thermal expansion α are obtained by substituting Eq. (1) into Eq. (2) as [11]

$$E = E(z) = E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^N = E_m + E_{cm} r^N$$

$$\nu = \nu(z) = \nu_m + (\nu_c - \nu_m)\left(\frac{2z+h}{2h}\right)^{N_1} = \nu_m + \nu_{cm} r^{N_1} \qquad (3)$$

Where

$$E_{cm} = E_c - E_m; r = \frac{2z+h}{2h}; \nu_{cm} = \nu_c - \nu_m; N \ge 0; N_1 \ge 0 \qquad (4)$$

It is evident from Eqs. (3), (4) that the upper surface of the plate ($z = -h/2$) is ceramic-rich, while the lower surface ($z = h/2$) is metal-rich, and the percentage of ceramic constituent in the panel is enhanced when N increases.

The plate – foundation interaction is represented by Pasternak model as [7]

$$q_e = k_1 w - k_2 \nabla^2 w \qquad (5)$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, w is the deflection of the plate, k_1 is Winkler foundation modulus and k_2 is the shear layer foundation stiffness of Pasternak model.

2.1. Theoretical formulation

For imperfect plates, the strain components on the reference surface with the geometrical nonlinearity in von Karman sense, are [1]

$$\begin{aligned}
\varepsilon_{0x} &= u_{,x} + \left(\mathrm{w}_{,x}\right)^2/2 + \mathrm{w}_{,x}\mathrm{w}^*_{,x};\\
\varepsilon_{0y} &= v_{,y} + \left(\mathrm{w}_{,y}\right)^2/2 + \mathrm{w}_{,y}\mathrm{w}^*_{,y};\\
\gamma_{0xy} &= u_{,y} + v_{,x} + \mathrm{w}_{,x}\mathrm{w}_{,y} + \mathrm{w}_{,y}\mathrm{w}^*_{,x} + \mathrm{w}_{,x}\mathrm{w}^*_{,y}
\end{aligned} \tag{6}$$

Where $u = u(x, y), v = v(x, y)$ and $w = w(x, y)$ are displacements along x, y and z respectively; $w^* = w^*(x, y)$ denotes an initial imperfections of plate. The quantity w^* is assumed small.

The strains across the plate thickness at a distance z from the mid-plane are [1]

$$\begin{aligned}
&\varepsilon_x = \varepsilon_{0x} + zk_x; \varepsilon_y = \varepsilon_{0y} + zk_y; \gamma_{xy} = \gamma_{0xy} + 2zk_{xy}\\
&k_x = -\mathrm{w}_{,xx}, k_y = -\mathrm{w}_{,yy}, k_{xy} = -\mathrm{w}_{,xy}
\end{aligned} \tag{7}$$

Hooke law for an FGM plate is defined as

$$\begin{aligned}
&(\sigma_x, \sigma_y) = \frac{E}{1-v^2}\left[(\varepsilon_x, \varepsilon_y) + v(\varepsilon_y, \varepsilon_x) - (1+v)\alpha\Delta T(1,1)\right]\\
&\sigma_{xy} = \frac{E}{2(1+v)}\gamma_{xy}
\end{aligned} \tag{8}$$

where we assume that the plate is subjected to a uniform temperature rise i.e. ΔT is a constant. So, the force and moment resultants are expressed a

$$\begin{aligned}
&(N_x, N_{,y}, N_{,xy}) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \sigma_{xy}\right) dz\\
&(M_x, M_y, M_{xy}) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \sigma_{xy}\right) z dz
\end{aligned} \tag{9}$$

Substituting relations (3), (6) $\div$ (8) into (9), after series of calculations, we have

$$\begin{aligned}
&\left(N_x, N_y\right) = \left(J_{10}, J_{20}\right)\varepsilon_{0x} + (J_{20}, J_{10})\varepsilon_{0y} + (J_{11}, J_{21})k_x + (J_{21}, J_{11})k_y + \left(1,1\right)\phi_1\Delta T\\
&N_{xy} = J_{30}\gamma_{0xy} + 2J_{31}k_{xy}\\
&\left(M_x, M_y\right) = (J_{11}, J_{21})\varepsilon_{0x} + (J_{21}, J_{11})\varepsilon_{0y} + (J_{12}, J_{22})k_x + (J_{22}, J_{12})k_y + \left(1,1\right)\phi_2\Delta T\\
&M_{xy} = J_{31}\gamma_{0xy} + 2J_{32}k_{xy}
\end{aligned} \tag{10}$$

$J_{\mathrm{ij}}(i=1,2,3;\ j=0,1,2), \phi_1, \phi_2$

defined as follows

$$J_{1j} = \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu^2(z)} z^j dz;$$

$$J_{2j} = \int_{-h/2}^{h/2} \frac{E(z)\nu(z)}{1-\nu^2(z)} z^j dz;$$

$$J_{3j} = \frac{1}{2}\int_{-h/2}^{h/2} \frac{E(z)}{1+\nu(z)} z^j dz = \frac{1}{2}(J_{1j} - J_{2j}) \qquad (11)$$

$$\phi_1 = -\int_{-h/2}^{h/2} \frac{E(z)\alpha(z)}{1-\nu(z)} dz;$$

$$\phi_2 = -\int_{-h/2}^{h/2} \frac{E(z)\alpha(z)}{1-\nu(z)} z dz$$

The equilibrium equations of a imperfect plate on elastic foundations are in the form [2÷4]:

$$N_{x,x} + N_{xy,y} = 0$$

$$N_{xy,x} + N_{y,y} = 0$$

$$M_{x,xx} + 2M_{xy,xy} + M_{y,yy} + N_x(w_{,xx} + w^*_{,xx})$$

$$+2N_{xy}(w_{,xy} + w^*_{,xy}) + N_y(w_{,yy} + w^*_{,yy}) - k_1 \mathrm{w} + k_2 \nabla^2 \mathrm{w} = 0 \qquad (12)$$

We introduce Airy's stress function $f = f(x, y)$ so that

$$N_x = f_{,yy}; N_y = f_{,xx}; N_{xy} = -f_{,xy} \qquad (13)$$

It is easy to see that the first two equations in (12) are automatically satisfied.

