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A combination of the identification algorithm and the modal superposition method for feedback active control of incomplete measured systems

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Abstract. In a previous paper [1], the identification algorithm is presented for feedback active controlled systems. However, this method can only be applied to complete measured systems. The aim of this paper is to present a combination of the identification algorithm and the modal superposition method to control the incomplete measured systems. The system response is expanded by modal eigenfunction technique. The external excitation acting on some first modes is identified with a time delay and with a small error depending on the locations of the sensors. Then the control forces will be generated to balance the identified excitations. A numerical simulation is applied to a building modeled as a cantilever beam subjected to base acceleration.

1. Introduction

The active control method can be applied to many problems such as robot control, ship autopilot, airplane autopilot, vibration control of vehicles or structures... Fig 1 provides a schematic diagram of an active control system.

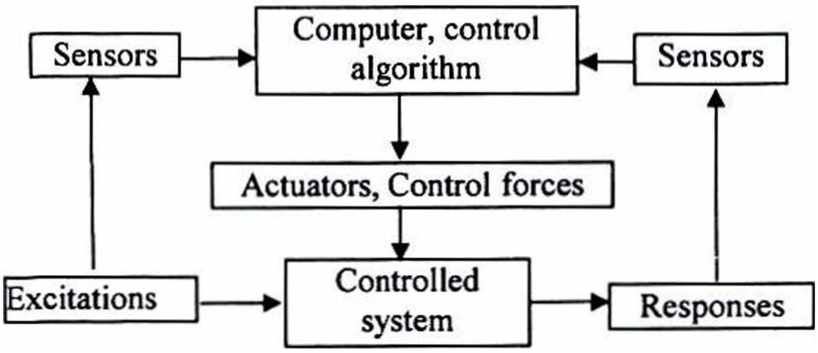

Fig. 1. Diagram of a structural control system.

It consists of 3 main parts: sensors to measure either external excitations or system responses or both; computer controller to process the measured information and to compute necessary control force

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based on a given control algorithm; actuators to produce the required forces. When only the responses can be measured, the method is called feedback active control. In recent years, the active control method has been widely used to reduce the excessive vibrations of civil structures due to environmental disturbances ([1-10]). One of the basic tasks of active structural control problem is to determine a control strategy that uses the measured structural responses to calculate an appropriate control signal to send to the actuator. Many control strategies have been proposed, such as LQR/LQG control [2,3], H_2/H_∞ control [4,5], sliding mode control [6], saturation control [7], reliability-based control [8], fuzzy control [9], neural control [10]... In fact, it is usually that one is unable to measure the external excitation while the structural response can often be measured. The identification algorithm presented in [1] is a method, which identifies the external excitation from the structural response measured. Although this version of identification algorithm can be applied even for the nonlinear structures, it requires knowledge of the entire state vector of the structure, which is not possible for large structures. Thus, the aim of this paper is to combine the identification algorithm and the modal superposition method for the linear structures with incomplete measurement, i.e only some components of state vector can be measured.

2. Problem formulation

Consider a multi-degree-of-freedom system described by the linear state equation

$$\dot{x}(t) = Ax(t) + u(t) + f(t), \quad x(0) = x_0 \tag{1}$$

Where, $x(t)$ is the n-dimensional state vector , $f(t)$ is the n-dimensional external force vector, $u(t)$ is the n-dimensional control vector, A is an $n \times n$ system matrix. Let $y(t)$ be the p-dimensional measurement (output) vector ($p \le n$) with:

$$y(t) = Cx(t) \tag{2}$$

Where, C is a $p \times n$ measurement matrix. The control force vector $u(t)$ is selected as a function of the measurement vector $y(t)$. The control problem is to find the active control force $u(t)$ necessary to reduce the norm state vector. It is seen obviously that the best control law is that

$$u(t) = -f(t) \tag{3}$$

Indeed with control law (3), the external excitation is totally eliminated. However, it is usually that one is unable to measure the external excitation, so the control law (3) cannot be realized in the practical application. The idea involved in the control law (3) may be used in a modified way, in which the history of the external excitation can be identified with a time delay by a so called identification process. The process identifying the entire external excitation is presented in [1] and is called the original identification algorithm here. The original identification algorithm requires the knowledge of the entire state vector to identify the entire excitation. However, when only the measurement vector in (2) can be measured, the excitation can not be identified all. In this paper, by using modal superposition method, the identification algorithm will be extended to identify some most important excitations base on measurement vector $y(t)$. The detail of this extension is presented in section 4.

