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Analysis of stress-strain relationship of titanium dioxide-epoxy composite tube under pressure and thermal load

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Abstract. Today, titanium dioxide particle is widely used in shipbuilding, petroleum industry and other fields. The goal of this paper is to study the effect that titanium dioxide particles have on the stress, strain of composite tubes under pressure and thermal load. From the obtained result, comments and suggestions are given on the manufacture and use of engineering tubes.

Key words: composite, engineering tube, titanium dioxide particle, thermal, pressure

1. Introduction

Additive and reinforced particles are often used to increase the abrasion resistant, crack resistant, fireproof, waterproof ability and strength of polymer materials [1]. In Vietnam, in the past few years, composite material with titanium dioxide particles is widely used in shipbuilding and other industries [2,3]. Thus, many researches concerning the physio-mechanical role of titanium dioxide particle in composite are carried out. Titanium dioxide/polystyren composite is studied experimentally in [4]. In [5], the authors investigate the optical and mechanical properties of composite membrane filled with titanium dioxide particles. In [6], the engineering moduli os 3-phase composite (glass fiber, titanium dioxide particle, polymer matrix) are determined by experiment and base on that, a bending analysis for composite plates used in shipbuilding is done [7].

One of the most common composite structures is engineering tube such as water tubes, oil tubes...[1]. Recently, some analysis for composite tubes have been done: in [8], the authurs study the stress-strain relationship for composite cylinder under unsteady, axisymmetric, plane temperature field. In [9], the inner/outer pressure is taken into account beside the heat transfer in an analysis for titanium dioxide/PVC composite. This paper's goal is to calculate the stress and strain for engineering tubes made of titanium dioxide/epoxy composite under pressure and thermal load with constant temperature increment.

The composite material is considered to have periodic structure with small particle volume ratio. The particles have the same diameter and the interaction between particle and matrix is neglected. By using the equation of theory of thermoelasticity and composite material, the authur has solved the stress-strain relationship of engineering tubes made of titanium dioxide/epoxy composite, thereby clarifying the role of titanium dioxide particle in improving the composite's mechanical properties.

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2. Problem formulation

2.1. Problem

Considering a composite cylinder with inner radius a and outer radius b filled with spherical particles. The cilynder is put under inner pressure, outer pressure p_2 and a temperature increment $\Delta T = T - T_0$ (T_0 is the initial temperature). The material is assumed to be elastic, isotropic and the interaction between particle and matrix is ignored (all partiles' radius are equal). The elastic moduli and thermal coefficient of the matrix are λ_1, μ_1, α_1, and those of the particle are λ_2, μ_2, α_2. The equavient properties of the composite cilynder are λ^*, μ^*, α^* or K^*, μ^*, α^*.

2.2. Basic equations

2.2.1. Determine the composite's elastic moduli

The problem of determining the elastic moduli for composite filled with spherical particles is studied by several authors [10,11]. In this paper, the result of Vanin-Nguyen Dinh Duc [12] is used:

$$E^* = \frac{9K^*G^*}{3K^* + G^*}, \tag{1}$$

here

$$K^* = K_1 \frac{1 + 4G_1 L (3K_1)^{-1} \xi}{1 - 4G_1 L (3K_1)^{-1} \xi}; \quad G^* = G_1 \frac{1 - (7 - 5\nu_1) H\xi}{1 + (8 - 10\nu_1) H\xi} \tag{2}$$

where

$$L = \frac{K_2 - K_1}{K_2 + \frac{4}{3}G_1}; \quad H = \frac{\left(\frac{G_1}{G_2} - 1\right)}{8 - 10\nu_1 + (7 - 5\nu_1)\frac{G_1}{G_2}} \tag{3}$$

Here $\xi = \frac{\sum_{i=1}^{N} V_i}{V}$ is the particle volume ratio (N is the total number of the particles, V_i is the volume of the i-th particle (i = 1, 2,...,N), V is the composite volume). μ^*, μ_1, μ_2 are the shear modulus and K^*, K_1, K_2 are the bulk modulus of composite, matrix and particle, respectively. ν_1, ν_2 are the Poisson's ratio of matrix and particle. The advantage of these results is that the interaction between particle and matrix is taken into account.

