VNU Journal of Science, Mathematics - Physics 23 (2007) 47-54

# Influence of intradot Coulomb interaction on transport properties of an Aharonov-Bohm interferometer

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Received 15 May 2007

**Abstract.** Using Greens function method and the equation of motion approach, we have investigated the electronic transport properties of an Aharonov-Bohm (AB) ring in the presence of a magnetic field with a quantum dot inserted in one arm of the ring. In particular, we consider the electron-electron Coulomb interaction within the quantum dot. We find that the current through the system is dependent on the magnetic flux via the AB phase and the Coulomb interaction within the quantum dot, in agreement with experiments. Furthermore, the intradot Coulomb interaction induces dephasing.

## 1. Introduction

For the nanodevices-such as a quantum ring and a quantum dot, the wave nature of the electrons contributes a crucial role. In the Aharonov-Bohm (AB) interferometer [1-3], the electron waves travel from the source to the drain along two different paths of the ring. The accumulated phase difference between these two waves can be changed by applying a magnetic field. Experiments show that a transport through the AB interferometer has the following striking features: (i) the AB phase increases sharply by $\pi$, (ii) the transmission amplitudes at the various resonances are in phase, (iii) the transport is partially coherent in the presence of a strong intradot Coulomb interaction [1-3]. Hackenbroich et al. calculated the entire scattering amplitude through the AB interferometer and reported theoretical results on the phase coherent transport through the quantum dot in the frame of the single-particle scattering theory [3, 4]. They focused on the AB phase and their results showed that as a function of the voltage on the dot, only the amplitude of the current oscillations is changed. They did not show, however, how the Coulomb interaction within the dot can affect the current. The AB phase is unaffected unless this amplitude changes sign. In this case, the AB phase suddenly jumps by $\pi$. To consider all possible effects that may influence the system, Tae-Suk Kim et al. [5] studied the thermoelectric effects of this system when the quantum dot lies in the Kondo regime and when it is directly connected with the two leads. Electrons can flow from one lead to the other through the two paths by direct tunneling (lead to lead) and a resonant tunneling via the quantum dot (lead to dot to lead). The interference between the resonant transport through the quantum dot and the direct channel gives rise to asymmetric line

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shapes in the linear conductance as a function of gate or bias voltage, the well-known Fano effect [6]. In the same context, Bogdan et al. [7] and Walter Hofstetter et al. [8] studied the combination of the Kondo effect and the Fano effect.

How the intradot Coulomb interaction influences the phase coherence of electronic transport through the AB interferometer has been the subject of debate. Several theoretical papers concluded that the intradot Coulomb interaction induces partial dephasing from the spin-flip process [9-11], while others argued that the intradot Coulomb interaction does not induce dephasing effect at all and the transport through the quantum dot is fully coherent [12-14]. We believe that this inconsistency may be solved by using a model that considers the role of the quantum ring that connects to the quantum dot, which was ignored in the previous works [9, 14], since the typology of the ring may play an important role in the transport property of the system. Also, a theoretical model that more closely resembles the experimental model needs to be utilized in order to obtain more realistic results.

## 2. Model and Calculation

As a first step to address these concerns, we investigate in this work the current through a two-terminal AB device which contains an AB ring with a quantum dot inserted in one of the ring's arms [1].