3. Original identification algorithm

The original identification algorithm is developed in [1]. Let T be the time duration of the action of external excitation. Let all the components of state $x(t)$ can be measured and all components of its first and second order derivatives can be calculated in a short time. The interval $[0, T]$ is divided into n small equal intervals of the length Δ where Δ is a small positive number whose value depends on computation speed and accuracy of computer. Thus one has:

$$T = q\Delta$$

For any given function vector $m(t)$, the following notation is introduced:

$$m^{[k]}(t) = \begin{cases} m(t) & (k-1)\Delta \le t \le k\Delta \\ 0 & \text{otherwise} \end{cases} \quad k = 1,2,\ldots,q \tag{4}$$

In $T_k = \left[(k-1)\Delta \le t \le k\Delta\right]$, the system response is described by the following equation:

$$\dot{x}^{[k]}(t) = Ax^{[k]}(t) + u^{[k]}(t) + f^{[k]}(t) \tag{5}$$

In this subinterval, we assume that the control force $u^{[k]}(t)$ can be known (by the control law (7) below), the state vector $x^{[k]}(t)$ is measured and its first derivatives is calculated. Thus, the external disturbance $f^{[k]}(t)$ can be calculated as

$$f^{[k]}(t) = \dot{x}^{[k]}(t) - Ax^{[k]}(t) - u^{[k]}(t) \tag{6}$$

So, at the end of the subinterval T_k, one can know all about $f(t)$ in this subinterval. Because the subinterval T_k ended, this information can be used only in the next subinterval T_{k+1} to calculate $u^{[k+1]}(t)$, This means that the information about $f(t)$ has a time delay Δ. Using the information of the delayed external excitation $f(t)$, the control algorithm is proposed as:

$$\begin{cases} u^{[1]}(t) = 0 \\ u^{[k]}(t) = -f^{[k-1]}(t-\Delta) = -\left[\dot{x}^{[k-1]}(t-\Delta) - Ax^{[k-1]}(t-\Delta) - u^{[k-1]}(t-\Delta)\right] \quad k = 2,3\ldots q \end{cases} \tag{7}$$

As we see, the control law (7) is established in the inductive way. With control law (7), the delayed external excitation $f(t-\Delta)$ is totally eliminated. As mentioned above, the disadvantage of the original identification algorithm is the requirement of the knowledge of entire state vector $x(t)$.

4. Combination of the identification algorithm and the modal superposition method

The incomplete measurement leads to the incomplete excitation identification. Two questions need to be addressed: which excitation is important and how to identify it? These questions are not easy to answer if the system is nonlinear. However, in case of linear system as modeled in (1), the answer can be found by well-known modal eigenfunction technique. Let A have distinct eigenvalues λ_j (j=1,..n) and corresponding eigenvectors η_j. Assuming that the eigenvalues λ_j are ordered such as:

$$|\lambda_1| \le |\lambda_2| \le \ldots \le |\lambda_n|$$

Define the $n\times p$ matrix Φ_c ,the $n\times(n-p)$ matrix Φ_r, the $p\times n$ matrix Ψ_c and the $(n-p)\times$n matrix Ψ_r by

$$\Phi_c = \begin{bmatrix}\eta_1 & \eta_2 & \ldots & \eta_p\end{bmatrix};\ \Phi_r = \begin{bmatrix}\eta_{p+1} & \eta_{p+2} & \ldots & \eta_n\end{bmatrix};\ \begin{bmatrix}\Phi_c & \Phi_r\end{bmatrix}^{-1} = \begin{bmatrix}\Psi_c \\ \Psi_r\end{bmatrix}$$