Substituting relations (13) in Eqs. (10), we obtain

$$\varepsilon_{0x} = I_1(J_{10} f_{,yy} - J_{20} f_{,xx} + I_2 \mathrm{w}_{,xx} + I_3 \mathrm{w}_{,yy} - I_4 \phi_1 \Delta T)$$

$$\varepsilon_{0y} = I_1(J_{10} f_{,xx} - J_{20} f_{,yy} + I_2 \mathrm{w}_{,yy} + I_3 \mathrm{w}_{,xx} - I_4 \phi_1 \Delta T)$$

$$\gamma_{0xy} = (2J_{31} \mathrm{w}_{,xy} - f_{,xy}) / J_{30} \qquad (13)$$

where

$$I_1 = 1/(J_{10}^2 - J_{20}^2), I_2 = J_{10}J_{11} - J_{20}J_{21}$$

$$I_3 = J_{10}J_{21} - J_{20}J_{11}, I_4 = J_{10} - J_{20} \qquad (15)$$

Substituting once again the expressions of Eq. (14) into the relations of internal moments M_{ij} in (10) we obtain

$$M_x = I_1 f_{,xx}(-J_{11}J_{20} + J_{21}J_{10}) + I_1 f_{,yy}(J_{11}J_{10} - J_{21}J_{20}) + I_1 \mathrm{w}_{,xx}(J_{11}I_2 + J_{21}I_3) + I_1 \mathrm{w}_{,yy}(J_{11}I_3 + J_{21}I_2) - J_{12}\mathrm{w}_{,xx} - J_{22}\mathrm{w}_{yy}$$

$$M_y = I_1 f_{,xx}(-J_{21}J_{20} + J_{11}J_{10}) + I_1 f_{,yy}(J_{21}J_{10} - J_{11}J_{20}) + I_1 \mathrm{w}_{,xx}(J_{21}I_2 + J_{11}I_3) + I_1 \mathrm{w}_{,yy}(J_{21}I_3 + J_{11}I_2) - J_{22}\mathrm{w}_{,xx} - J_{12}\mathrm{w}_{yy}$$

$$M_{xy} = \frac{J_{31}}{J_{30}}\left(2J_{31}\mathrm{w}_{,xy} - f_{,xy}\right) - 2J_{32}\mathrm{w}_{,xy} \tag{16}$$

The substituting (16) into the third equation of (12) we have

$$A_1\nabla^4 f + A_2\nabla^4\mathrm{w} + f_{,yy}(\mathrm{w}_{,xx} + \mathrm{w}^*_{,xx}) - 2f_{,xy}(\mathrm{w}_{,xy} + \mathrm{w}^*_{,xy}) + f_{,xx}(\mathrm{w}_{,yy} + \mathrm{w}^*_{,yy}) - k_1\mathrm{w} + k_2\nabla^2\mathrm{w} = 0 \tag{17}$$

where

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 y^2} + \frac{\partial^4}{\partial y^4}$$

$$A_1 = I_1 I_3;\, A_2 = I_1(J_{11}I_2 + J_{21}I_3) - J_{12}$$

The equation (17) includes two unknowns functions w and f , so it is necessary to find a second equation relating to these two unknowns functions by using the compatible equation:

$$\varepsilon_{0x,yy} + \varepsilon_{0y,xx} - \gamma_{0xy,xy} = (\mathrm{w}_{,xy})^2 - \mathrm{w}_{,xx}\mathrm{w}_{,yy} \tag{18}$$

The substituting the above expressions of ε_{ij} in (14) into Eqs. (18), we obtain

$$\nabla^4 f + A_3\nabla^4\mathrm{w} - A_4\left[\left(\mathrm{w}_{,xy}\right)^2 - \mathrm{w}_{,xx}\mathrm{w}_{,yy} + 2w_{,xy}\mathrm{w}^*_{,xy} - \mathrm{w}_{,xx}\mathrm{w}^*_{yy} - \mathrm{w}_{,yy}\mathrm{w}^*_{,xx}\right] = 0 \tag{19}$$

in wich

$$A_3 = \frac{I_3}{J_{10}};\, A_4 = \frac{1}{I_1 J_{10}}$$

The couple of Eqs. (17) and (19) are the governing equations used to investigate the nonlinear stability of imperfect FGM plates with the Poisson's Ratio $\nu = \nu(z)$ resting on elastic foundation.

In the case $\mathrm{w}^* = 0$, from (17) and (19) we obtain the basic stability equations for perfect FGM plates.