2.2.2 The effective thermal expansion coefficient of composite

The current paper uses the result from [13] for calculating the thermal expansion coefficient:

$$\alpha^* = \alpha_1 + (\alpha_2 - \alpha_1)\frac{K_2(3K_1 + 4G_1)\xi}{K_1(3K_2 + 4G_1) + 4(K_2 - K_1)G_1\xi} \tag{4}$$

In which α^* is the effective thermal expansion coefficient of composite; α_1, α_2 are the thermal expansion coefficients of matrix and particle, respectively.

2.2.3. The Hooke's law for the thermoelastic problem

We study the thermoelastic deformation of the material under mechanical and thermal loads (the composite is being heated from T_0 to T).

The mechanical stress-strain relationship follows Hooke's law

$$\varepsilon_{ij}^{(s)} = \frac{1}{2\mu^*}(\sigma_{ij} - \frac{3\lambda^*}{3\lambda^* + 2\mu^*}\sigma\,\delta_{ij}). \tag{5}$$

Thereby, the thermoelastic relationship between strain and stress is [14]

$$\varepsilon_{ij} = \frac{1}{2\mu^*}(\delta_{ij} - \frac{3\lambda^*}{3\lambda^* + 2\mu^*}\sigma\,\delta_{ij}) + \alpha^*_T(T - T_0)\delta_{ij} \tag{6}$$

or

$$\varepsilon_{ij} = \frac{1+\nu^*}{E^*}\sigma_{ij} - \frac{3\nu^*}{E^*}\sigma\,\delta_{ij} + \alpha^*_T(T - T_0)\delta_{ij}$$

The inverse expression for stress is

$$\sigma_{ij} = \lambda^*\theta\delta_{ij} + 2\mu^*\,\varepsilon_{ij} - (3\lambda^* + 2\mu^*)\alpha^*_T(T - T_0)\delta_{ij} \tag{7}$$

This is the stress-strain relationship in thermoelasticity theory.

2.3. Calculating the stress and strain

When the cylinder is long enough we have the plane stress stage. The equations written for the cylindrical coordinate system (*r, θ, z*) are [14]

From the symmetric property, all points only have radial displacement. The displacement filed has the following form

$$u_r = u_r(r),\ u_z = u_\theta = 0 \tag{8}$$

The Cauchy strains are

$$\varepsilon_{rr} = \frac{du_r}{dr},\ \varepsilon_{\theta\theta} = \frac{u_r}{r};\quad \theta = \frac{du_r}{dr} + \frac{u_r}{r} \tag{9}$$

The stress-strain relationship is

$$\sigma_{rr} = \lambda^*\theta + 2\mu^*\varepsilon_{rr} - (3\lambda^* + 2\mu^*)\alpha^*\Delta T \tag{10a}$$

$$\sigma_{\theta\theta} = \lambda^*\theta + 2\mu^*\varepsilon_{\theta\theta} - (3\lambda^* + 2\mu^*)\alpha^*\Delta T \tag{10b}$$

$$\sigma_{zz} = \lambda^*\theta - (3\lambda^* + 2\mu^*)\alpha^*\Delta T,\ \sigma_{rz} = \sigma_{z\theta} = \sigma_{r\theta} = 0 \tag{10c}$$

The equibrilium equation is

$$\frac{d\sigma_{rr}}{dr} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0 \tag{11}$$

Introduce (10a) and (10b) into (11) we get the differential equation for u_r

$$\frac{d^2u_r}{dr^2} + \frac{du_r}{dr} - \frac{u_r}{r^2} = 0 \tag{12}$$

This Euler differential equation with the boundary condition:

$$\sigma_{rr}\big|_{r=a} = -p_1,\ \sigma_{rr}\big|_{r=b} = -p_2$$

The solution of (12) is

$$u_r = \frac{1}{2(\lambda^* + \mu^*)}\left[\frac{p_1a^2 - p_2b^2}{(b^2 - a^2)} + (3\lambda^* + 2\mu^*)\alpha^*\Delta T\right]r - \frac{(p_2 - p_1)a^2b^2}{2\mu^*(b^2 - a^2)}\frac{1}{r} =$$

$$= \frac{1}{2\left(K^* + \dfrac{\mu^*}{3}\right)}\left(\frac{p_1a^2 - p_2b^2}{b^2 - a^2} + 3K^*\alpha^*\Delta T\right)r - \frac{(p_2 - p_1)a^2b^2}{2\mu^*(b^2 - a^2)}\frac{1}{r} \quad (13)$$

