A NEW VIEW ON AN OLD PROBLEM IN QUANTUM CHROMODYNAMICS

Nguyen Suan Han

Department of Physics, College of Science, VNU

Abstract. We suggested a new infrared mechanism of dimensional transmutation that is omitted in the conventional approach and leads effectively to the stochastization of the Faddeev-Popov functional. We have proved possibilities of such a stochastization in the Abelian version of the collective excitation and showed that the quantization of infrared fields $\overrightarrow{k}^2 = 0$ leads to one of the versions of the "confinement propagator" .

1. Introduction

Quantum Chromodynamics (QCD)has arisen [1[has fruitfully developed [2] as theory version of the quark-parton model after the discovery of asymptotic freedom phenomenon with the help of renormalization group method [3].

QCD has been constructed by analogy with quantum electrodynamics (QED), all physical consequences of which can be got from the first principles of symmetry and quantization . The main task for QCD up to now was foundation of its working hypotheses from first principles. One can roughly separate those hypotheses into parts related of the low (i) and high (ii) energies: i/ The hypotheses of the short-distance action of gluon forces, that concern the $PCAC$ (F_π) and hadron spectra (α'); ii/ The principle of the local quark-hadron duality ($LQUD$) and its modifications.

QCD inherits the principle $LQHD$ from the Feynman naive parton model , Feynman justified this principle with the help of the unitary conditions $SS^+ = 1$ $(S = 1 + iT)$

$$\sum_k < i|T|h >< h|T^*|j >= 2Im < i|T|j > . \qquad (1)$$

In the left-hand of $Eq.(1)$ it is sum over the complete set of hadron physical states. For the calculation of the right-hand side of $Eq.(1)$ Feynman proposed that it is a quantum field theory for partons which do not contribute to the physical states on the left -hand side and for which the usual free propagators perturbation theory are valid.This assumption allows interpretation of the inclusive - processes cross-sections in terms of the imaginary parts of the quark-parton diagrams and hence determination of the quark quantum numbers [3] (which are the basis for the construction of unification theory).

Now the principle $LQHD$ is the basis of the QCD-phenomenology [2,7]. The experimental momentum distributions of hadrons in the left-hand side of $Eq.(1)$ entirely reproduce the distributions of partons (quarks , antiquarks, gluons), whose dynamics is completely controlled by the right-hand side of $Eq.(1)$ (i.e. QED-perturbation theory). Here the following paradox arises: in the right-hand of $Eq.(1)$ one uses the free of quarks

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

and gluons in the mass shell regime and simultaneously one proposes that quarks and gluons do not contribute to the observable physical states of the left-hand side of $Eq.(1)$. Just the absence of the quark and gluon states in the left-hand side $Eq.(1)$ is called the confinement hypothesis.

The proof of confinement has to satisfy the principle of accordance with the parton model . (Any attempts to support the confinement with the quark propagator modification by removing their poles in the scaling region, simultaneously removes the very possibilities of the quark-parton interpretation of deep -inelastic processes).

The QCD hypotheses now are explained by the asymptotic freedom phenomenon [5]

$$\alpha(q^2) = \frac{1}{\beta log\left(\frac{q^2}{\Lambda^2}\right)}, \tag{2}$$

where $\beta = \frac{1}{4\pi}\left(11 - \frac{2}{3}n_f\right)$; n_f is the number of flavours of quarks, Λ is the infrared boundary conditions for solutions of the renormalizable equation. this formula of asymptotic freedom defines the ranges of validity of perturbation theory as at $q = \Lambda$ the coupling constant is infinite. Attempts are made to bind Λ with the scale of the $PCAC$ and hadron mass spectra.

At this stage the ideology of potential confinement arise. Its essence consists in the aspiration to got confinement gluon propagator (or quark-quark potential) by an approximate summation of the Feynman diagrams by means of renormalization group equations or the Schwinger - Dyson ones [8].

As a model of such a confinement one proposes the following gluon propagator

$$\frac{M^2}{q^4}; M^2\delta^4(q); \left(V(r) = \alpha' r; \frac{r^2}{V}\right).$$

Further calculations on the basis of such a propagator are found in the main on solutions of the Schwinger-Dyson or the Bether -Salpeter equations of the type

$$\sum(p) = \frac{e^2}{(2\pi)^4 i}\int d^4q D_{\mu\nu}(q)\gamma_\mu \frac{1}{\widehat{p} - \widehat{q} - m - \sum(p-q)}\gamma_\nu \tag{3}$$

and the results of the calculations are the hadron mass spectrum condensates F_π, etc, [8,9].