2.2. Boundary conditions and the solution of the problem

Suppose that three cases boundary conditions for a rectangular plate will be considered follow as [5, 6]:

Case 1. The edges of plate are simply supported and freely movable (FM). The associated boundary conditions are:

$$\begin{aligned} &\mathrm{x} = 0, x = a; \mathrm{w} = M_x = N_{xy} = 0; N_x = N_{x0} \\ &\mathrm{y} = 0, y = b;\ \mathrm{w} = M_y = N_{xy} = 0; N_y = N_{y0} \end{aligned} \tag{20}$$

Case 2. The edges of plate are simply supported and immovable (IM). The associated boundary conditions are:

$$\begin{aligned} &\mathrm{x}=0, x=a; \mathrm{w}=M_x=u=0; N_x=N_{x0} \\ &\mathrm{y}=0, y=b; \mathrm{w}=M_y=v=0; N_y=N_{y0} \end{aligned} \tag{21}$$

Case 3. The edges of plate are simply supported. Uniaxial edge loads are applied in the direction of the x -coordinate. The edges $x=0$, $x=a$ are considered freely movable, the remaining two edges being unloaded and immovable. The boundary conditions, for this case, are

$$\begin{aligned} &\mathrm{x}=0, x=a; \mathrm{w}=M_x=N_{xy}=0; N_x=N_{x0} \\ &\mathrm{y}=0, y=b; \mathrm{w}=M_y=v=0; N_y=N_{y0} \end{aligned} \tag{22}$$

where u, v are the displacement components x, y directions, respectively, N_{xy}, M_x, M_y are force and moment resultants. Moreover, N_{x0} , N_{y0} are prebuckling force resultants in the direction x and y respectively, for Case 1 and the first of Case 3 or are fictitious compressive edge loads at immovable edges (Case 2 and the second of Case 3).

Approximate solutions of basic equations (17) and (19) are assumed as [5, 6]:

$$\begin{aligned} &\mathrm{w}=\mathrm{W}\sin\lambda_m x\sin\delta_n y \\ &\mathrm{w}^*=\mu h\sin\lambda_m x\sin\delta_n y \\ &f=C_1\cos 2\lambda_m x+C_2\cos 2\delta_n y+C_3\sin\lambda_m x\sin\delta_n y+ \\ &C_4\cos 2\lambda_m x\cos 2\delta_n y+\frac{1}{2}N_{x0}y^2+\frac{1}{2}N_{y0}x^2 \end{aligned} \tag{23}$$

which fulfill exactly the out-of-plane boundary conditions and satisfy in-plane boundary conditions in an average sense. Moreover, $\lambda_m=m\pi/a; \delta_n=n\pi/b; m,n=1,2,...$ W is amplitude of deflection and μ is imperfection parameter $(0\le\mu\le 1)$.

By substituting expression (23) into Eq. (19), we obtain

$$\begin{aligned} &C_1=\frac{A_4\mathrm{W}\delta_n^2(\mathrm{W}+2\mu h)}{32\lambda_m^2} \\ &C_2=\frac{A_4\mathrm{W}\lambda_m^2(\mathrm{W}+2\mu h)}{32\delta_n^2} \\ &C_3=-A_3\mathrm{W} \\ &C_4=0 \end{aligned} \tag{24}$$

Introducing Eqs. (24) and (23) into the left side of Eq. (17) and the applying Galerkin method we obtain equation for determining buckling loads and postbuckling curves of rectangular FGM plates subjected to mechanical, thermal and combined loads.

$$\left[\left(A_2 - A_1A_3\right)\left(\lambda_m^2 + \delta_n^2\right)^2 - k_1 - k_2(\lambda_m^2 + \delta_n^2)\right]\mathrm{W} -$$
$$(N_{x0}\lambda_m^2 + N_{y0}\delta_n^2)(\mathrm{W} + \mu h) - \frac{32}{3}\frac{A_3\lambda_m\delta_n}{ab}\mathrm{W}(\mathrm{W} + \mu h) \tag{25}$$
$$-\frac{16}{3}\frac{A_1A_4\lambda_m^2\delta_n^2}{mn\pi^2}\mathrm{W}(\mathrm{W} + 2\mu h) - \frac{A_4(\lambda_m^4 + \delta_n^4)}{8}\mathrm{W}(\mathrm{W} + \mu h)(\mathrm{W} + 2\mu h) = 0$$

where m, n are odd numbers. This equation will be used to analyze the buckling and postbuckling behaviors of thick FGM plates under mechanical, thermal and thermomechanical loads on elastic foundation.

3. Nonlinear stability analysis FGM plate on elastic foundation

3.1. Mechanical stability analysis

Consider a rectangular imperfect FGM plate being simply supported at its edges and freely movable (Case. 1) and subjected to the in-plane compressive only loads P_x uniformly distributed along the edges $x = 0, a$. In this case, the prebuckling force resultants are given [7]

$$N_{x0} = -P_x h, N_{y0} = 0 \tag{26}$$

Substituting this expression (26) into Eq. (25) we receive :

$$P_x = \left[\frac{\overline{D}\pi^2(B_a^2m^2 + n^2)^2}{B_a^2B_h^2m^2} + \frac{K_1\overline{D}B_a^2}{B_h^2m^2\pi^2} + \frac{K_2\overline{D}(m^2B_a^2 + n^2)}{B_h^2m^2}\right]\frac{\overline{\mathrm{W}}}{\overline{\mathrm{W}}+\mu} + \frac{32}{3}\frac{\overline{A_3}\overline{\mathrm{W}}n}{B_h^2m} + \frac{16}{3}\frac{\overline{A_1}\overline{A_4}n\overline{\mathrm{W}}(\overline{\mathrm{W}}+2\mu)}{B_h^2m(\overline{\mathrm{W}}+\mu)}$$
$$+\frac{1}{8}\frac{\overline{A_4}\pi^2(m^4B_a^4 + n^4)\overline{\mathrm{W}}(\overline{\mathrm{W}}+2\mu)}{B_a^2B_h^2m^2} \tag{27}$$