The $p\times p$ diagonal matrix Λ_c and the $(n-p)\times(n-p)$ diagonal matrix Λ_r is also defined by:

$$\Lambda_c = diag\begin{bmatrix}\lambda_1 & \lambda_2 & \dots & \lambda_p\end{bmatrix};\ \Lambda_r = diag\begin{bmatrix}\lambda_{p+1} & \lambda_{p+2} & \dots & \lambda_n\end{bmatrix}$$

Then

$$A = \begin{bmatrix}\Phi_c & \Phi_r\end{bmatrix}\begin{bmatrix}\Lambda_c & 0 \\ 0 & \Lambda_r\end{bmatrix}\begin{bmatrix}\Psi_c \\ \Psi_r\end{bmatrix}$$

Applying the modal transformation

$$\begin{bmatrix}x_c(t) \\ x_r(t)\end{bmatrix} = \begin{bmatrix}\Psi_c \\ \Psi_r\end{bmatrix}x(t)$$

The state equation (1) is decoupled

$$\dot{x}_c = \Lambda_c x_c + u_c + f_c \tag{8}$$

$$\dot{x}_r = \Lambda_r x_r + u_r + f_r. \tag{9}$$

Where

$$x_c = \Psi_c x;\ x_r = \Psi_r x;\ u_c = \Psi_c u;\ u_r = \Psi_r u;\ f_c = \Psi_c f;\ f_r = \Psi_r f$$

The measurement vector $y(t)$ is also rewritten in modal space:

$$y = C_c x_c + C_r x_r \tag{10}$$

Where

$$C_c = C\Phi_c; C_r = C\Phi_r$$

As one knows, the vibrational modes corresponding to large eigenvalues often contribute insignificantly to the response [11], so attention needs to be paid only to a few vibrational modes. Thus, the important excitation is f_c and we need to identify it. The identification process here is implemented in the same manner of the process in section 3. The interval [0, T] is also divided into n small equal intervals of the length Δ. Using the notation (4), in $T_k = \left[(k-1)\Delta \le t \le k\Delta\right]$, the equation (8) has form:

$$\dot{x}_c^{[k]}(t) = \Lambda_c x_c^{[k]}(t) + u_c^{[k]}(t) + f_c^{[k]}(t)$$

Using (10), we have

$$f_c^{[k]}(t) = C_c^{-1}\dot{y}^{[k]}(t) - C_c^{-1}C_r\dot{x}_r^{[k]}(t) - \Lambda_c C_c^{-1}y^{[k]}(t) + \Lambda_c C_c^{-1}C_r x_r^{[k]}(t) - u_c^{[k]}(t)$$

$$\Rightarrow f_c^{[k]}(t) + E^{[k]}(t) = C_c^{-1}\dot{y}^{[k]}(t) - \Lambda_c C_c^{-1}y^{[k]}(t) - u_c^{[k]}(t) \tag{11}$$

Where

$$E^{[k]}(t) = C_c^{-1}C_r\dot{x}_r^{[k]}(t) - \Lambda_c C_c^{-1}C_r x_r^{[k]}(t) \tag{12}$$

In the subinterval T_k, we assume that the control force $u_c^{[k]}(t)$ can be known (by the control law (13) below), the measurement vector $y^{[k]}(t)$ is known and its first derivatives is calculated. But the error term $E^{[k]}(t)$ introduced through the truncation process is still unknown. Thus, from (11), we can not know the exact excitation $f_c^{[k]}(t)$, but only an estimate of $f_c^{[k]}(t)$ with an error $E^{[k]}(t)$. To attenuate this error term, the sensors should be located to obtain a significant contribution of the information of x_c. This means a large norm of C_c in comparison with the norm of C_r. Because the subinterval T_k ended, the information known can be used only in the next subinterval T_{k+1} to calculate $u^{[k+1]}(t)$. Using the delayed information, the control force u_c acting on the significant modes x_c is proposed as:

$$\begin{cases} u_c^{[1]}(t) = 0 \\ u_c^{[k]}(t) = -\left\{ f_c^{[k-1]}(t-\Delta) + E^{[k-1]}(t-\Delta) \right\} \\ \qquad = -\left[C_c^{-1}\dot{y}^{[k-1]}(t-\Delta) - \Lambda_c C_c^{-1} y^{[k-1]}(t-\Delta) - u_c^{[k-1]}(t-\Delta) \right] \quad k = 2,3...q \end{cases} \tag{13}$$

Besides, because it is unnecessary to control the insignificant vibrational mode x_r, we choose u_r=0 for the entire time duration. At last, we determine $u(t)$ by transformation from modal space to state space:

$$u = \Phi_c u_c + \Phi_r u_r = \Phi_c u_c \tag{14}$$

The control law using the combination of the identification algorithm and the modal superposition method is described as (13) and (14).

5. Numerical simulation

Considering a base excited building modeled as a vertical cantilever beam as showed in Fig 2.

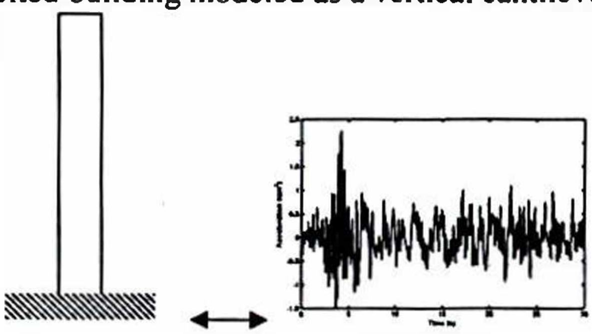


Fig. 2. Model of a cantilever beam subjected to base acceleration.

The characteristics of the beam are taken from [12]. The beam has a square cross-section with the dimension of 21m x 21m. The total mass is 153,000 tons, the total height is 306m, the modulus of elasticity is 40 GPa and the damping ratios for all modes are assumed to be 2%. Using the method of separation of variables, the governing partial differential equation of the beam is represented by a system of infinite ordinary differential equations. After that, the system of infinite equations is truncated to derive the state equation [11]. In this calculation, the truncated system retains five differential equations. We assume that there is only one sensor measuring the displacement of a certain point of the beam. Because the velocity can be calculated from the displacement, the measurement vector contains 2 components: the displacement and the velocity of the point, where the sensor is located on. That means the measurement matrix C in (2) has 2 rows. The state vector of the beam has 10 components, in which only 2 first modes are controlled by the identification algorithm. The numerical simulations are taken when the sensor is placed at the distances $L/4$, $L/2$ and L from the base. In Fig 3, the shapes of the 1st mode, the 3rd mode and the 5th mode are drawn from left to right. As we see, if the sensor locates at the distance $L/4$, the contribution to the measurement information of the 1st mode (which is retained) is smaller than that of the higher modes (which are truncated). Thus, in this case, the error produced through the truncation process in (12) might be large.

