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The dependence of the parametric transformation coefficient of acoustic and optical phonons in doped superlattices on concentration of impurities

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Abstract. The parametric transformation of acoustic and optical phonons in doped superlattices is theoretically studied by using a set of quantum kinetic equations for the phonons. The analytic expression of parametric transformation coefficient of acoustic and optical phonons in doped superlattices is obtained, that depends non-linear on the concentration of impurities. Numerical computations of theoretical results and graph are performed for GaAs:Si/GaAs:Be doped superlattices.

Keywords: parametric transformation, doped superlattices.

1. Introduction

It is well known that in presence of an external electromagnetic field, an electron gas becomes non-stationary. When the conditions of parametric resonance are satisfied, parametric resonance and transformation (PRT) of same kinds of excitations such as phonon-phonon, plasmon-plasmon, or of different kinds of excitations, such as plasmon-phonon will arise; i.e., the energy exchange process between these excitations will occur [1-9]. The physical picture can be described as follows: due to the electron-phonon interaction, propagation of an acoustic phonon with a frequency $\omega_{\vec{q}}$ accompanied by a density wave with the same frequency Ω. When an external electromagnetic field with frequency is presented, a charge density waves (CDW) with a combination frequency $\upsilon_{\vec{q}} \pm l\Omega$ (l=1,2,3,4...) will appear. If among the CDW there exits a certain wave having a frequency which coincides, or approximately coincides, with the frequency of optical phonon, $\upsilon_{\vec{q}}$, optical phonons will appear. These optical phonons cause a CDW with a combination frequency of $\upsilon_{\vec{q}} \pm l\Omega$, and when $\upsilon_{\vec{q}} \pm l\Omega \cong \omega_{\vec{q}}$, a certain CDW causes the acoustic phonons mentioned above. The PRT can speed up the camping process for one excitation and the amplification process for another excitation. Recently, there have been several studies on parametric excitation in quantum approximation. The parametric

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interactions and transformation of acoustic and optical phonons has been considered in bulk semiconductors [1-5], for low-dimensional semiconductors (doped superlattices, quantum wells, quantum wires), the dependence of the parametric transformation coefficient of acoustic and optical phonons on temperature T and frequency Ω is has been also studied [6-9]. In order to improve the PRT theoretics for low-dimensional semiconductors, we, in the paper, examine dependence of the parametric transformation coefficient of acoustic and optical phonons in doped superlattices on concentration of impurities.

2. Model and quantum kinetic equation for phonons

We use model for doped superlattice with electron gas is confined by the superlattice potential along the z direction and electrons are free on the xy plane. If a laser field $\vec{E}(t)=\vec{E}_0 sin(\Omega t)$ irradiates the sample in direction which is along the z axis, the electromagnetic field of laser wave will polarize parallels the x axis and y axis, and its strength is expressed as a vector potential $\vec{A}(t) = \frac{c}{\Omega}\vec{E}_0 cos(\Omega t)$ (c is the light velocity; Ω is EMW frequency; E_0 is electric field intensity).

The Hamiltonian of the electron-acoustic phonon-optical phonon system in doped superlattice can be written as (in this paper, we select $\hbar=1$):

$$H(t)=\sum_{\alpha}\varepsilon_{\alpha}(t)a_{\alpha}^{+}a_{\alpha}+\sum_{\vec{q}}\omega_{\vec{q}}b_{\vec{q}}^{+}b_{\vec{q}}+\sum_{\vec{q}}\upsilon_{\vec{q}}c_{\vec{q}}^{+}c_{\vec{q}}+$$
$$+\sum_{\vec{q}}\sum_{\alpha\alpha'}C_{\vec{q}}I_{n,n'}(\vec{q})a_{\alpha'}^{+}a_{\alpha}(b_{\vec{q}}+b_{-\vec{q}}^{+})\sum_{\vec{q}}\sum_{\alpha\alpha'}D_{\vec{q}}I_{n,n'}(\vec{q})a_{\alpha'}^{+}a_{\alpha}(c_{\vec{q}}+c_{-\vec{q}}^{+}) \tag{1}$$

where $\varepsilon_{\alpha}(t)=\varepsilon_n(\vec{k}_{\perp}+\frac{e}{c}\vec{A}(t))$ is energy spectrum of an electron in external electromagnetic field, a_{α}^{+}, (a_{α}) is the creation (annihilation) operator of an electron for state $|n,\vec{k}_{\perp}\rangle$, $b_{\vec{q}}^{+}$, $b_{\vec{q}}$ ($c_{\vec{q}}^{+}$, $c_{\vec{q}}$) is the creation operator and annihilation operator of an acoustic (optical) phonon for state have wave vector $\vec{q}$.