where :

$$\overline{A_1} = \frac{A_1}{h}; \overline{A_3} = \frac{A_3}{h^2}; \overline{A_2} = \frac{A_2}{h^3}; \overline{A_4} = \frac{A_4}{h}; B_h = \frac{b}{h}; D = -A_2 + A_1A_3;$$
$$\overline{D} = \frac{D}{h^3}; K_1 = \frac{k_1a^4}{D}; K_2 = \frac{k_2a^2}{D}; \overline{W} = W/h; B_a = b/a \tag{28}$$

for perfect FGM plate we have

$$\mu = 0$$
$$P_x = \left[\frac{\overline{D}\pi^2(B_a^2m^2 + n^2)^2}{B_a^2B_h^2m^2} + \frac{K_1\overline{D}B_a^2}{B_h^2m^2\pi^2} + \frac{K_2\overline{D}(m^2B_a^2 + n^2)}{B_h^2m^2}\right] + \frac{32}{3}\frac{\overline{A_3}\overline{\mathrm{W}}n}{B_h^2m} + \frac{16}{3}\frac{\overline{A_1}\overline{A_4}n\overline{\mathrm{W}}}{B_h^2m}$$
$$+\frac{1}{8}\frac{\overline{A_4}\pi^2(m^4B_a^4 + n^4)\overline{\mathrm{W}}^2}{B_a^2B_h^2m^2} \tag{29}$$

If $v = const$ we have $A_1 = 0$, $A_3 = 0$

$$P_x = \left[\frac{\overline{D_1}\pi^2 (B_a^2 m^2 + n^2)^2}{B_a^2 B_h^2 m^2} + \frac{K_1^{'} \overline{D_1} B_a^2}{B_h^2 m^2 \pi^2} + \frac{K_2^{'} \overline{D_1} (m^2 B_a^2 + n^2)}{B_h^2 m^2} \right] \frac{\overline{W}}{\overline{W} + \mu} + \frac{1}{8} \frac{\overline{A_4}\pi^2 (m^4 B_a^4 + n^4) \overline{W}(\overline{W} + 2\mu)}{B_a^2 B_h^2 m^2}$$

(30)

If $v = const; \mu = 0$, for perfect FGM plate we receive :

$$P_x = \left[\frac{\overline{D_1}\pi^2 (B_a^2 m^2 + n^2)^2}{B_a^2 B_h^2 m^2} + \frac{K_1^{'} \overline{D_1} B_a^2}{B_h^2 m^2 \pi^2} + \frac{K_2^{'} \overline{D_1} (m^2 B_a^2 + n^2)}{B_h^2 m^2} \right] + \frac{1}{8} \frac{\overline{A_4}\pi^2 (m^4 B_a^4 + n^4) \overline{W}^2}{B_a^2 B_h^2 m^2} \quad (31)$$

Where

$$D_1 = -A_2; \overline{D_1} = \frac{D_1}{h^3}, K_1^{'} = \frac{k_1 a^4}{D_1}; K_2^{'} = \frac{k_2 a^2}{D_1} \quad (32)$$

Looking at the expression (27) and ($29 \div 31$) we see the beneficial effects of elastic foundation to rainbow load capacity and after rainbow.

From the equation (29) we can see that for the perfect plate ($\mu = 0$) the function $P_x(\overline{W})$ will reach a minimum at $\overline{W} = 0$ and $P_x(0)$ is the lowest point of the deflection-load graph.

3.2. Numerical results and discussions

The purpose of this section is to explore the dependence of the critical force on the coefficients K_1, K_2 of the elastic foundation in some cases when the plate is perfect and imperfect.

To illustrate the present approach for buckling and postbuckling analysis of thick FGM plates resting on elastic foundations, consider a square ceramic–metal plate consisting of aluminum and alumina with the following properties [3,8,9,10]

$$E_m = 70GPa; v_m = 0.3177$$

$$E_c = 380GPa; v_c = 0.24$$

In figures, W/h denotes the dimension-less maximum deflection and the FGM plate foundation interaction is ignored, unless otherwise stated.