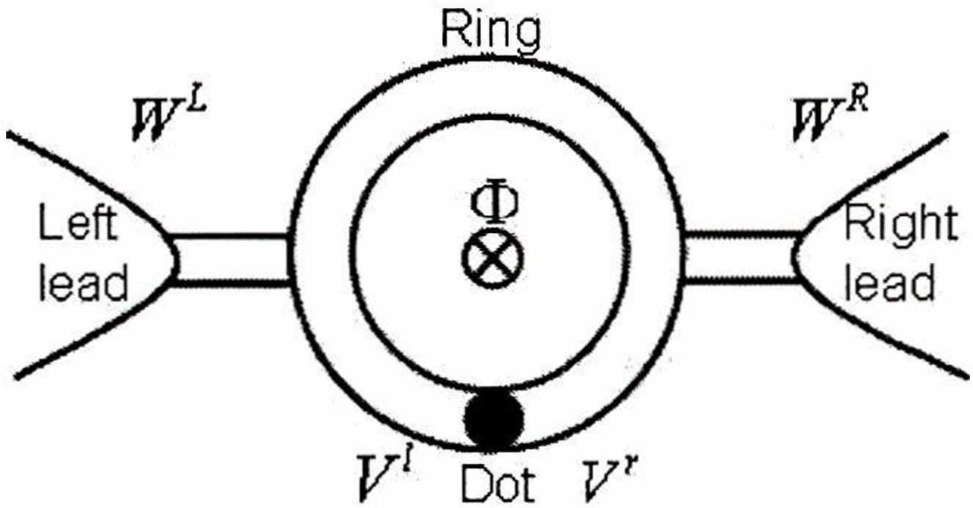


Figure 1. A schematic description of the AB device.

We consider the system (Figure 1) where an intradot Coulomb interaction exists without the spin-flip process and where the tunneling probability through the dot is considerably small. We investigate the system with an indirect tunneling channel (lead to ring to lead) and a resonant tunneling channel via the dot (lead to ring to dot to ring to lead). In this AB device, the quantum dot can be considered as an impurity based on the Anderson model [15]. Hence, we want to study the transport as a function of the impurity characteristics and derive a reliable expression for the Green function on the quantum dot. To study the transport, we obtain the total current through the AB device using the current formulation for interacting systems [16]. A ubiquitous method to derive an analytical expression for the Green function is to use the equations of motion method [16]. We choose this method because it gives results equivalent to those using the perturbation method providing that the Kondo effect is not included (in the present paper, the Kondo effect is not included) [17]. Our results show that the coherent currents, with different Coulomb interactions, are in phase [1-3] and the intradot Coulomb interaction can induce dephasing under an appropriate condition.

We can express the Hamiltonian for the present system (with the quantum dot as an impurity) as follows

$$H = H_0 + H_T + H_c \tag{1}$$

Here, $H_0$ describes the totally isolated subsystems of two leads, AB ring, and quantum dot, and is given explicitly by

$$H_0 = \sum_{k\sigma,\alpha=L,R} \varepsilon_{k\alpha} C^{\alpha+}_{k\sigma} C^{\alpha}_{k\sigma} + \sum_{p\sigma} \varepsilon_p C^{+}_{p\sigma} C_{p\sigma} + \sum_{\sigma} \varepsilon_d d^{+}_{\sigma} d_{\sigma} \tag{2}$$

Where $\alpha$ stands for the left (L) and the right (R) leads, while $k$ and $\varepsilon_{k\alpha}$ are the longitudinal wave number and the corresponding energy of the electron. The energies of the single particle states within the ring and within the quantum dot are $\varepsilon_p$ and $\varepsilon_d$, respectively. $C^{\alpha+}_{k\sigma}$, $C^{+}_{p\sigma}$, $d^{+}_{\sigma}$ ($C^{\alpha}_{k\sigma}$, $C_{p\sigma}$, $d_{\sigma}$) are the creation (annihilation) operators for the electron in the lead, the ring, and the dot, respectively, while $\sigma$ is the spin index. The tunneling part $H_T$ consists of the couplings between the subsystems, and is given by

$$H_T = \sum_{\substack{kp\sigma \\ \alpha=L,R}} (W^{\alpha}_{kp\sigma} C^{\alpha+}_{k\sigma} C_{p\sigma} + h.c) + \sum_{p\sigma} \left[ \left( V^{l}_{pd} C^{+}_{p\sigma} d_{\sigma} + h.c \right) + \left( V^{r}_{pd} e^{-i\phi} C^{+}_{p\sigma} d_{\sigma} + h.c \right) \right] \tag{3}$$