The non-zero strains are

$$\varepsilon_{rr} = \frac{1}{2(\lambda^* + \mu^*)}\left[\frac{p_1a^2 - p_2b^2}{(b^2 - a^2)} + (3\lambda^* + 2\mu^*)\alpha^*\Delta T\right] + \frac{(p_2 - p_1)a^2b^2}{2\mu^*(b^2 - a^2)}\frac{1}{r^2}$$

$$= \frac{1}{2\left(K^* + \dfrac{\mu^*}{3}\right)}\left(\frac{p_1a^2 - p_2b^2}{b^2 - a^2} + 3K^*\alpha^*\Delta T\right) + \frac{(p_2 - p_1)a^2b^2}{2\mu^*(b^2 - a^2)}\frac{1}{r^2} \quad (14a)$$

$$\varepsilon_{\theta\theta} = \frac{1}{2(\lambda^* + \mu^*)}\left[\frac{p_1a^2 - p_2b^2}{(b^2 - a^2)} + (3\lambda^* + 2\mu^*)\alpha^*\Delta T\right] - \frac{(p_2 - p_1)a^2b^2}{2\mu^*(b^2 - a^2)}\frac{1}{r^2}$$

$$= \frac{1}{2\left(K^* + \dfrac{\mu^*}{3}\right)}\left(\frac{p_1a^2 - p_2b^2}{b^2 - a^2} + 3K^*\alpha^*\Delta T\right) - \frac{(p_2 - p_1)a^2b^2}{2\mu^*(b^2 - a^2)}\frac{1}{r^2} \quad (14b)$$

The non-zero stress are

$$\sigma_{rr} = \frac{p_1a^2 - p_2b^2}{b^2 - a^2} + \frac{(p_2 - p_1)a^2b^2}{b^2 - a^2}\frac{1}{r^2} \quad (15a)$$

$$\sigma_{\theta\theta} = \frac{p_1a^2 - p_2b^2}{b^2 - a^2} - \frac{(p_2 - p_1)a^2b^2}{b^2 - a^2}\frac{1}{r^2} \quad (15b)$$

$$\sigma_{zz} = \frac{p_1a^2 - p_2b^2}{b^2 - a^2}\frac{\lambda^*}{\lambda^* + \mu^*} - \frac{(3\lambda^* + 2\mu^*)\mu^*}{\lambda^* + \mu^*}\alpha^*\Delta T$$

$$= \frac{p_1a^2 - p_2b^2}{b^2 - a^2}\frac{3K^* - 2\mu^*}{3K^* + \mu^*} - \frac{9\mu^*K^*}{3K^* + \mu^*}\alpha^*\Delta T \quad (15c)$$

From equation (13)÷(15c), we can see that the displacement, strain, stress depend on not only pressure, elastic moduli, the thickness of the tube but also temperature.

3. Numerical results

Consider a composite cylinder having the following properties

Epoxy matrix [4]: $E_1 = 2.75$ (GPa), $\nu_1 = 0.35$, $\alpha_1 = 54\times10^{-6}\,/^{o}C$. (16)

Titian dioxide particle [4]: $E_2 = 147$ (GPa), $\nu_2 = 0.2$, $\alpha_2 = 4\times10^{-6}\,/^{o}C$ (17)

Assume that the tube has inner radius a = 13 (cm), outer radius b = 14 (cm). It is put under inner pressure $p_1 = 50$ (MPa) and outer pressure V, initial temperature $T_0 = 25^{0}C$, current temperature $T = 125^{0}C$.