2. A new view on QCD

Remarkable success in constructing the consistent quantum gravitational theory (superstring $E_8 \times E_8$ [10] gives reasons to recomprehend a new both solved and unsolved QCD problems. In particular , now one undertakes construction of a finite unification field theory (without divergences)[11] constraining QCD as a part, It should be noted, in the theory without ultraviolet divergences the renormalization group equations turn into identities and have no any physical information including the asymptotic freedom

The asymptotic freedom phenomenon in such theory may be only a consequence of the trivial summation of the Feynman diagrams in the one-log approximation

$$\alpha(q^2) = \frac{\alpha(M_s^2)}{1+\beta\alpha(M_s^2)log\left(\frac{q^2}{\Lambda_s^2}\right)} = \frac{1}{\beta log\left(\frac{q^2}{\Lambda^2}\right)},$$

where M_s is the scale of the supersymmetry breaking in the ultra-relativistic region of the asymptotic "desert" ($M_s \sim 10^{15} m_p$). The parameter Λ

$$\Lambda^2 = M_s^2 exp\left(-\frac{1}{\beta\alpha(M_s^2)}\right)$$

here really does not relate to the infrared gluon interaction (as one proposes in the renormalization group version of the dimensional transmutation).

A new point view forced us to find another infrared mechanism for justifying the QCD hypotheses (i,ii).

As has been shown in ref. [12], for confinement of colour fields the infrared degeneration of the gauge (phase) factors are sufficient: due to destructive interference of these factors the amplitudes with colour particles disappear and do not contribute to the left - hand side of the $Eq.(1)$

Here, we consider the dynamics of the infrared fiels $\partial_i^2 A_j(\overrightarrow{x},t) = 0$ that omitted in the canonical relativistic covariant method of quantization of gauge fields [13-15]. It is well-known that this method is based on the transverse commutation relation [15,16]

$$i\left[E_i^{T\,a}(\overrightarrow{x},t), A_j^{T\,b}(\overrightarrow{y},t)\right] = \delta^{ab}\delta_{ij}^T\delta^3(\overrightarrow{x}-\overrightarrow{y}) \quad (4)$$

$$\left((\delta)_{ij}^T = \delta_{ij} - \partial_i \frac{1}{\partial_k}^2 \partial_j \equiv \delta_{ij} + \frac{1}{4\pi}\frac{\partial}{\partial x_i}\int \frac{d^3z}{|x-z|}\frac{\partial}{\partial x_j}\right),$$

that is given on the function class

$$\partial_i^2 A_j(\overrightarrow{x},t) \neq 0; \int d^3x A_j(\overrightarrow{x},t) = 0. \quad (5)$$

The infrared dynamics fields

$$\partial_i^2 b_j^a(\overrightarrow{x},t) = 0 \quad (6)$$

are omitted by the communication relations (4). In QCD this omission is physically justified, as these quantum fields are unobservable due to the finite energy arrangement resolution [12].

In QCD we have no such justification. Moreover, the including of the gluon fields (6) may be justified by the nonlinearity of the theory and the strong coupling of fields in the infrared limit (that leads as a role to collective excitation of infrared gluons correlated in the whole volume of the space they occupy, $V = \int d^3x$). There is a trivial generalization of the commutation relations (4) with the space - constant fields b_i^a included

$$i\left[E_i{}^a(\overrightarrow{x},t), A_j{}^b(\overrightarrow{y},t)\right] = \delta^{ab}\left[\delta_{ij}^T\delta^3(\overrightarrow{x}-\overrightarrow{y}) + \frac{\delta_{ij}}{V}\right], \quad (7)$$

where $A_i^a(\overrightarrow{x},t) = A^{T}{}_i^a(\overrightarrow{x},t) + b_i^a(t)$.