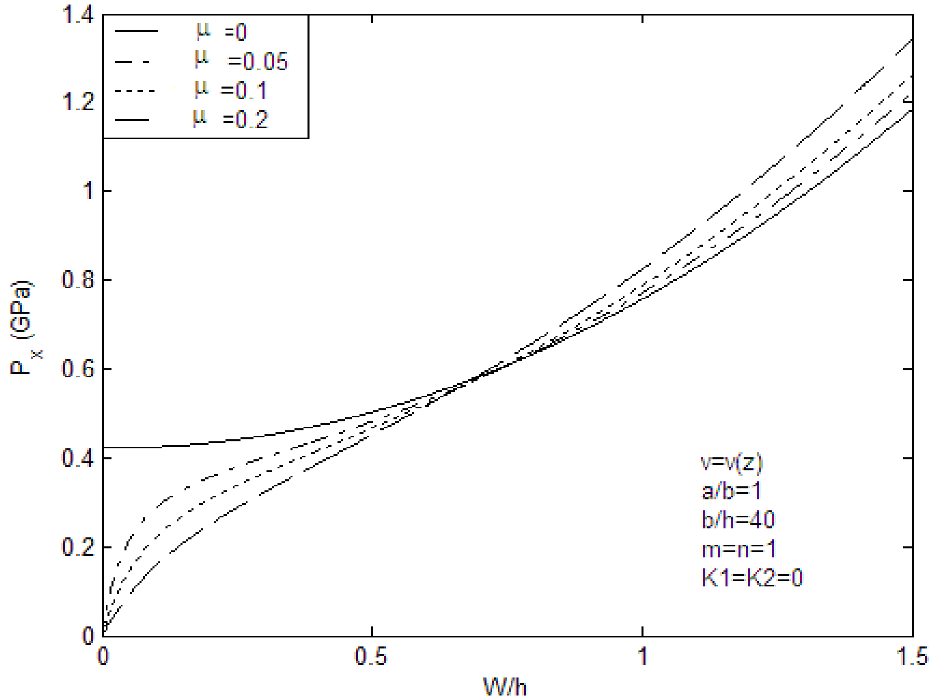

Fig.2. The influence of imperfections on the stability of FGM plates under compression

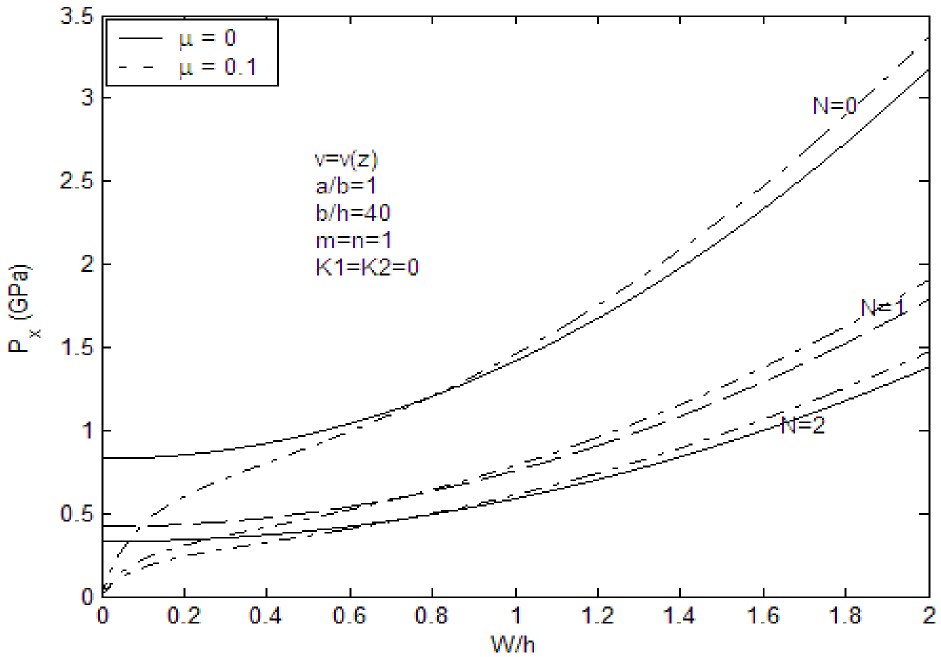

Fig.3. The effects of the area ratio coefficient N on the stability of FGM plates under compression

Fig.3 shows the variation of postbuckling for FGM plates with the ratio $b/h = 40$ under compression on one side with three different values of the ratio of the area $N = (0,1,2)$. As we can see, the curves after rainbow becomes lower representing less carrying capacity load of the plate when the area ratio coefficient N decreases. This is true because of the elastic modulus of ceramic E is much larger than the metal's when area ratio percentage of ceramic components in the plate decreases when N increases.