3.1. The dependence of K^*, μ^*, α^* on particle volume ratio ξ

3.1.1. Analytical expression

The relationship between the elastic module for component materials [14]

$$K = \frac{E}{3(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}$$

Thus, bulk modulus and Poisson's ratio for matrix and particle are

$$K_1 \approx 3.0556 \text{ (GPa)}, \; K_2 = 81.6667 \text{ (GPa)}$$

$$\mu_1 = 1.0185 \text{ (GPa)}, \; \mu_2 = 61.25 \text{ (GPa)}$$

The effective moduli for composite cylinder can be calculted using (2) and (4)

$$K^* \approx \frac{3.0556(1+0.4222\xi)}{(1-0.4222\xi)} \text{(GPa)} \tag{18}$$

$$\mu^* \approx \frac{1.0185(1+1.1025\xi)}{1-0.945\xi} \text{(GPa)} \tag{19}$$

$$\alpha^* \approx \frac{0.0412-0.0378\xi}{762.2154+320.7288\xi} /^0\text{C} \tag{20}$$

3.1.2. Graphs

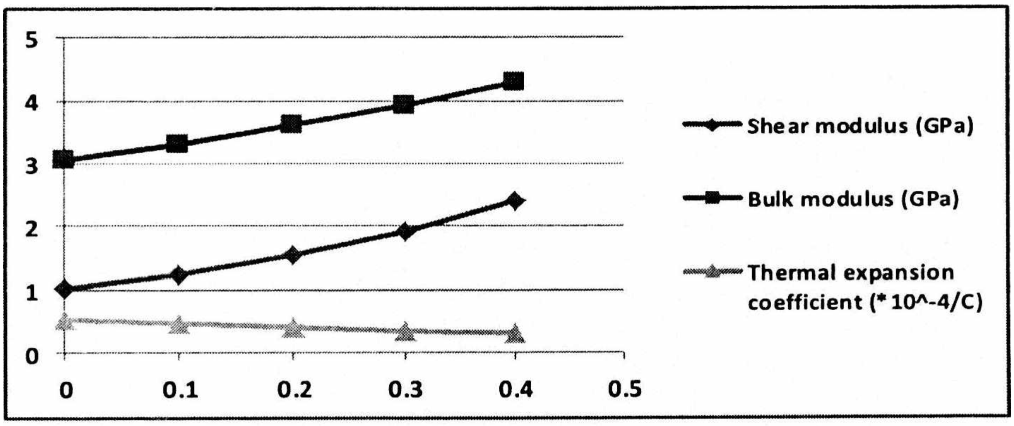

Fig. 1. Effective moduli of composite material.

From Figure 1, it can be seen that increasing the particle volume ratio leads to decrease of the thermal expansion coefficient and increase of the elastic moduli, which means improvement of the mechanical and thermal properties of the composite cylinder.

3.2. The dependence of stress, strain, displacement on particle volume ratio ξ and radius r

3.2.1. Analytical expression

The stress components are calculated as in (15a), (15b) and (15c)

$$\sigma_{rr} = -0.1952 + \frac{24.5362}{r^2} \text{ (GPa)} \tag{21a}$$

$$\sigma_{\theta\theta} = -0.1952 - \frac{24.5362}{r^2} \text{ (GPa)} \tag{21b}$$

$$\sigma_{zz} = -\frac{0.235[9.18(1+0.4222\xi)(1-0.945\xi)-2.04(1+1.1025\xi)(1-0.4222\xi)]}{9.18(1+0.4222\xi)(1-0.945\xi)+1.02(1+1.1025\xi)(1-0.4222\xi)} -$$

$$-100.\frac{9.3,06.1,0185(1+0.4222\xi)(1+1.1025\xi)(0.0412-0.0378\xi)}{(762.2154-320.7288\xi)[9.18(1+0.4222\xi)(1-0.945\xi)+1.02(1+1.1025\xi)(1-0.4222\xi)]} \quad (21c)$$

Strain components are calculated from (14a) and (14b)

$$\varepsilon_{rr} = \left[\frac{-0.2925(1-0.4222\xi)(1-0.945\xi)}{9.18(1+0.4222\xi)(1-0.945\xi)+1.02(1+1.1025\xi)(1-0.4222\xi)} + \frac{1377(1+0.4222\xi)(1-0.0945\xi)(0.0412-0.0378\xi)}{9.18(1+0.4222\xi)(1-0.945\xi)+1.02(1+1.1025\xi)(1-0.4222\xi)(762.2154+320.7288\xi)}\right] + \frac{12.0453(1-0.945\xi)}{(1+1.1025\xi)}\frac{1}{r^2} \quad (22a)$$

$$\varepsilon_{\theta\theta} = \left[\frac{-0.2925(1-0.4222\xi)(1-0.945\xi)}{9.18(1+0.4222\xi)(1-0.945\xi)+1.02(1+1.1025\xi)(1-0.4222\xi)} + \frac{1377(1+0.42\xi)(1-0.0945\xi)(0.0412-0.0378\xi)}{9.18(1+0.4222\xi)(1-0.945\xi)+1.02(1+1.1025\xi)(1-0.4222\xi)(762.2154+320.7288\xi)}\right] - \frac{12.0453(1-0.945\xi)}{(1+1.1025\xi)}\frac{1}{r^2} \quad (22b)$$