where the tunneling matrix elements W describe the coupling between the ring and the leads, while the tunneling matrix elements $V^l$ ($V^r$) describe the coupling -between the left (right) side of the dot and the ring. We attach the magnetic flux on the right hand side of the dot, hence $V^{l}_{pd}$ carries a phase factor $exp(-i\phi)$, where $\phi = 2\pi\Phi e/h$ and $\Phi$ is the magnetic flux enclosed by the ring which is formed by the arms. Finally, the intradot Coulomb interaction Hamiltonian is given by,

$$H_c = U n_{\uparrow} n_{\downarrow} \tag{4}$$

where $n_\sigma$ is the number operator, $n_\sigma = d^{+}_{\sigma} d_{\sigma}$. The upper arm (reference arm) current is defined by the following formular [16],

$$I_{upper\_arm} = \frac{ie}{\hbar} \sum_{p\sigma} \int \frac{d\varepsilon}{2\pi} \frac{\Gamma^L(\varepsilon)\Gamma^R(\varepsilon)}{\Gamma^L(\varepsilon) + \Gamma^R(\varepsilon)} ((f_L(\varepsilon) - f_R(\varepsilon)) \times \ [G^{r}_{p\sigma}(\varepsilon) - G^{a}_{p\sigma}(\varepsilon)]) \tag{5}$$

where the Greens function for the reference arm is written as

$$G^{r,a}_{p\sigma}(\varepsilon) = \frac{1}{\varepsilon - \varepsilon_p - \sum_{k\alpha} \frac{|W^{\alpha}_{kp\sigma}|^2}{\varepsilon - \varepsilon_{k\alpha} \pm i\delta}} \tag{6}$$

The line width of the leads is $\Gamma^\alpha = 2\pi \sum_k W^{\alpha *}_{pk\sigma} W^{\alpha}_{pk\sigma} \delta(\varepsilon - \varepsilon_{k\alpha})$ ($\alpha = L,\ R$), and $f_\alpha(\varepsilon)$ is the Fermi distribution function of the leads. Similarly, the lower arm current can be written as

$$I_{lower\_arm} = \frac{ie}{\hbar} \sum_{p\sigma\sigma'} \int \frac{d\varepsilon}{2\pi} \frac{\Gamma^L(\varepsilon)\Gamma^R(\varepsilon)}{\Gamma^L(\varepsilon) + \Gamma^R(\varepsilon)} ((f_L(\varepsilon) - f_R(\varepsilon)) \times [G^{r}_{p\sigma\sigma'}(\varepsilon) - G^{a}_{p\sigma\sigma'}(\varepsilon)]) \tag{7}$$

In order to compute $I_{lower\_arm}$, we calculate the retarded and advanced Green functions (i.e. and) for the ring-dot using the equations of motion method [17]. The equations of motion method of the dot Green function gives rise to an infinite order system of higher-order Green functions. An approximation procedure is applied to truncate the system of equations, thus producing the closed set of equations. To do the truncation, we approximate the Green functions up to the second order of U. This truncation ensures that most of the interaction effects are captured. In this paper, we do not consider the Kondo effect. Thus, the truncation up to the second order of U can be acceptable (if the Kondo effect is expected to include, the higher-order truncation need to be developed).

Equations (8), (9), and (10) constitute the system of equations which gives the relationship among the Greens functions of the quantum dot $G_{\sigma\sigma'}(\varepsilon)$, the ring-dot $G_{p\sigma\sigma'}(\varepsilon)$, and the dot-leads $G^{a}_{k\sigma\sigma'}(\varepsilon)$;

$$(\varepsilon-\varepsilon_d)G_{\sigma\sigma'}(\varepsilon)=\frac{\delta_{\sigma\sigma'}}{2\pi}+(V^{l*}_{pd}+V^{r*}_{pd}e^{i\phi})G_{p\sigma\sigma'}(\varepsilon)+ \\ +\frac{\frac{1}{2\pi}\delta_{\sigma\sigma'}U<n_{\bar{\sigma}}>}{\varepsilon-\varepsilon_d-U-\sum\limits_{p}\frac{(V^{l*}_{pd}+V^{r*}_{pd}e^{i\phi})(V^{l}_{pd}+V^{r}_{pd}e^{-i\phi})}{\varepsilon-\varepsilon_p-\sum\limits_{k\alpha=L,R}\frac{W^{\alpha\ 2}_{kp\sigma}}{\varepsilon-\varepsilon_{k\alpha}}}} \tag{8}$$