The electron-acoustic and optical phonon interaction coefficients take the forms[10]

$$|C_{\vec{q}}|^2=\frac{q\xi^2}{2\rho v_s V}; \qquad |D_{\vec{q}}|^2=\frac{e^2 v_{\vec{q}}}{2V\chi q^2}(\frac{1}{\chi_\infty}-\frac{1}{\chi_0}) \tag{2}$$

here V, ρ, v_s, and ξ are the volume, the density, the acoustic velocity and the deformation potential constant, respectively. χ is the electronic constant, χ_∞, χ_0 are the static and high-frequency dielectric constants, respectively. The electron form factor, $I_{nn'}(\vec{q})$ is written as [11]:

$$I_{nn'}(\vec{q})=\sum_{j=1}^{s_0}\int_0^d e^{iqd}\Phi_n(z-jd)\Phi_{n'}(z-jd)dz \tag{3}$$

here, $\Phi_n(z)$ is the eigenfunction for a single potential well, and s_0 is the number of doped superlattices periods, d is the period.

Energy spectrum of electron in doped superlattic [12]:

$$\varepsilon_n(\vec{k}_\perp)=\varepsilon_n+\frac{\vec{k}_\perp^2}{2m}; \qquad \varepsilon_n=(\frac{4\pi e^2 n_D}{\chi m})^{\frac{1}{2}}(n+\frac{1}{2}) \tag{4}$$

here, n_D is concentration of impurities, m and e are the effective mass and the charge of the electron, respectively and ε_n are the energy levels of an individual well.

In order to establish a set of quantum kinetic equations for acoustic and optical phonons, we use equation of motion of statistical average value for phonons

$$i\frac{\partial}{\partial t}<b_{\vec{q}}>_t=\langle[b_{\vec{q}},H(t)]\rangle_t; \qquad i\frac{\partial}{\partial t}<c_{\vec{q}}>_t=\langle[c_{\vec{q}},H(t)]\rangle_t \tag{5}$$

where $\langle X\rangle_t$ means the usual thermodynamic average of operator X.

Using Hamiltonian in Eq.(1) and realizing operator algebraic calculations, we obtain a set of coupled quantum kinetic equations for phonons. The equation for the acoustic phonons can be formulated as:

$$\frac{\partial}{\partial t}\langle b_{\vec{q}}\rangle_t+i\omega_{\vec{q}}\langle b_{\vec{q}}\rangle_t=-\sum_{nn'\vec{k}_\perp}\sum_{-\infty}^{\infty}J_\nu(\frac{\lambda}{\Omega})J_\mu(\frac{\lambda}{\Omega})[f_n(\vec{k}_\perp-\vec{q})-f_{n'}(\vec{k}_\perp)]\times$$
$$\times\int_{-\infty}^{t}dt_1\{|C_{\vec{q}}I_{nn'}|^2(\langle b_{\vec{q}}\rangle_t+\langle b^+_{-\vec{q}}\rangle_t)+C_{-\vec{q}}D_{\vec{q}}I^2_{nn'}(\langle c_{\vec{q}}\rangle_t+\langle c^+_{-\vec{q}}\rangle_t)\}\times$$
$$\times exp\{i[\varepsilon_{n'}(\vec{k}_\perp)-\varepsilon_n(\vec{k}_\perp-\vec{q})](t_1-t)-i\nu\Omega t_1+i\mu\Omega t\} \tag{6}$$

A similar equation for the optical phonons can be obtained in which $\langle c_{\vec{q}}\rangle_t$, $\langle b_{\vec{q}}\rangle_t$, $\upsilon_{\vec{q}}$, $\omega_{\vec{q}}$, $C_{\vec{q}}$, $D_{\vec{q}}$ are replaced by $\langle b_{\vec{q}}\rangle_t$, $\langle c_{\vec{q}}\rangle_t$, $\omega_{\vec{q}}$, $\upsilon_{\vec{q}}$, $D_{\vec{q}}$, $C_{\vec{q}}$, respectively.