Displacement is calculated from (13)

$$u_r = \left[\frac{-0.2925(1-0.4222\xi)(1-0.945\xi)}{9.18(1+0.4222\xi)(1-0.945\xi)+1.02(1+1.1025\xi)(1-0.4222\xi)} + \frac{1377(1+0.4222\xi)(1-0.0945\xi)(0.0412-0.0378\xi)}{9.18(1+0.4222\xi)(1-0.945\xi)+1.02(1+1.1025\xi)(1-0.4222\xi)(762.2154+320.7288\xi)}\right] r - \frac{12.0453(1-0.945\xi)}{(1+1.1025\xi)}\frac{1}{r} \quad (23)$$

3.2.2. Graphs

Stress components are presented in Fig 2 and Fig 3.

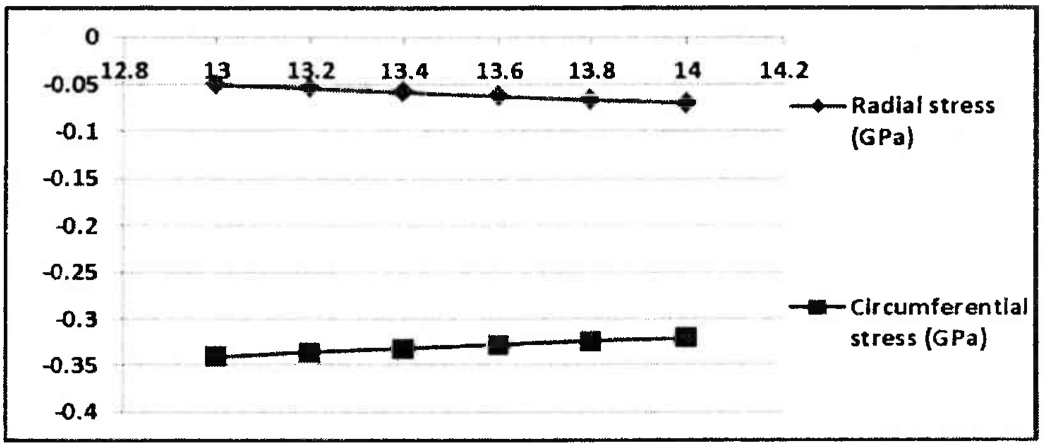

Fig. 2. Stress components σ_{rr} and $\sigma_{\theta\theta}$.

