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The bending analysis of thin composite plate under steady temperature field

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Abstract. Composite material with polymer matrix - titanium particle is widely applied in Vietnam, today. In this paper, the bending of thin polymer composite plate with Titanium particles under the effect of the steady temperature field is investigated. The paper has established the boundary problem of a simply supported composite plate reinforced by particles under the influence of steady heat transfer process. Using the Navier's method to solve the problem, an analytic solution of the bending plate given in the form of double trigonometric series is obtained. Base on the analytic result, a numerical test is done to find suitable material and clarify the influence of the particles.

Keywork: Composite plate; bending; steady heat transfer; polymer PVC; Titanium particle.

1. Introduction

Today, polymer composite materials with Titanium particles are widely applied in Vietnam and the world. In Vietnam, it is widely applied in shipbuilding industry, oil and gas pipeline, chemical and more recently in bio-chips, as well as luminescent materials OLED.

Titanium particles have a signification role in improving physical and mechanical features of materials. In [1], calculating stresses - strains of the composite pipe leading chemical and gas under the effect of unsteady temperature field was studied. In this paper, we studied the bending behavior of the structure of polymer - titanium composite plate under steady heat transfer condition.

Material components are considered homogeneous, isotropic. Titanium particles are mixed by volumn ratio ξ.

Then, according to [2, 3, 4], we can completely determine the moduluses of elasticity, and according to [5], we determine the coefficient of thermal expansion of the composite material depending on the properties of the material components and volumn ratio between them. Here, we choice the fomulas determined in [4, 5], then, we get:

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$$K = K_m + \frac{(K_c - K_m)\xi}{1 + (K_c - K_m)/(K_m + \frac{4}{3}G_m)},$$
$$G = G_m - \frac{15(1-\nu_m)(G_m - G_c)\xi}{7 - 5\nu_m + (8 - 10\nu_m)\frac{G_c}{G_m}}, \tag{1}$$

$$\alpha = \alpha_m + (\alpha_c - \alpha_m)\frac{K_c(3K_m + 4G_m)\xi}{K_m(3K_c + 4G_m) + 4(K_c - K_m)G_m\xi}. \tag{2}$$

where,

$$\xi = \frac{\sum_{i=1}^{N} V_i}{V}$$

is volumn ratio of the particles. $G_i, K_i, \nu_i, \alpha_i$ ($i = m, c$) are the shear moduluses, the bulk moduluses, Poission's ratios, the thermal expansion coefficients of the background phase, particle phase, respectively.

2. Governing equations

In [7], Since there is heat transfer through the thickness, so the heat conduction equation is:

$$\frac{d^2T}{dz^2} = 0 \tag{3}$$

With the boundary conditions:

$$\begin{cases} k\dfrac{\partial T}{\partial z} = \beta_1 (T - T_1), & z = -\dfrac{h}{2} \\ -k\dfrac{\partial T}{\partial z} = \beta_2 (T - T_2), & z = \dfrac{h}{2} \end{cases} \tag{4}$$

where, T_1, β_1 are the environmental temperature, the convective heat transfer coefficient, respectively at $z = -h/2$; T_2, β_2 are the environmental temperature, the convective heat transfer coefficient,respectively at $z = h/2$; k is the conductive coefficient of the material, and $k = (1-\xi)k_m + \xi k_c$.

Let :

$$\gamma_1 = \frac{\beta_1}{k}, \quad \gamma_2 = \frac{\beta_2}{k} \tag{5}$$

Then, the boundary conditions (4) become [7]:

$$\begin{cases} \dfrac{\partial T}{\partial z} = \gamma_1 (T - T_1), & z = -\dfrac{h}{2} \\ -\dfrac{\partial T}{\partial z} = \gamma_2 (T - T_2), & z = \dfrac{h}{2} \end{cases} \tag{6}$$

According to Cauchy expression , the strains of plates are determined as the following form [8, 9]:

$$\begin{aligned} e_{xx} &= \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \\ e_{yy} &= \frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} \\ e_{xy} &= \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - z\frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{7}$$

where, u, v, w denote displacements of the middle surface point along x, y, z respectively.
Hooke's law for the stresses - strains when there is temperature [8]

$$\begin{aligned} \sigma_{xx} &= \frac{E}{1-\nu^2}\left(e_{xx} + \nu e_{yy} - (1+\nu)\,\alpha\Delta T\right), \\ \sigma_{yy} &= \frac{E}{1-\nu^2}\left(e_{yy} + \nu e_{xx} - (1+\nu)\,\alpha\Delta T\right), \\ \sigma_{xy} &= \frac{E}{1+\nu} e_{xy} . \end{aligned} \tag{8}$$