In Eq.(6), $f_n(\vec{k}_\perp)$ is the distribution function of electrons in the state $|n,\vec{k}_\perp\rangle$, $J_\mu(\frac{\lambda}{\Omega})$ is the Bessel function, and $\lambda=\frac{e\vec{E}_0\vec{q}}{m\Omega}$

3. The parametric transformation coefficient of acoustic and optical phonon in doped superlattices

In order to establish the parametric transformation coefficient of acoustic and optical phonon, we use standard Fourier transform techniques for statistical average value of phonon operators: $\langle b_{\vec{q}}\rangle_t$, $\langle b^+_{-\vec{i}}\rangle_t$, $\langle c_{\vec{q}}\rangle_t$, $\langle c^+_{-\vec{q}}\rangle_t$. The Fourier transforms take the form :

$$\Psi_{\vec{q}}(\omega)=\int_{-\infty}^{\infty}\langle\Psi_{\vec{q}}\rangle_t e^{i\omega t}dt; \qquad \langle\Psi_{\vec{q}}\rangle_t=\frac{1}{2\pi}\int_{-\infty}^{\infty}\Psi_{\vec{q}}(\omega)e^{-i\omega t}d\omega \tag{7}$$

One finds that the final result consists of coupled equations for the Fourier transformations $C_{\vec{q}}(\omega)$ and $B_{\vec{q}}(\omega)$ of $\langle c_{\vec{q}}\rangle_t$ and $\langle b_{\vec{q}}\rangle_t$.

For instance, the equation for $C_{\vec{q}}(\omega)$ can be written as:

$$(\omega-\upsilon_{\vec{q}})C_{\vec{q}}(\omega)=2\sum_{nn'}\sum_{l=-\infty}^{\infty}|I_{nn'}|^2D^2_{\vec{q}}\upsilon_{\vec{q}}\frac{C_{\vec{q}}(\omega-l\Omega)}{\omega-l\Omega+\upsilon_{\vec{q}}}P_l(\vec{q},\omega)+$$

$$+2\sum_{nn'}\sum_{l=-\infty}^{\infty}|I_{nn'}|^2 D_{-\vec{q}}C_{\vec{q}}\omega_{\vec{q}}\frac{B_{\vec{q}}(\omega-l\Omega)}{\omega-l\Omega+\omega_{\vec{q}}}P_l(\vec{q},\omega) \tag{8}$$

In the similar equation for $B_{\vec{q}}(\omega)$, functions such as $C_{\vec{q}}(\omega)$, $C_{\vec{q}}(\omega-l\Omega)$, $B_{\vec{q}}(\omega-l\Omega)$, $\upsilon_{\vec{q}}$, $\omega_{\vec{q}}$, $C_{\vec{q}}$, $D_{\vec{q}}$ are replaced by $B_{\vec{q}}(\omega)$, $B_{\vec{q}}(\omega-l\Omega)$, $C_{\vec{q}}(\omega-l\Omega)$, $\omega_{\vec{q}}$, $\upsilon_{\vec{q}}$, $D_{\vec{q}}$, $C_{\vec{q}}$, respectively.

In Eq.(8), we have:

$$P_l(\vec{q},\omega)=\sum_{\mu=-\infty}^{\infty}J_\mu(\frac{\lambda}{\Omega})J_{\mu+l}(\frac{\lambda}{\Omega})\Gamma_{\vec{q}}(\omega+\mu\Omega) \tag{9}$$

$$\Gamma_{\vec{q}}(\omega+\mu\Omega)=\sum_{\vec{k}_\perp}\frac{[f_{n'}(\vec{k}_\perp)-f_n(\vec{k}_\perp-\vec{q})]}{\varepsilon_{n'}(\vec{k}_\perp)-\varepsilon_n(\vec{k}_\perp)-(\omega+\mu\Omega)-i\delta} \tag{10}$$

where, the quantity δ is infinitesimal and appears due to the assumption of an adiabatic interaction of the electromagnetic wave (EMW).