$$(\varepsilon-\varepsilon_p)G_{p\sigma\sigma'}(\varepsilon)=\sum_{k\alpha}W^{\alpha*}_{kp\sigma}G^{\alpha}_{k\sigma\sigma'}(\varepsilon)+(V^{l}_{pd}+V^{r}_{pd}e^{-i\phi})G_{\sigma\sigma'}(\varepsilon) \tag{9}$$

$$(\varepsilon-\varepsilon_{k\alpha})G^{\alpha}_{k\sigma\sigma'}(\varepsilon)=\sum_{p'}W^{\alpha}_{kp'\sigma}G_{p'\sigma\sigma'}(\varepsilon) \tag{10}$$

Solving for $G_{\sigma\sigma'}(\varepsilon)$ and $G_{p\sigma\sigma'}(\varepsilon)$, with the assumption that the matrix elements W are independent on k, p, and $\sigma$, we obtain

$$G_{\sigma\sigma'}(\varepsilon)=\frac{B}{\varepsilon-\varepsilon_d-(V^{l*}_{pd}+V^{r*}_{pd}e^{i\phi})A} \tag{11}$$

and

$$G_{p\sigma\sigma'}(\varepsilon)=\frac{AB}{\varepsilon-\varepsilon_d-(V^{l*}_{pd}+V^{r*}_{pd}e^{i\phi})A} \tag{12}$$

where

$$A=\frac{1}{\varepsilon-\varepsilon_p}\left\{\sum_{k\alpha}\frac{|W^{\alpha}|^2}{\varepsilon-\varepsilon_{k\alpha}}\left(\frac{\sum\limits_{p}\frac{V^{l}_{pd}+V^{r}_{pd}e^{-i\phi}}{\varepsilon-\varepsilon_p}}{1-\sum\limits_{p}\frac{1}{\varepsilon-\varepsilon_p}\sum\limits_{k\alpha}\frac{|W^{\alpha}|^2}{\varepsilon-\varepsilon_{k\alpha}}}\right)+(V^{l}_{pd}+V^{r}_{pd}e^{-i\phi})\right\} \tag{13}$$

and

$$B=\frac{\delta_{\sigma\sigma'}}{2\pi}+\frac{\frac{\delta_{\sigma\sigma'}}{2\pi}U<n_{\bar{\sigma}}>}{\varepsilon-\varepsilon_d-U-\sum\limits_{p}\frac{|(V^{l}_{pd}+V^{r}_{pd}e^{-i\phi})|^2}{\varepsilon-\varepsilon_p-\sum\limits_{k\alpha}\frac{|W^{\alpha}|^2}{\varepsilon-\varepsilon_{k\alpha}}}} \tag{14}$$

$<n_{\sigma}>$ is the average occupied number of the electron level in the dot given by

$$< n_\sigma >= -\frac{1}{\pi}\sum_{\sigma\sigma'}\int_{-\infty}^{E_F} d\varepsilon\, \mathrm{Im}\, G_{\sigma\sigma'}(\varepsilon + i\delta) \tag{15}$$

where $\delta$ is an infinitesimal quantity.

To check formulas (11) and (12), we determine the dot Greens function in the atomic, non-interacting, and strongly interacting limits:

(i) In the atomic limit, where the tunneling matrix elements V go to zero, we arrive at

$$\underset{Atomic}{G_{\sigma\sigma'}}(\varepsilon) = \frac{\delta_{\sigma\sigma'}}{2\pi}\left\{\frac{1 - < n_{\bar{\sigma}} >}{\varepsilon - \varepsilon_d} + \frac{< n_{\bar{\sigma}} >}{\varepsilon - \varepsilon_d - U}\right\} \tag{16}$$

