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Charge parity violation in the minimal supersymmetric standard model (MSSM) and some new interactions

Ha Huy Bang[1], Nguyen Thi Thu Huong[1,*], Nguyen Chinh Cuong[2]

[1]*Faculty of Physics, College of Science, VNU*
334 Nguyen Trai, Thanh Xuan, Hanoi, Vietnam
[2]*Faculty of Physics, Hanoi University of Education, 136 Xuan Thuy, Cau Giay, Hanoi, Vietnam*

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Abstract. We present the appearance of some new interaction in the MSSM with CP violation. We give analytic formulae and perform an evaluation of the effects of these new interactions on some processes concerning the productions and decays of squarks. We find that these effects are typically of -3.5% to $+3\%$ depending on each process.
Keyworks: MSSM, CP violation.

1. Introduction

Test of the discrete symmetries, charge conjugation C, parity P, and time-reversal T, have played an important role in establishing the structure of Standard Model (SM). In particular, CP violation has been observed in the electroweak sector of the SM in the K and B systems. It is linked to a single phase in the unitary Cabbibo-Kobayashi-Maskawa (CKM) matrix describing transitions between the three generations of quarks; see e.g. [1] for a detailed review. It is important to note that this source of CP violation is strictly flavour non-diagonal.

The strong sector of the SM also allows for CP violation through a dimension-four term $\theta G\tilde{G}$, which is of topological origin. Such a term would lead to flavour-diagonal CP violation and hence to electric dipole moments (EDMs). The current experimental limits on the EDMs of atoms and neutrons [2-4]

$$|d_{Tl}| < 9.10^{-25} e.cm (90\% C.L.)$$

$$|d_{Hg}| < 2.10^{-28} e.cm (95\% C.L.)$$

$$|d_n| < 6.10^{-26} e.cm (90\% C.L.)$$

however constrain the strong CP phase to $|\theta| < 10^{-9}$! A comprehensive discussion of this issue can be found in [5].While θ appears to be extremly tuned, the CKM contribution to the EDMs is several orders of magnitude below the experimntal bounds, e.g. $d_n^{CKM} \sim 10^{-32} e.cm$. Therefore, while providing important constraints, the current EDM bounds still leave ample room for new sources of CP violation beyond the SM.

* Corresponding author. Tel.: 0983199870
E-mail: nguyenhuong1982@yahoo.com

Such new sources of CP violation are indeed very interesting in point of view of the observed baryon asymmetry of the Universe

$$\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.14 \pm 0.25).10^{-10}$$

with n_B, $n_{\bar{B}}$ and n_γ the number densities of baryons, antibaryons and photons, respectively; see [6,7] for recent reviews. The necessary ingredients for baryogenesis [8] i) baryon number violation, ii) C and CP violation and iii) departure from equilibrium are in principle present in the SM, however not with sufficient strength. In particular, the amount of CP violation is not enough. This provides a strong motivation to consider CP violation in extensions of the SM, as reviewed e.g. in [9].

In general, CP violation in extensions of the SM can be either explicit or spontaneous. Explicit CP violation occurs through phases in the Lagrangian, which cannot be rotated away by field redefinitions. This is the standard case in the MSSM, on which I will concentrate in the following. Spontaneous CP violation, on the other hand, occurs if an extra Higgs field develops a complex vacuum expectation value. This can lead to a vanishing ¦È term as well as to a complex CKM matrix. Spontaneous CP violation is a very interesting and elegant idea, but difficult to realize in SUSY and obviously not possible in the MSSM (where the Higgs potential conserves CP). There has, however, been very interesting new work on left-right symmetric models and SUSY GUTs. For instance, models based on supersymmetric SO(10) may provide a link with the neutrino seesaw and leptogenesis. I do not follow this further in this talk but refer to [9] for a review.

It is noted that CP violation in the MSSM alone is a large field with a vast amount of literature; In this paper, we present the appearance of new vertices concerning with interaction of squarks in the MSSM once CP violation taken into account. Then we give analytic formulae and numerical results to evaluate the effects of these new interactions to some processes concerning with squarks productions and decays.

2. CP violation in the MSSM and the appearance of some new squark interaction vertices

The Minimal Supersymmetric Standard Model (MSSM) is considered the most attractive extension of the Standard Model. Many phenomenological studies on SUSY particle searches have been performed in the MSSM with real SUSY parameters. In general, however, some of the SUSY parameters may be complex, in particular the higgsino mass parameter μ, the gaugino mass parameters $M_{1,2,3}$ and the trilinear scalar coupling parameters A_f of the sfermions $\tilde{f}$,

$$\mu = |\mu|e^{i\phi_\mu} = |\mu|e^{i\phi_1}, A_q = |A_q|e^{i\phi_q} = |A_q|e^{i\phi_2}, \widetilde{M} = |\widetilde{M}|e^{i\phi_M} = |\widetilde{M}|e^{i\phi_3}. \quad (1)$$

thus inducing explicit CP violation in the model. Not all of the phases in eq. (1) are, however, physical. The physical combinations indeed are $Arg(M_i\mu)$ and $Arg(A_f\mu)$. They can

-affect sparticle masses and couplings through their mixing,

-induce CP mixing in the Higgs sector through radiative corrections,

-influence CP-even observables like cross sections and branching ratios,

-lead to interesting CP-odd asymmetries at colliders.