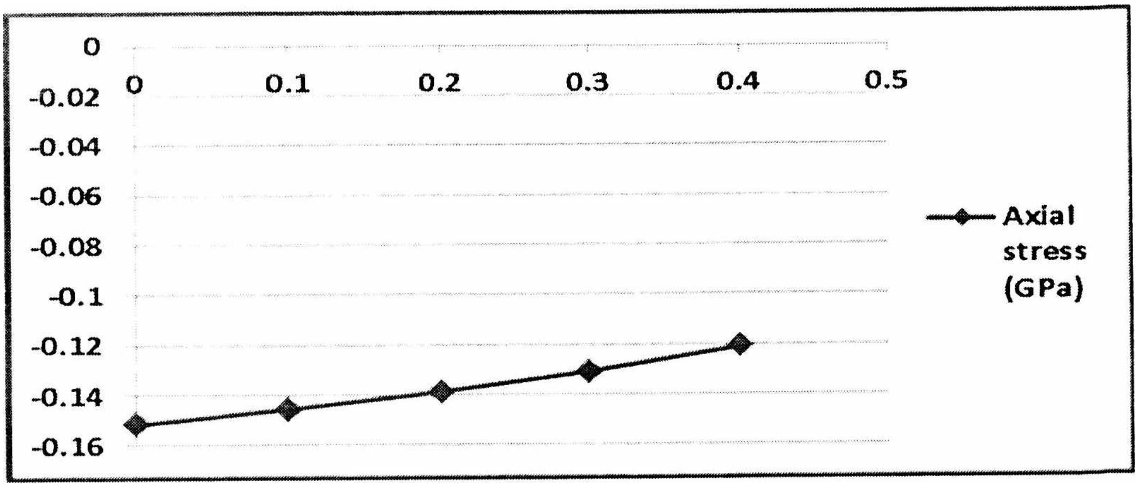

Fig. 3. Stress component σ_{zz}.

From (21a), (21b), (21c), it can be seen that σ_{zz} only depends on ξ while σ_{rr}, $\sigma_{\theta\theta}$ only depend on r. On Figure 2, σ_{rr} and $\sigma_{\theta\theta}$ vary slowly in different directions (when r increases, σ_{rr} decreases but $\sigma_{\theta\theta}$ increased), but they both have negative values. On the other hand, σ_{zz} has the same value for any point in the cylinder. On Figure 3, σ_{zz} increases when r increases and it always have negative values. Thus, the particle help to reduce the stress concentration inside the tube.

Strain components ε_{rr}, $\varepsilon_{\theta\theta}$ are presented in Fig 4 and Fig 5.

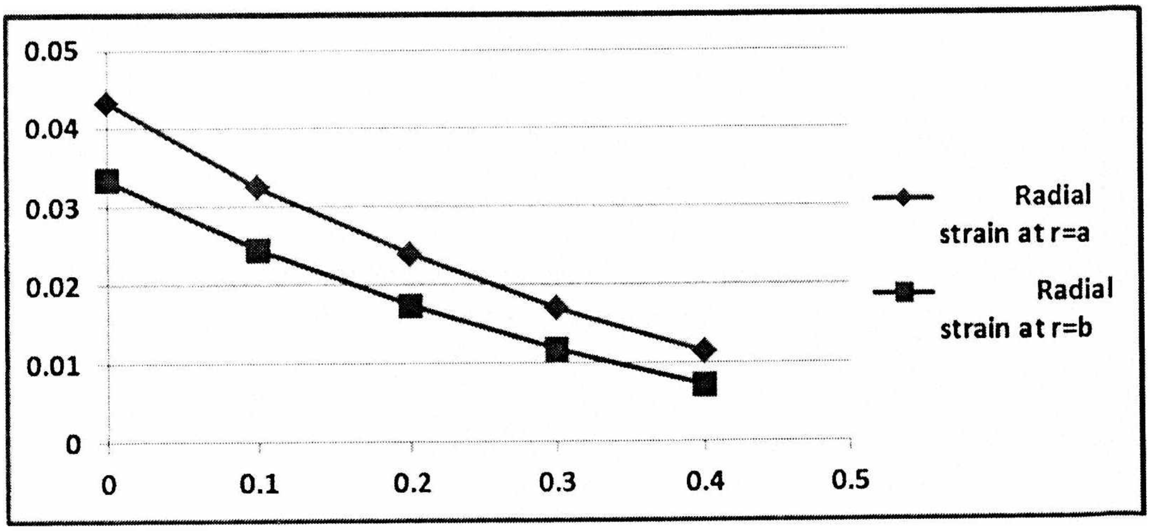

Fig. 4. Strain component ε_{rr} at r=a và r=b.