In Eq.(8), the first term on the right-hand side is significant just in case l=0. If not, it will contribute more than second order of electron-phonon interaction constant. Therefore, we have

$$(\omega-\upsilon_{\vec{q}})C_{\vec{q}}(\omega)=2\sum_{nn'}|I_{nn'}|^2|D_{\vec{q}}|^2\upsilon_{\vec{q}}\frac{C_{\vec{q}}(\omega-l\Omega)}{\omega-l\Omega+\upsilon_{\vec{q}}}P_l(\vec{q},\omega)$$

$$+2\sum_{nn'}\sum_{l=-\infty}^{\infty}|I_{nn'}|^2 D_{\vec{q}}C_{\vec{q}}\omega_{\vec{q}}\frac{B_{\vec{q}}(\omega-l\Omega)}{\omega-l\Omega+\omega_{\vec{q}}}P_l(\vec{q},\omega) \tag{11}$$

Transforming Eq.(11) and using the parametric resonant condition $\omega_{\vec{q}}+m\Omega\cong\upsilon_{\vec{q}}$, the parametric transformation coefficient is obtained

$$\frac{C_{\vec{q}}(\upsilon_{\vec{q}})}{B_{\vec{q}}(\omega_{\vec{q}})}=\frac{\sum_{nn'}|I_{nn'}|^2 D_{-\vec{q}}C_{\vec{q}}P_{-l}(\vec{q},\omega)}{\delta-i\sum_{nn'}|I_{nn'}|^2|D_{\vec{q}}|^2 ImP_0(\vec{q},\upsilon_{\vec{q}})}=K_l \tag{12}$$

Consider the case of $l=1$; and assign $\gamma_0=\sum_{nn'}|I_{nn'}|^2|D_{\vec{q}}|^2 ImP_0(\vec{q},\upsilon_{\vec{q}})$

Note that $\delta\ll\gamma_0$, we have

$$K_1=\frac{\sum_{nn'}|I_{nn'}|^2 D_{-\vec{q}}C_{\vec{q}}P_{-1}(\vec{q},\omega)}{i\gamma_0} \tag{13}$$

Using Bessel function, Fermi-Dirac distribution function for electron and energy spectrum of electron in Eq.(4), we have

$$K_1=|\frac{\Gamma}{2\gamma_0}| \tag{14}$$

Where

$$\Gamma=\frac{\lambda}{\Omega}\sum_{nn'}|I_{nn'}|^2 D_{\vec{q}}C_{\vec{q}}Re\Gamma_{\vec{q}}(\omega_{\vec{q}}) \tag{15}$$

$$\gamma_0=\sum_{nn'}|I_{nn'}|^2|D_{\vec{q}}|^2 Im\Gamma_{\vec{q}}(\upsilon_{\vec{q}}) \tag{16}$$

$$Re\Gamma_{\vec{q}}(\omega_{\vec{q}}) = \frac{f_0 m}{2\pi\beta A_1}[exp[-\beta(\frac{4\pi e^2 n_D}{\chi m})^{1/2}(n+1/2)] - exp[-\beta(\frac{4\pi e^2 n_D}{\chi m})^{1/2}(n'+1/2)]] \quad (17)$$

$$Im\Gamma_{\vec{q}}(\upsilon_{\vec{q}}) = \frac{f_0 m}{2\pi\vec{q}}\sqrt{\frac{2m\pi}{\beta}}exp(-\beta\frac{mA_2^2}{2\vec{q}^2})exp[-\beta(\frac{4\pi e^2 n_D}{\chi m})^{1/2}(n+1/2)] \times exp(\frac{\beta\upsilon_{\vec{q}}}{2})sinh(\frac{\beta\upsilon_{\vec{q}}}{2}) \quad (18)$$

$$A_1 = \frac{\vec{q}^2}{2m} + (\frac{4\pi e^2 n_D}{\chi m})^{1/2}(n-n') + \omega_{\vec{q}}; \; A_2 = \frac{\vec{q}^2}{2m} + (\frac{4\pi e^2 n_D}{\chi m})^{1/2}(n-n') + \upsilon_{\vec{q}} \quad (19)$$

In Eqs.(17) and (18), $\beta = 1/k_b T$ (k_b is Boltzmann constant), f_0 is the electron density in doped superlattices.