(ii) In the non-interacting limit, the intradot Coulomb interaction U becomes zero, hence we get

$$\underset{U\to 0}{G_{\sigma\sigma'}}(\varepsilon) = \frac{\delta_{\sigma\sigma'}}{2\pi}\frac{1}{\varepsilon - \varepsilon_d - (V_{pd}^{l*} + V_{pd}^{r*}e^{i\phi})A} \tag{17}$$

(iii) In the strongly interacting limit, the intradot Coulomb interaction U progresses to infinity, thus we obtain

$$\underset{U\to\infty}{G_{\sigma\sigma'}}(\varepsilon) = \frac{\delta_{\sigma\sigma'}}{2\pi}\frac{1 - < n_{\bar{\sigma}} >}{\varepsilon - \varepsilon_d - (V_{pd}^{l*} + V_{pd}^{r*}e^{i\phi})A} \tag{18}$$

We find that the expressions (16), (17), and (18) are in good agreement with the results [17, 19]. The only difference lies in the self energy since our system is different. To determine the retarded and advanced Greens functions of the ring-dot we introduce the following relationship

$$G^{r,a}_{p\sigma\sigma'}(\varepsilon) = G_{p\sigma\sigma'}(\varepsilon + i\delta) \tag{19}$$

By substituting $G^{r,a}_{p\sigma\sigma'}$ into (7), we then obtain the lower arm current expression (with an assumption that the matrix elements V are independent on p, and that the dot is always kept symmetric) as follows

$$\begin{aligned} I_{lower\_arm} = & \frac{e}{\hbar}\frac{\Gamma^L\Gamma^R}{\Gamma}\frac{1}{4\pi}\sum_p (f_L(\varepsilon_p) - f_R(\varepsilon_p)) \times \left\{ \frac{V_d(1+\cos\phi)(\varepsilon_p - \varepsilon_d)}{(\varepsilon_p - \varepsilon_d)^2 + [\Gamma_d(1+\cos\phi)]^2} + \right. \\ & + \frac{V_d(1+\cos\phi)\,U < n_{\bar{\sigma}} > (\varepsilon_p - \varepsilon_d)(\varepsilon_p - \varepsilon_d - U)}{[(\varepsilon_p - \varepsilon_d)^2 + [\Gamma_d(1+\cos\phi)]^2]\left[(\varepsilon_p - \varepsilon_d - U)^2 + \left[4V_d^2(1+\cos\phi)/_\Gamma\right]^2\right]} + \\ & \left. + \frac{4V_d^3(1+\cos\phi)^3\,\Gamma_d\, U < n_{\bar{\sigma}} >}{\Gamma\,[(\varepsilon_p - \varepsilon_d)^2 + [\Gamma_d(1+\cos\phi)]^2]\left[(\varepsilon_p - \varepsilon_d - U)^2 + \left[4V_d^2(1+\cos\phi)/_\Gamma\right]^2\right]} \right\} \end{aligned} \tag{20}$$

Here, $\Gamma_d = 2\pi V_d^2\delta(\varepsilon - \varepsilon_p)$ and $\Gamma = \Gamma^L + \Gamma^R$.

## 3. Discussion

In our calculations, the dot Green function was approximated up to the second order of U. The energy diagram of the dot and its density of states are then described in Figure 2.