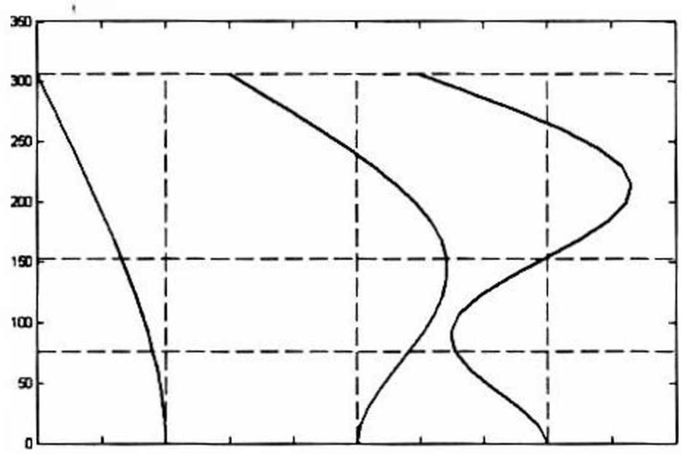
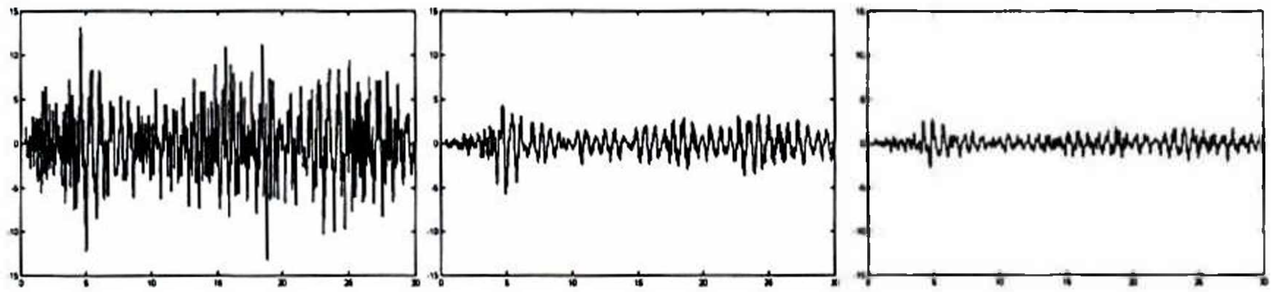


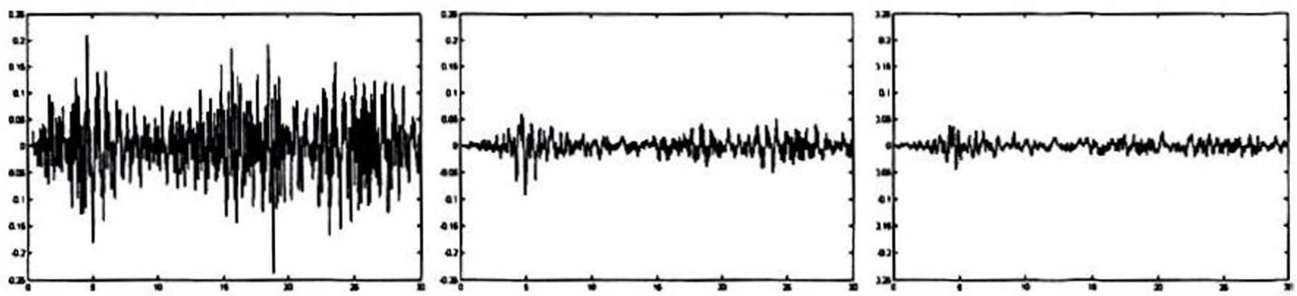
Fig. 3. The 1st, 3rd and 5th mode shapes of the beam.

To see more clearly, we plot the history of the error term. Since the measurement matrix C has 2 rows, the error term $E(t)$ in (12) is a 2-dimensional vector. The histories of 2 components of $E(t)$ are plotted in Fig 4 and 5 for each case of the location of sensor .



a) b) c)

Fig. 4. The history of the 1st component of error term $E(t)$, sensor locates at the distance $L/4$ (a), $L/2$ (b) and L (c).