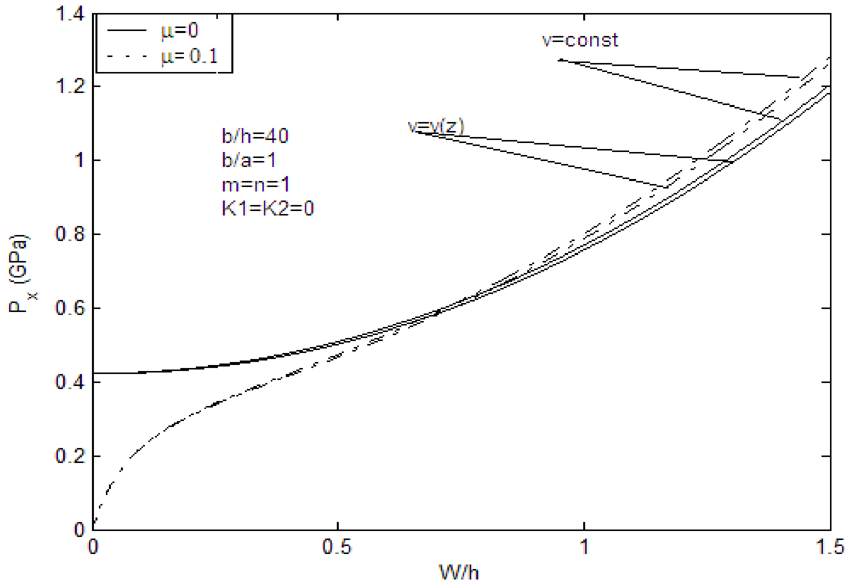

Fig.4. Effect of Poisson's ratio on posbuckling of FGM plates

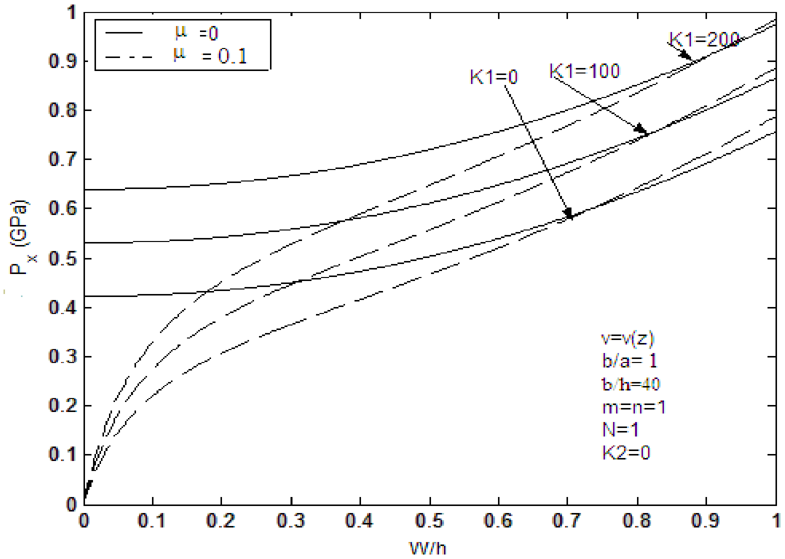

Fig.5. The effects of the linear Winkler foundation model to the postbuckling of FGM plates under axial compression load.

Figure 5 shows the effects of the linear Winker foundation model to the postbuckling of FGM plates under axial compression load when $K_2 = 0$ and the value K_1 changes.

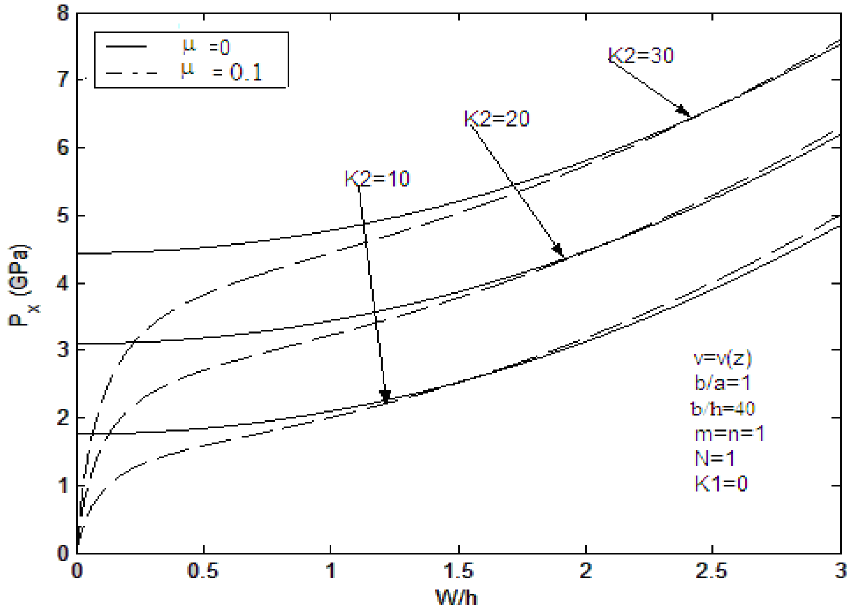

Fig.6. The effects of Pasternak foundation model to the postbuckling of FGM plates under axial compression load.