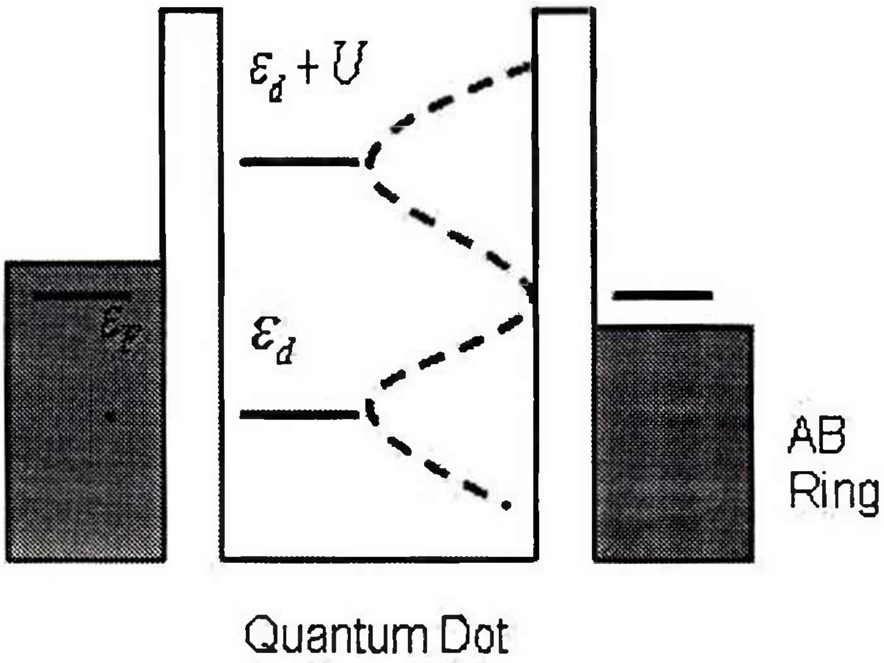


Figure 2. Energy diagram of the QD. The density of states is represented by the dashed line.

For simplicity, we introduce a symmetric model with $\varepsilon_d = -U/2$ and assume that the Fermi levels in the dot and in the ring are the same. Resonant tunneling between the two leads occurs and the current flows when an energy level in the dot is aligned with the Fermi level in the ring and in the leads. When one of the dot levels drops below the Fermi level in the ring, this level becomes occupied and the number of electrons increases by one in the dot. In order to scan the levels in the dot over the Fermi level, Yacoby et al. used a plunger gate [1]. By changing the plunger gate voltage, they were able to change as well as U.

In the Coulomb blockade regime and for $V_d < \Gamma << U$ and $\varepsilon_p << U$, the tunneling probability through the dot is very small, hence, the flux depending AB oscillations of the current are dominated by the lowest harmonics. All higher harmonics corresponding to electrons traveling two or more times around the ring are suppressed. As a result, it may be regarded that the Coulomb interaction in the dot does not influence the upper arm current. The behavior of the total current as a function of the AB phase can then be analyzed by means of the lower arm current, which is shown in Figure 3 for three different values of U (i.e. 0.1 (V), 0.4 (V), and 0.6 (V)).

Figure 3 shows that as U increases, the amplitude of the AB oscillation of the current decreases. This means that the intradot Coulomb interaction can suppress the interference. The figure also shows that the AB oscillations are all in phase. We can also find these features from the approximated expression for (20) for $V_d < \Gamma << U$ and $\varepsilon_p << U$. Under these conditions, (20) can be written as

$$I \approx \frac{V_d(1+\cos\varphi)}{U} + \frac{V_d^3(1+\cos\varphi)^3}{U^3} \tag{21}$$

This expression shows that: (i) the AB oscillation of the current is a function of cos$\varphi$, hence, the oscillations are periodic and all the oscillations for various U values are in phase, and (ii) the amplitude of the current is inversely proportional to U.

It is necessary to consider the system under as many physical phenomena as possible. In our study, the dot can be regarded as a magnetic impurity [19], hence, the essential extension is to investigate the transport property of the system in the spin-flip process and in the Kondo regime. The Kondo