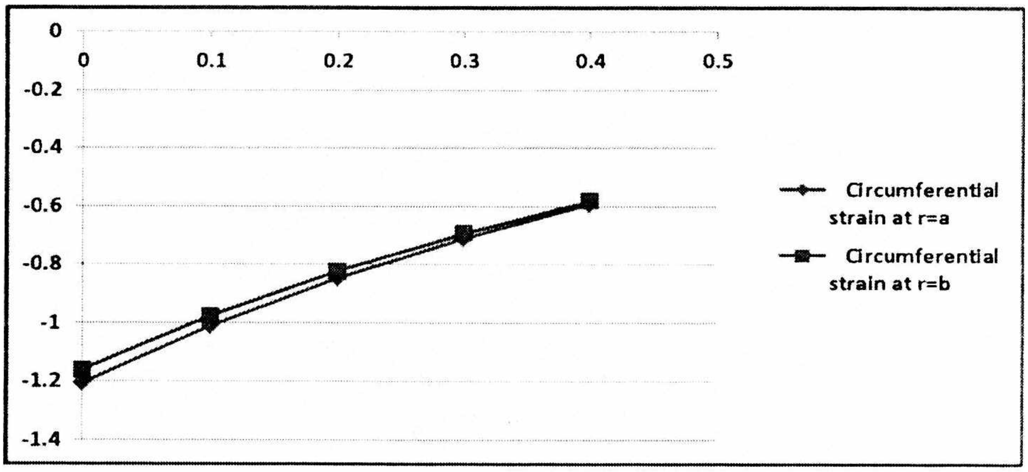

Fig. 5. Strain component $\varepsilon_{\theta\theta}$ at $r = a$ and $r = b$.

Displacements u_r at $r = a$ and $r = b$ are presented in Fig 6.

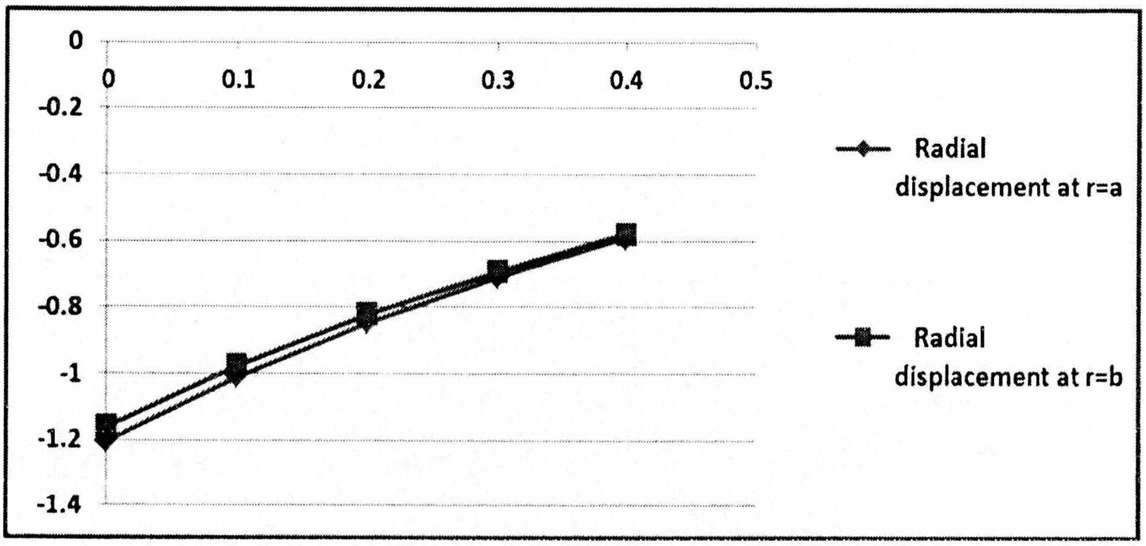

Fig. 6. Displacement u_r at $r = a$ and $r = b$.

From Figure 4,5 and 6, under mechanical and thermal loads, the strain components ε_{rr}, $\varepsilon_{\theta\theta}$ and displacement u_r at the same radius vary slowly in different directions when particles are added. When ξ increases, the radial strain decreases (and has positive values) while radial displacement and angular strain increase (and have positive values).

Thus, under mechanical and thermal loads, increasing the particle volume ratio can help improve the mechanical and thermal properties of the composite cylinder.

4. Conclusions

In the current paper, the author has studied the following points

1. The dependence of the composite's elastic moduli on the titanium dioxide particle volume ratio.

2. Base on the thermoelasticity theory and the mechanics of composite structure, the problem of calculating stress, strain for a composite cylinder under pressure and thermal loads was set up and solved.

3. The numerical results show that adding the titanium dioxide particle can help to improve the mechanical and thermal properties of composite material. The elastic moduli and thermal expansion coefficient are experessed explicitly from the component materials' properties and volume ratios. This can be the scientific base for the problem of optimization in design and manufacture. Composite materials with desired properties can be manufactured by changing the matrix and the particle with proper volume ratios.

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