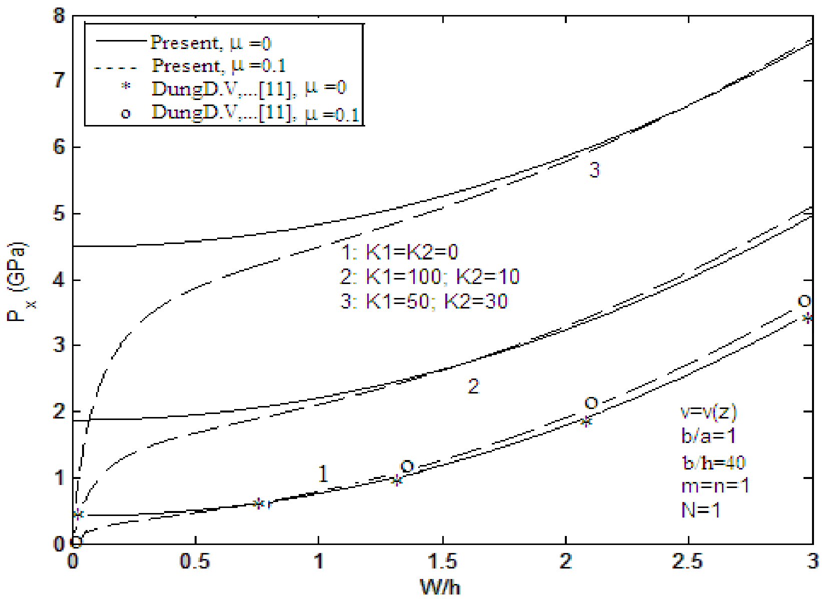

Fig.7. The effects of elastic foundations on postbuckling for FGM plates

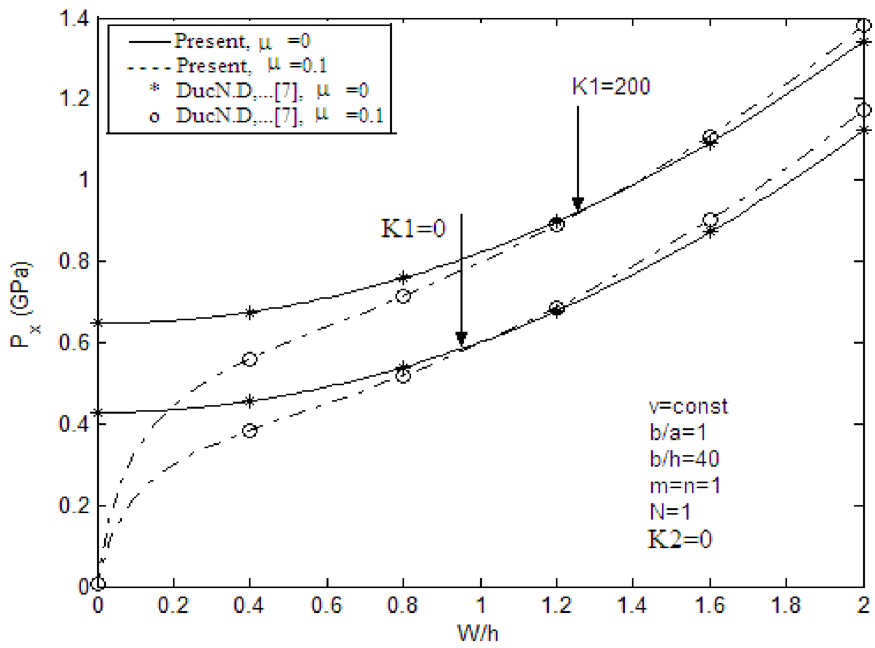

Fig.8. The effects of Winkler's elastic foundations on postbuckling for FGM plates

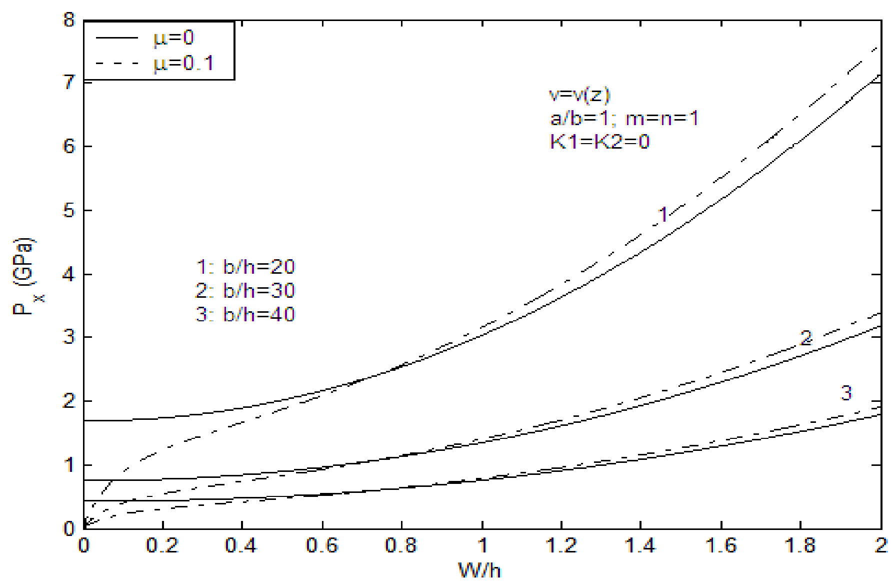

Fig.9. The effect of the ratio b/h on the stability of FGM plates under compression.

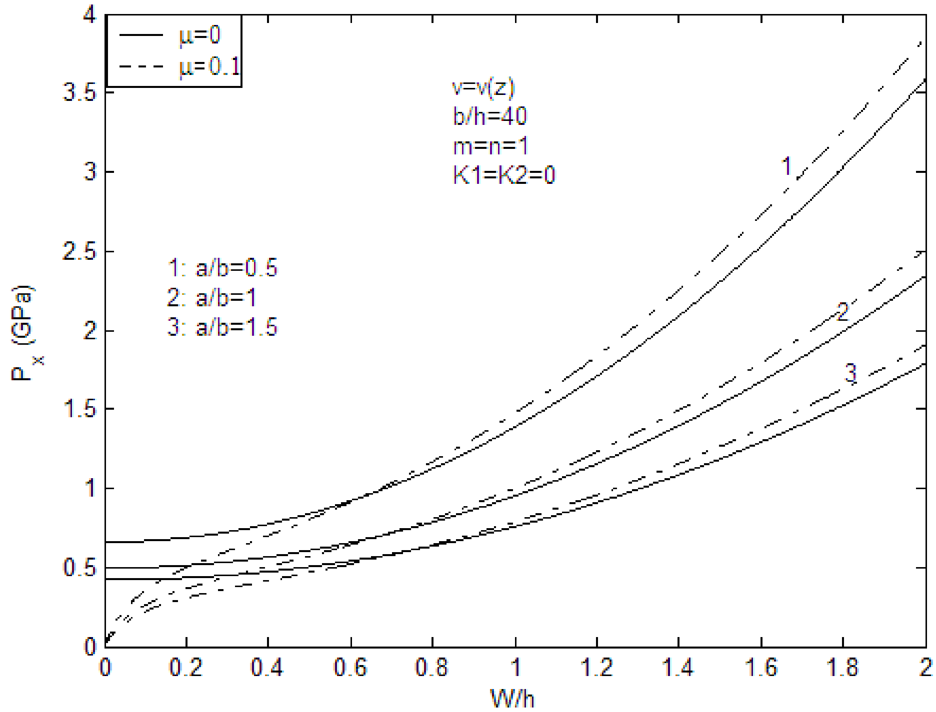

Fig.10. The effects of the ratio a/b to the stability of FGM plates

4. Concluding remarks

The content of the article has evaluated the effects of elastic foundation to the buckling and postbuckling of FGM plates under compression load when both elastic modulus of materials, Young's module and Poisson's ratio, are graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of constituents.