The most consistent canonical quantization of the theory

$$L = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \overline{\psi} i\gamma_\mu \nabla_\mu \psi - m\overline{\psi}\psi; S = \int d^4L \tag{8}$$

$$\nabla_\mu = \partial_\mu + \widehat{A}_\mu; \widehat{A}_\mu = g + \frac{\tau^a A^a}{2i}; \widehat{F}_{\mu\nu} = \partial_\mu \widehat{A}_\nu - \partial_\nu \widehat{A}_\mu + [\widehat{A}_\mu \widehat{A}_\nu]$$

with the constraint equation equation $\delta S/\delta A_0^a = 0$ which has been made in [13].

According to this paper the quantization of only the transverse fields A_μ^T and the fields ψ leads to the effective potential for the fields

$$Z(b_i|0,0) = \int d^4 A_\mu^a d\psi d\overline{\psi} \delta(\partial_i A_i^a) det\Big[\nabla_i (A_i + b_i \partial_i)\Big] \times$$

$$\times \exp\Big\{ iS(A_0, A_i + b_i) + i\int d^4x(\overline{\psi}\eta + \overline{\eta}\psi)\Big\}|_{\eta=\overline{\eta}=0} = \exp\Big\{ iV\int dt\Big[\frac{1}{2}(\partial_0 b)^2 + \phi(b)\Big]\Big\} \tag{9}$$

where

$$V\int dt\phi(b) = -\frac{1}{4}\int d^4(F^a(b)_{ij}^2) - itrlogdet\Big[\nabla_i(A+b)\partial_i\Big] + ... \tag{10}$$

is the potential of the infrared fields (6), induced by all interactions in the whole volume V.

The quantization of the infrared fields in the limit of the infinite volume V can be reduced to the stochastization of the Green function generating functional

$$Z(\eta,\overline{\eta}) = \exp\Big\{\frac{1}{2}M^2(\frac{\partial}{\partial b_i^a})^2\Big\}\Bigg[\frac{Z(b|\eta,\overline{\eta})}{Z(b|0,0)}\Bigg]|\, b = 0, \tag{11}$$

where M is the infrared dimensional transmutation parameter - the analog of the arrangement energy resolution in QED. (Recall that the old renormalization group QCD parameter Λ was also defined by a nonperturbative interaction in the infrared region where the renormalization group method was invalid).

The relativistic covariant version of $Eq.(11)$ has the form

$$Z^l(\eta,\overline{\eta}) = \exp\Big\{\frac{1}{2}M^2\Big(\frac{\partial}{\partial b_i^a}\Big)^2\Big\}\Bigg[\frac{Z^l(b^l|\eta,\overline{\eta})}{Z^l(b^l|0,0)}\Bigg] \tag{12}$$

$$Z^l(b^l|\eta,\overline{\eta}) = \int d\overline{\psi} d\psi d^4 A_\mu^a \delta(\partial_\mu^l A^{l\mu}) det(\nabla_\mu^l \partial^{\mu l}) \times$$

$$\times \exp\Big\{ iS[A_\mu + b_\mu^l] + i\int d^4x(\overline{\psi}\eta + \overline{\eta}\psi)\Big\}, \tag{13}$$

where $B_\mu^l = B_\mu - l_\mu(B_\nu l^\nu)$, $B^l = (b^l, A^l, \partial^l, \nabla^l)$; l_μ is the time axis of quantization .

The possibilities of such a stochastization and its physical meaning can be seen on the simplest example of the Abelian theory given on the function class (6)

$$L = \int d^3x \left[-\frac{1}{4}F_{\mu\nu}^2(b) - \frac{\mu^2}{2}b_i^2 + b_i j_i(\vec{x},t) \right] =$$

$$= \frac{1}{2}V \left[(\partial_0 b_i^2 - \mu^2 b_i^2) \right] + b_i \int d^3x j_i(\vec{x},t). \tag{14}$$

We introduced here the mass μ for the infrared regularization of the propagator of the quantum field b_i

$$p_i = \frac{\partial L}{\partial(\partial_0 b_i)}; i[p_j, b_i] = \delta_{ji}; \left(i[(\partial_0 b_j, b_i)] = \frac{\delta_{ji}}{V} \right) \tag{15}$$

$$D_{ij}(t) = \frac{1}{i} < 0|T(b_i(t)b_j(0))|0 >= \frac{\delta_{ij}}{2\pi V} \int dq_0 \frac{e^{iq_0 t}}{q_0^2 - \mu^2 + i\epsilon} = i\frac{e^{i\mu|t|}}{2\mu V}. \tag{16}$$