K_1 is analytic expression of parametric transformation coefficient of acoustic and optical phonon in doped superlattices when the parametric resonant condition $\omega_{\vec{q}} \pm l\Omega \cong \upsilon_{\vec{q}}$ is satisfied

4. Numerical results and discussions

In order to clarify the mechanism for parametric transformation coefficient of acoustic and optical phonons in doped superlattices, in this section we perform numerical computations and graph for GaAs:Si/GaAs:Be doped superlattices. The parameters used in the calculation [6,7] $\xi = 13.5eV$, $\rho = 5.32gcm^{-3}$, $v_s = 5378ms^{-1}$, $\chi_\infty = 10.9$, $\chi_0 = 12.9$, $d = 10nm$, $m = 0.066m_0$, m_0 being the mass of free electron, $\hbar\omega_0 = 36.25meV$, $E_0 = 10^6 V/m$ (E_0 is electric field intensity), $k_b = 1.3807\times10^{-23} j/K$, $f_0 = 10^{23} m^{-3}$, $e = 1.60219\times10^{-19}C$, $\hbar = 1.05459\times10^{-34} j.s$, T=300K, $q = 3.2\times10^7 m^{-1}$.

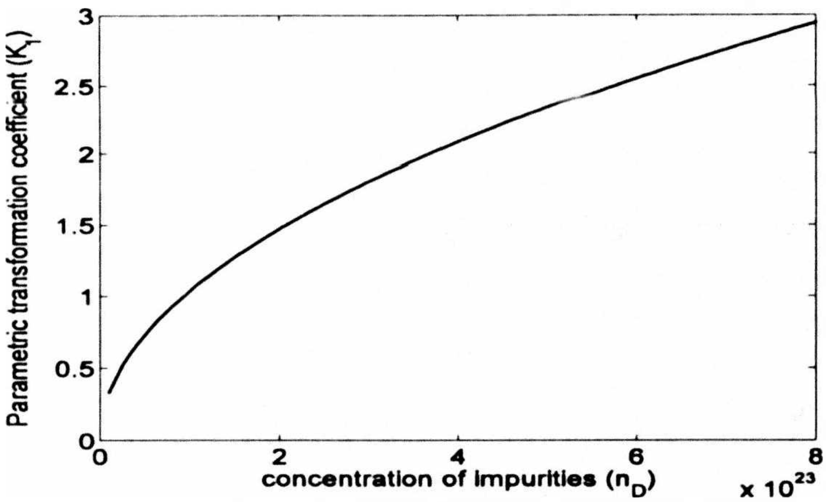

Fig. 1. Dependence of the K_1 on n_D.

Fig 1 shows the parametric transformation coefficient K1 as a function of concentration of impurities n_D. It is seen that the parametric transformation coefficient of acoustic and optical phonons in doped superlattices depends non-linearly on concentration of impurities n_D. Especially, when concentration of impurities n_D tend toward zero, value of the parametric transformation coefficient of acoustic and optical phonons in doped superlattices will turn back nearly equal that in bulk semiconductors [5] ($K_1 \simeq 0.35$), when concentration of impurities raises, the parametric transformation coefficient also to increase. This can be explained as follows: when concentration of impurities n_D tend toward zero, the doped superlattices as a normal bulk semi- conductors. When concentration of impurities raises large enough, the doped superlattices is as low-dimensional semiconductor.

5. Conclusions

In this paper, we obtain analytic expression of the parametric transformation coefficient of acoustic and optical phonons in doped superlattices in presence of an external electromagnetic field K_1, Eqs.(14)-(19). It is seen that K_1 depends on concentration of impurities. Numerical computations and graph are performed for GaAs:Si/GaAs:Be doped superlattices, fig 1. The results is seen non-linear dependence of parametric transformation coefficient on concentration of impurities. when concentration of impurities n_D tend toward zero, value of the parametric transformation coefficient of acoustic and optical phonons in doped superlattices will turn back nearly equal that in bulk semiconductors [5] ($K_1 \simeq 0.35$), when concentration of impurities raises, the parametric transformation coefficient also to increase. This can be explained as follows: when concentration of impurities n_D tend toward zero, the doped superlattice as a normal bulk semiconductors. When concentration of impurities raises large enough, the doped superlattice is a low-dimensional semiconductor.

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