Substitute (7) into (8), we get the relation of the stresses - displacements:

$$\begin{aligned} \sigma_{xx} &= \frac{E}{1-\nu^2}\left[\frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} - z\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) - (1+\nu)\,\alpha\Delta T\right], \\ \sigma_{yy} &= \frac{E}{1-\nu^2}\left[\frac{\partial v}{\partial y} + \nu\frac{\partial u}{\partial x} - z\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) - (1+\nu)\,\alpha\Delta T\right], \\ \sigma_{xy} &= \frac{E}{2(1+\nu)}\left[\frac{\partial u}{\partial y} + \nu\frac{\partial v}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y}\right] . \end{aligned} \tag{9}$$

Then, the momens and membrane forces are:

$$\begin{aligned} N_x &= \frac{Eh}{1-\nu^2}\left(\frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y}\right) - \frac{N_T}{1-\nu}, \\ N_y &= \frac{Eh}{1-\nu^2}\left(\frac{\partial v}{\partial y} + \nu\frac{\partial u}{\partial x}\right) - \frac{N_T}{1-\nu}, \\ N_{xy} &= \frac{Eh}{2(1+\nu)}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right), \end{aligned} \tag{10}$$

$$\begin{aligned} M_x &= -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) - \frac{M_T}{1-\nu}, \\ M_y &= -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) - \frac{M_T}{1-\nu}, \\ M_{xy} &= -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}, \end{aligned} \tag{11}$$

where,

$$N_T = \alpha E \int_{-h/2}^{h/2} \Delta T dz, M_T = \alpha E \int_{-h/2}^{h/2} z\Delta T dz, D = \frac{Eh^3}{12(1-\nu^2)}. \tag{12}$$

The basic equations for determining the deflection of a plate are the equilibrium equations [6, 8, 9]:

$$\begin{aligned} &\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \\ &\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0, \\ &\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x\frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y\frac{\partial^2 w}{\partial y^2} = 0. \end{aligned} \tag{13}$$

ported plate

ns (3), (6), we get the solution form:

$$\ldots\gamma_1\left(1+\gamma_2\frac{h}{2}\right)+T_2\gamma_2\left(1+\gamma_1\frac{h}{2}\right)+\gamma_1\gamma_2(T_2-T_1)z\Big] \tag{14}$$

$$\ldots T_1\gamma_1\left(1+\gamma_2\frac{h}{2}\right)+T_2\gamma_2\left(1+\gamma_1\frac{h}{2}\right)+\gamma_1\gamma_2(T_2-T_1)z\Big]\ . \tag{15}$$

re in natural condition.

ace no strains, then, (10) becomes:

$$\begin{aligned} N_x &= -\frac{N_T}{1-\nu},\\ N_y &= -\frac{N_T}{1-\nu},\\ N_{xy} &= 0, \end{aligned} \tag{16}$$

15)) into the expansions of N_T, M_T in (12), integrate, and notice = M_0.With the found forces and moments and (11) substituted into tomatically satisfied, the third equation has the form

$$D\nabla^2\nabla^2 w - N_0\nabla^2 w = 0\ ,$$

$$\nabla^2\left(\nabla^2 w - \frac{N_0}{D}w\right)=0\ . \tag{17}$$

ly supported plate:

$$\begin{aligned} &\ldots\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\Big)-\frac{M_0}{1-\nu}=0, \quad \text{at edges } x=0, x=a.\\ &\ldots\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\Big)-\frac{M_0}{1-\nu}=0, \quad \text{at edges } y=0, y=b. \end{aligned} \tag{18}$$

$$\begin{cases} \left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)f=0\\ \left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)w-\dfrac{N_0}{D}w=f \end{cases} \tag{19}$$

r the boundary conditions as the following:

$$\begin{aligned} &\ldots=0, \quad \frac{\partial^2 w}{\partial y^2}=0, \quad \text{at } x=0, x=a,\\ &\ldots=0, \quad \frac{\partial^2 w}{\partial x^2}=0, \quad \text{at } y=0, y=b. \end{aligned} \tag{20}$$

Thereby, using the boundary conditions (20), we obtain the bo
function f:

$$f+\frac{M_0}{D(1-\nu)}=0,\quad \text{at } x=0,x$$

$$f+\frac{M_0}{D(1-\nu)}=0,\quad \text{at } y=0,y$$

From these above equations, we obtain the unknown function f ir

$$f=-\frac{M_0}{D(1-\nu)},$$

Then, the second equation of system (19) becomes:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)w-\frac{N_0}{D}w=-\frac{M}{D(1}$$

this is the equation for determining the deflection w of the plate. Bc
if w is expressed by Fourier series [8]:

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi}{b}$$

where m,n are natural numbers. We can also express M_0 under t

$$M_0=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi}{}$$

where,

$$a_{mn}=\begin{cases}\dfrac{16M_0}{\pi^2 mn}, & m,n=1,3,5\\ 0 & m,n=2,4,6,...\end{cases}$$

Substitute (22) and (23) into equation (21), and since the equatioı

$$w_{mn}=\begin{cases}\dfrac{a_{mn}}{D(1-\nu)}\cdot\dfrac{1}{\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}+\frac{N_0}{D}} & m\\ 0 & m,n=2,4,6,...\end{cases}$$

Thus, we get the solution of the problem:

$$w=\sum_{m=1,3,5,...}\sum_{n=1,3,5,...}\frac{a_{mn}}{D(1-\nu)}\cdot\frac{1}{\frac{m^2\pi^2}{a^2}+\frac{n^2\pi^2}{b^2}+}$$

where, a_{mn} is determined according to (24), the coefficients are c

4. Numberical result

To study a specific case, we consider composite plate with
components having the characters respectively as the following [5

PVC matrix : $E_m=3.10^9(Pa),\nu_m=0.2,\alpha_m=8.10^{-}5/.$

Titanium particle: $E_c=100.10^9(Pa),\nu_c=0.34,\alpha_c=4.8$

Suppose that the plate has the length $a=2.25m$, the w
$0.02m$. The surrounding medium is air with the convective heat tr

temperature, respectively at ($z = -\frac{h}{2}$) are $\beta_1 = 60(W/m^2.K)$, $T_1 = 330^0K$ and at ($z = \frac{h}{2}$) are $\beta_2 = 40(W/m^2.K)$, $T_2 = 300^0K$, respectively. $T_0 = 293^0K$.

Substitute the above data into the expression determining the deflection of the plate, and use Matlab 7.1 for the calculation, the authors obtained the following results:

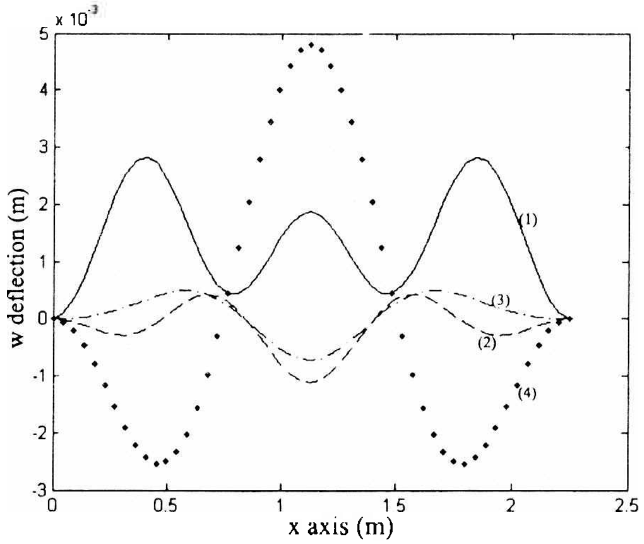

Fig. 1. Deflection of the plate along the length $(1)\xi = 0.1$ $(2)\xi = 0.15$ $(3)\xi = 0.2$ $(4)\xi = 0.3$.

From the result on the figure expresses deflection of the plate along the length with specific volumn ratio of Titanium particles under the influence of steady heat transfer through the thickness of the plate (Fig. 1), it can be seen that when we mix Titanium particles with different volumn ratios, Titanium particles has a significant effect on the bending behavior of the plate. When we mix particles with volumn ratio less than or equal to 20% ($\xi \leq 0.2$), then deflection at points of the plate decreases and among the points is more uniform when the volumn ratio of the particles is increased. Thus, with increasing volumn ratio ($\leq 20\%$), Titanium particles cause the plate increased the capability of bending resistance, cracking resistance and heat resistance. But when the volumn ratio of particles is 30% ($\xi = 0.3$), then bending behavior of the plate has large change, the deflection at points of the plate is large and among points has large difference. Therefore, when the particles are mixed with suitable volumn ratio, Titanium particles play the signification role in increasing the capability of bending resistance, cracking resistance and heat resistance.

5. Conclusion

The paper estabishted the boundary problem of a simply supported thin composite plate reinforced by particles under the influence of steady heat transfer process, from which solves the problem and an analytic solution of the deflection of the plate given in the form of double trigonometric series is obtained.

Base on the obtained analytic solution, a numerical test is done to study the deflection of composite plate made of PVC matrix material and Titanium particles. The obtained results confirm that with suitable volumn ratio, particles play the signification role in increasing the capability of bending resistance, cracking resistance and heat resistance of the plate.

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