We see that there is a limit of the infinite volume

$$V \to \infty; \mu \to 0; 2\mu V = M^{-2} \neq 0, \tag{17}$$

with the nonzero propagator (16)

$$\lim_{V\to\infty,\mu\to 0} D_{ij}(t) = iM^2 \neq 0 \tag{18}$$

The evolution operator for the theory (14) has the form

$$\lim_{V\to\infty,\mu\to 0} < e^{-iTH} >= \exp\left\{ -\frac{M^2}{2}\left(\frac{\partial}{\partial b_i}\right)^2 \right\} e^{ib_i j_i} \mid b = 0; j_i = \int d^4x j_i.$$

It is clear that in the limit (17) the propagator of the total field $A_i = A_i^T + b_i$ has the form of the sum of the usual transverse propagator and expression (18)

$$D_{ij}(x) = \frac{1}{i} < 0|T[A_i(x), A_j(0)]|0 >= D_{ij}^T(x) + iM^2 \tag{19}$$

or in the momentum representation

$$D_{ij}(q) = \left(\delta_{ij} - q_i \frac{1}{q^2} q_j \right) \frac{1}{q_\mu^2} + i(2\pi)^4 \delta^4(q) M^2. \tag{20}$$

So, we have got one of versions of the confinement propagator [9] that reflects the collective excitation of the infrared fields (6) in the whole space they occupy. In the light of this fact the attempts to got the confinement propagator by analytical calculation in the framework of the of the convention perturbation theory given only in the function class (5) [8,9] look very doubtful.

For the generation function of the Green functions for the Abelian theory with the communication relations like (7) in the limit (19), we got the expression of the type of (11)

$$Z(\eta,\overline{\eta}) = \exp\left\{\frac{1}{2}M^2\left(\frac{\partial}{\partial b_i^a}\right)^2\right\}\left[Z(b|\eta,\overline{\eta})\right]|\, b=0, \qquad (21)$$

$$Z(b_i|\eta,\overline{\eta}) = \int d^4A_\mu^a d\psi d\overline{\psi}\delta(\partial_i A_i^a)\exp\{iS\left[A_0, A_i + b_i\right] + i\int d^4x(\overline{\psi}\eta + \overline{\eta}\psi)\},$$

where $S[A_\mu]$ is the usual QED action . As has been shown in ref. [13] the correct transformation properties of the operator formalism [15] can be restored in terms of the functional integral if in it one explicit takes into account the time dependent axis l_μ of quantization

$$Z^l(b^l|\eta,\overline{\eta}) = \exp\left\{\frac{1}{2}M^2\left(\frac{\partial}{\partial b_i^a}\right)^2\right\}\int d\overline{\psi}d\psi d^4A_\mu^a\delta(\partial_\mu^l A^{l\mu})\times$$

$$\times\exp\left\{iS\left[A_\mu + b_\mu^l\right] + i\int d^4x(\overline{\psi}\eta + \overline{\eta}\psi)\right\}|midb = 0$$

where $A_\mu^l = A_\mu - l_\mu(l_\nu A^\nu)$.

If we neglect the interaction with the transverse fields, we can exactly calculate the function fermion Green function and the corrector of two currents

$$G(p_0, \overrightarrow{p} = 0) = \exp\left\{\frac{1}{2}M^2\left(\frac{\partial}{\partial b_i^a}\right)^2\right\}[\widehat{p} - eb_i\gamma_i - m]^{-1} =$$

$$= \frac{\widehat{p}+m}{e^2M^2}\left\{-1 + \sqrt{\pi\delta}e^\delta\left[1 - \phi(\sqrt{b})\right]\right\}; \delta = \frac{m^2 - p^2}{2e^2M^2}; \phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x dte^{-t^2}, \qquad (22)$$

$$< j(q)j(-q) >= \exp\left\{\frac{1}{2}M^2\left(\frac{\partial}{\partial b_i^a}\right)^2\right\}\int d^4ptr\gamma_\mu[\widehat{p} + \widehat{q} - eb_i\gamma_i - m]^{-1}\gamma_\nu[\widehat{p} - eb_i\gamma_i - m]^{-1}$$

$$= \int d^4ptr\gamma_\mu[\widehat{p} - \widehat{q} - m]^{-1}\gamma_\nu[\widehat{p} - m]^{-1}. \qquad (23)$$