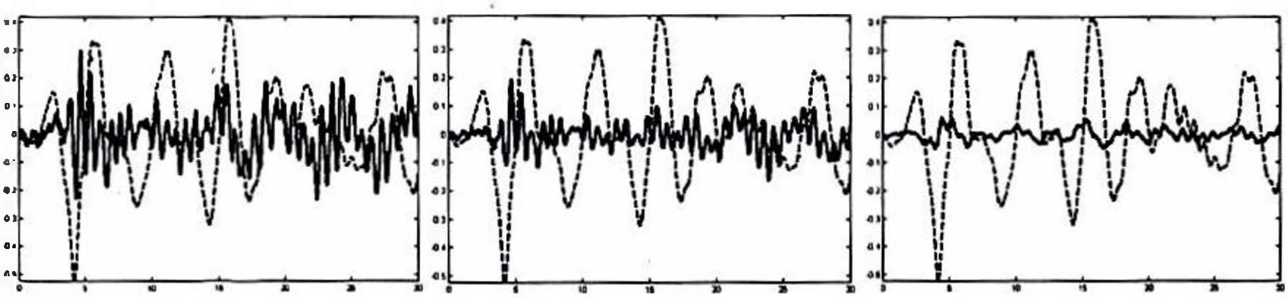
a) b) c)

Fig. 5. The history of the 2nd component of error term $E(t)$, sensor locates at the distance $L/4$ (a), $L/2$ (b) and L (c).

It can be seen that, locating the sensor at the distances $L/2$ and L is better than at the distance $L/4$. However, more investigate need to be done in the future to find the method seeking the optimal locations of the sensors. The time delay is taken with 1/500 and 1/800 of total duration time T. Some of the controlled results are shown in table 1 and Fig 6 and 7. In Fig 6 and 7, thin and dotted lines are uncontrolled responses

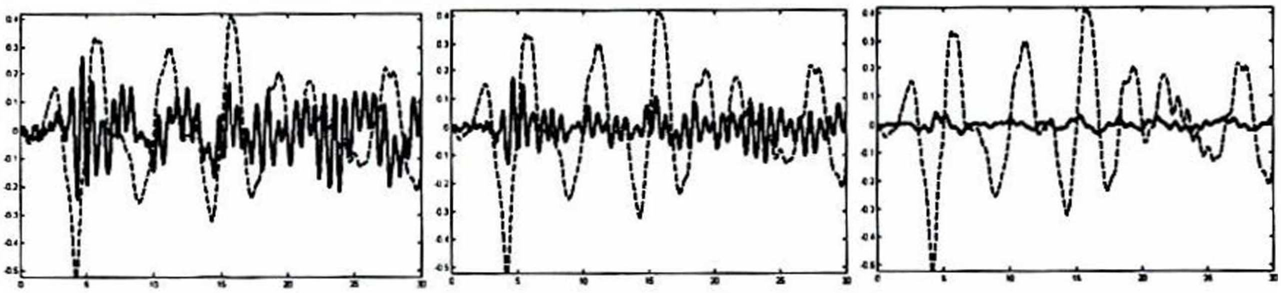
Table 1: The peak displacement in the numerical simulation

Distance locate the sensor		L/4		L/2		L	
Time delay (% of total time)		0.2	0.125	0.2	0.125	0.2	0.125
Top point displacement (cm)	Controlled	29.9	26.45	19.34	17.43	6.02	4.16
	Uncontrolled	52.22					



a) b) c)

Fig. 6. The history of top point displacement, Δ= 0.2%T, sensor locates at the distance L/4 (a), L/2 (b) and L (c).



a) b) c)

Fig. 7. The history of top point displacement, Δ= 0.125%T, sensor locates at the distance L/4 (a), L/2 (b) and L (c).

As we see, locating the sensor at the distances $L/2$ and L leads to the smaller response than locating at the distance $L/4$. Return to figures 4 and 5, this situation can be understood because the effect of identification algorithm depends on the error term $E(t)$.

6. Conclusion

This paper proposes a combination of the identification algorithm and the modal superposition method for feedback active control of incomplete measured systems. The system is expanded to the modal space. A limited number of sensors are used to measure some components of the state vector. Using this incomplete information, an algorithm is presented to identify the external excitation acting on some first modes. The excitation is identified with a time delay and a small error term. The magnitude of the error term depends on the number and the locations of the sensors. The numerical simulation is applied to a base excited cantilever beam to illustrate the algorithm. The effects of the time delay and the location of sensor are considered.

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