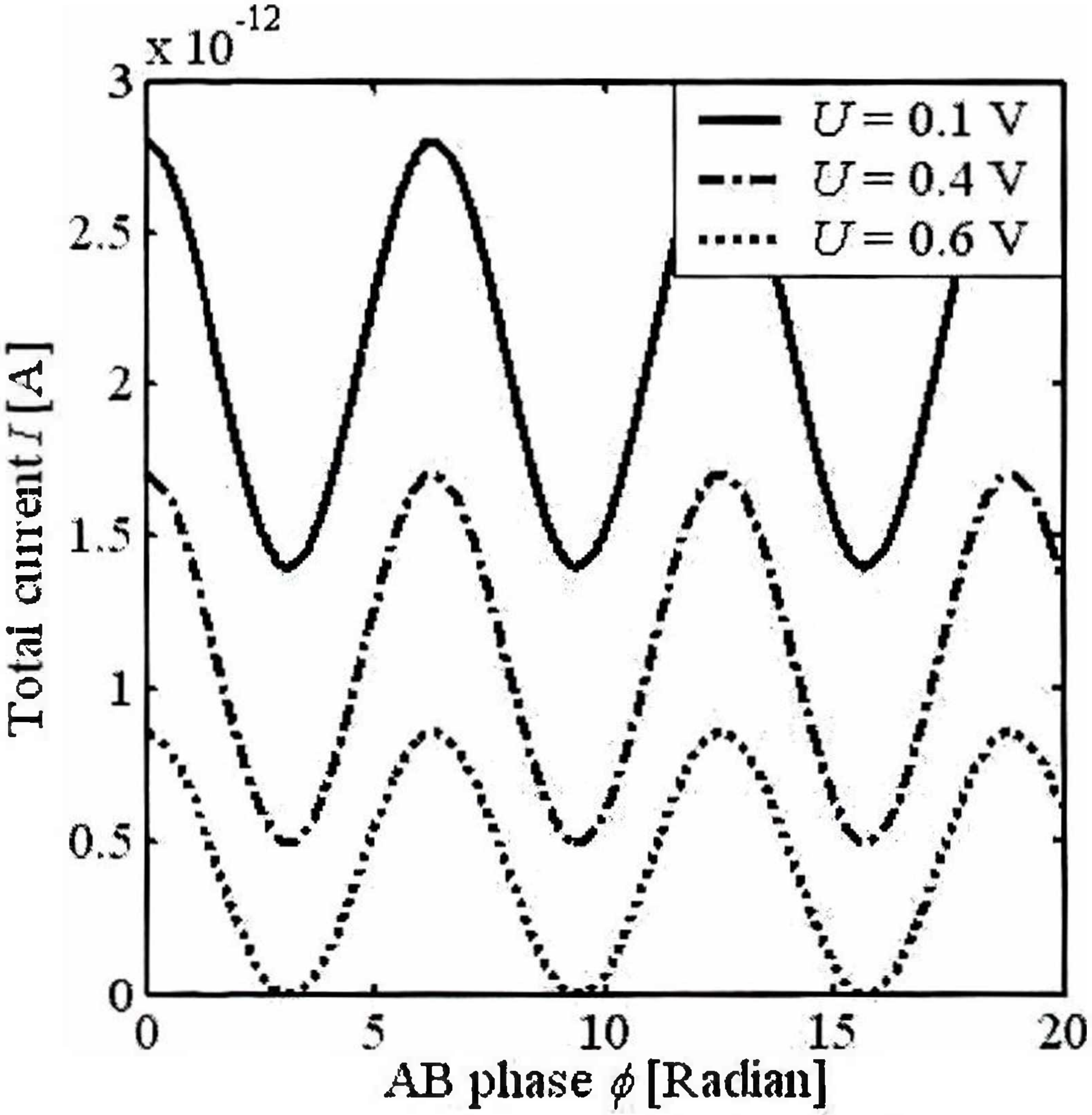


Figure 3. The current through the system as a function of AB phase for U=0.1, 0.4, 0.6 V.

effect provides clues to understanding the electronic properties of a variety of materials where the interactions between electrons are particularly strong. The Kondo effect only occurs when the defects are magnetic or when the total spin of the electron in an impurity atom is non-zero. These electrons coexist with the itinerant electrons in the host metal (e.g. the AB ring in the AB interferometer), which behave like a Fermi sea. In such a Fermi sea, all the states with energies below the Fermi level are occupied, while the higher energy states are empty. If we scan this level over the Fermi level, one electron from the ring should jump into the dot.

## 4. Conclusion

In conclusion, we obtained the current expression as a function of the intradot Coulomb interaction and the AB phase. The AB oscillations of the current are qualitatively in good agreement with [1, 3]. The results showed that the intradot Coulomb interaction induces dephasing.

**Acknowledgments.** The authors thank the fundamental research program 406206 for support.

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