In the Abelian version of the collective excitation analytical properties of the correlator (28) do not change the Green function (22) its pole. Note that in the potential version of confinement the physical consequences of the propagator $\delta^4(q)$ [9] are obtained with the help of the Schwinger-Dyson equation of the type of $Eq.(3)$

$$\sum(p) = -\widehat{p}A(p^2) + B(p^2);$$

$$B(p^2) - \widehat{p}A(p^2) = 3e^2M^2[\widehat{p}(1 + A) + (1 + B)]^{-1}.$$

It is easy to convince oneself that the solution of this equation does not concern the exact expression (22) . In QCD the expression (12) leads to the infinite power series in momenta M^2/q^2 , that disappear in limit M^2 or $q^2 \to 0$. In this limit we get the usual QCD. The constant fields $b_i(\overrightarrow{k}^2)$ take part only in hadronization of the colour fields in the low-energy region.

3. Conclusion

From 1974 till 1984 the renormalization group idea of asymptotic freedom dominate in QCD. Constructively this idea consists in the introduction of the QCD parameter Λ as the infrared boundary condition of the renormalization group equation in the region, where this equation is in valid. In this sense the parameter Λ reflects the result of an infrared nonperturbative interaction denoted by dimensional transmutation.

The 1984 theoretical revolution led to consistent unification theories without ultraviolet divergences, where the renormalization group became identities and lost their physical meaning . We can see in such a theory that the mysterious infrared dimensional tramutation is absent and the parameter Λ sooner reflects the ultraviolet scale of the supersymmetry breaking in the asymptotic desert region than the infrared nonperturbative interaction.

We suggested a new infrared mechanism of dimensional transmutation that is omitted in the conventional approach and leads effectively to the stochastization of the Faddeev-Popov functional. We have proved the possibilities of such a stochastization in the Abelian version of the collective excitation and showed that the quantization of infrared fields $\vec{k}^2 = 0$ leads to one of the version of the "confinement propagator"

Acknowledgements. We are grateful to Profs. B. M. Barbashov, Yu. L. Kalinovski, and V. N. Pervushin for useful discussions. This work was supported in part by Vietnam National Research Programme in National Sciences.

References

1. H.Fretzsch, M. Gell-Mann, Leutwyler, *Phys. Lett.*, **B47**(1973) 375.
2. A.V. Efremov, A.V. Radushkin, *Riv. Nuovo Cimento*, **3**(1980) 2.
3. R.P. Feynman, *Hadron Interaction*, New York, N.Y. 1972.
4. N.N. Bogolubov, B.V. Struminski, A.N. Tavkhelidze, *JINR, D-1986*, Dubna, 1965.
5. D. Gross, F. Wilezek, *Phys. Rev.*, **D8**(1973) 13633.
6. N.N. Bogolubov, D.V. Shirkov, *Introduction to Theory of Quantized Fields*, Moscow, 1976, Nauka.
7. P.D.B. Collins, A.D. Martin, *Hadron Interactions*, Adam Hilder Ltd. Bristol, 1984.
8. A.I. Alekceev, B.A. Arbuzov, *V,A, Baikov, TMF,* **52**(1982) 197.
9. R.L. Stuller, *Phys.Rev.* **D13**(1976) 513.
10. E. Witten, *Nucl. Phys.* **B258**(1985) 75.
11. D.V. Shirkov, *Foundation of Physics*, **16**(1986) 27.
12. Nguyen Suan Han, V.A. Pervushin, *Forschritte Der Physik,* **N8** (1989) 611.
13. Nguyen Suan Han, *J. Communications in Theoretical Physics*, China, **37**(2002) 167.
14. L.D. Faddeev, A.A. Slavnov, *Introduction in Quantum Theory of Gauge Fields,* Nauka 1976.
15. J. Schwinger, *Phys. Rev.,* **127**(1962) 324.
16. E.S. Abers, B.W. Lee, *Phys. Reports,* **9**(1973) 1.