The results show that elastic media, especially Pasternak type foundations have a beneficial influence on the buckling loads and post buckling load carrying capacity of FGM plates, and effects of Poisson's ratio ν is small.

In the case $\nu = const$ or the cases with out elastic foundation, the results of present paper return to the previous well-known results.

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References

[1] Brush D.O., Almroth B.O, *Buckling of Bars, Plates and Shells*, McGraw-Hill, New York, 1975.

[2] Samsam Shariat B.A., Javaheri R., Eslami M.R. "Buckling of imperfect functionally graded plates under in-plane compressive loading", *Thin-Walled Struct*. 43, pp. 1020-1036, 2005.

[3] Samsam Shariat B.A., Eslami M.R., "Thermal buckling of imperfect functionally graded plates", *Int. J. Solids Struct*. 43, pp. 4082-4096, 2006.

[4] Samsam Shariat B.A., Eslami M.R., "Effect of initial imperfection on thermal buckling of functionally graded plates", *J. Thermal Stresses* 28, pp. 1183-1198, 2005.

[5] Librescu L., Stein M., "A geometrically nonlinear theory of transversely isotropic laminated composite plates and its use in the post-buckling analysis", *Thin-Walled Struct*. 11, pp. 177-201, 1991.

[6] Nguyen Dinh Duc, Hoang Van Tung, Mechanical and thermal postbuckling of higher order shear deformable functionally graded plates on elastic foundations. *J. Composite Structures,* Vol. 93, p2874-2881, 2011.

[7] Nguyen Dinh Duc, Do Nam, Hoang Van Tung, Effects of elastic foundation on nonlinear stability of FGM plates under compressive and thermal loads. *Proceedings of* X^{th} *National Conference on Mechanics of Deformed Solid*, Thai Nguyen, Nov. 2010, p 191-197, 2010.

[8] Javaheri R., Eslami M.R., Buckling of functionally graded plates under in-plane compressive loading, *ZAMM* 82(4). pp. 277-283, 2002.

[9] Javaheri R., Eslami M.R., Thermal buckling of functionally graded plates, *AIAA J.* 40(1), pp. 162-169, 2002.

[10] Lanhe W. , Thermal buckling of a simply supported moderately thick rectangular FGM plate, *Compos. Struct*. 64(2), pp. 211-218, 2004.

[11] Dao van Dung, Nguyen Thi Nga, Nonlinear Stability Analysis of Imperfect Functionally Graded Plates with the Poisson's Ratio v=v(z) Subjected to Mechanical and Thermal Loads. *Proceedings of* X^{th} *National Conference on Mechanics of Deformed Solid*, Thai Nguyen, Nov. 2010, pp. 142-154, 2010.

[12] Do Nam, Stability of FGM plates on elastic foundations. Master's thesis, Hanoi, 2011.

[13] Hoang Van Tung, Stability of FGM plates and shells. PhD's thesis, Hanoi, 2010.

[14] Zhao X, Lee YY, Liew KM. Mechanical and thermal buckling analysis of functionally graded plates. *J. Compos Struct*; 90:161–71, 2009.

[15] Liew KM, Jang J, Kitipornchai S. Postbuckling of piezoelectric FGM plates subject to thermo-electro-mechanical loading. *Int J Solids Struct* ;40:3869–92, 2003.

[16] Yang J, Liew KM, Kitipornchai S. Imperfection sensitivity of the post-buckling behavior of higher-order shear deformable functionally graded plates. *Int J Solids Struct* ;43:5247–66, 2006.

[17] Shen H-S. Postbuckling of FGM plates with piezoelectric actuators under thermo–electro-mechanical loadings. *Int J Solids Struct* ;42:6101–21, 2005.

[18] Shen H-S. Thermal postbuckling behavior of shear deformable FGM plates with temperature-dependent properties. *Int J Mech Sci* ;49:466–78, 2007.

[19] Lee YY, Zhao X, Reddy JN. Postbuckling analysis of functionally graded plates subject to compressive and thermal loads. *Comput Methods Appl Mech Eng* ;199:1645–53, 2010.

[20] Librescu L, Lin W. Postbuckling and vibration of shear deformable flat and curved panels on a non-linear elastic foundation. *Int J Non-Lin Mech* ;32(2):211–25, 1997

[21] Huang ZY, Lu CF, Chen WQ. Benchmark solutions for functionally graded thick plates resting on Winkler–Pasternak elastic foundations. *J. Compos Struct* ;85:95–104, 2008.

[22] Zenkour AM. Hygro–thermo-mechanical effects on FGM plates resting on elastic foundations. *J. Compos Struct* ;93:234–8, 2010.

[23] Shen H-S, Wang Z-X. Nonlinear bending of FGM plates subjected to combined loading and resting on elastic foundations. *J.Compos Struct* ;92:2517–24, 2010