Non-trivial phases, although constrained by EDMs, can hence significantly influence the collider phenomenology of Higgs and SUSY particles, and also the properties of neutralino dark matter.

In the MSSM every quark has two scalar partners, the squarks $\tilde{q}_L$ and $\tilde{q}_R$, corresponding to the left and the right helicity states of a quark. In general $\tilde{q}_L$ and $\tilde{q}_R$ mix to form mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$ (with $m_{\tilde{q}_1} < m_{\tilde{q}_2}$), the size of the mixing being proportional to the mass of the corresponding quark q [10], and so neligible except for the third generation. The mass matrix in the basis $(\tilde{q}_L, \tilde{q}_R)$ is given by [10]

$$M_{q^2} = \begin{pmatrix} m^2_{q_L} & a_q m_q \\ a_q m_q & m^2_{q_R} \end{pmatrix} = \left(R^q\right)^\dagger \begin{pmatrix} m^2_{q_1} & 0 \\ 0 & m^2_{q_2} \end{pmatrix} R^q. \tag{2}$$

with

$$m^2_{q_L} = M^2_Q + m^2_Z \cos 2\beta (I_3^{q_L} - e_q \sin^2\theta_W) + m^2_q, \tag{3}$$

$$m^2_{q_R} = M^2_{\{U,D\}} + m^2_Z \cos 2\beta e_q \sin^2\theta_W) + m^2_q, \tag{4}$$

$$a_q = A_q - \mu\{\cot\beta, \tan\beta\}. \tag{5}$$

Here I_3^q is the third component of the weak isospin and e_q the electric charge of the quark q. $M_{\tilde{Q},\tilde{U},\tilde{D}}$ and $A_{t,b}$ are soft SUSY-breaking parameters and $\tan\beta = v_2/v_1$ with v_1 (v_2) being the vacuum expectation value of the Higgs field H_1^0 (H_2^0). According to eq (2), M_q^2 is diagonalized by a unitary matrix R^q. The weak eigenstates $\widetilde{q}_1$ and $\widetilde{q}_2$ are thus related to their mass eigenstates $\widetilde{q}_L$ and $\widetilde{q}_R$ by

$$\begin{pmatrix} \widetilde{q}_1 \\ \widetilde{q}_2 \end{pmatrix} = R^q \begin{pmatrix} \widetilde{q}_L \\ \widetilde{q}_R \end{pmatrix},$$

With complex parameters, we have

$$R^q = \begin{pmatrix} e^{\frac{1}{2}\phi_q}\cos\theta_q & e^{-\frac{1}{2}\phi_q}\sin\theta_q \\ -e^{\frac{1}{2}\phi_q}\sin\theta_q & e^{-\frac{1}{2}\phi_q}\cos\theta_q \end{pmatrix}. \tag{6}$$

Let us turn to the impact of the CP phase $\phi_{\tilde{q}}$ to the squark interaction. Our terminology and notation are as in Ref.[11]. The relevant parts of the Lagrangian for squark interactions $\widetilde{q}_i^\alpha \widetilde{q}_j^\beta \gamma$ and $\widetilde{q}_i^\alpha \widetilde{q}_j^\beta g$ (α and β are flavor indices) are given by

$$\mathcal{L}_{qq\gamma} = iee_q A_\mu (R^q_{i1} R^q_{j1} + R^q_{i2} R^q_{j2}) \widetilde{q}_j^* \overleftrightarrow{\partial}^\mu \widetilde{q}_i = iee_q A_\mu \widetilde{\delta}_{ij} \widetilde{q}_j^* \overleftrightarrow{\partial}^\mu \widetilde{q}_i \tag{7}$$

$$\mathcal{L}_{qqg} = ig_s T^a_{rs} (R^q_{i1} R^q_{j1} + R^q_{i2} R^q_{j2}) G^a_\mu \widetilde{q}_{jr}^* \overleftrightarrow{\partial}^\mu \widetilde{q}_{is} = ig_s T^a_{rs} \widetilde{\delta}_{ij} G^a_\mu \widetilde{q}_{jr}^* \overleftrightarrow{\partial}^\mu \widetilde{q}_{is} \tag{8}$$

where $\widetilde{\delta}_{ij} = R^q_{i1} R^q_{j1} + R^q_{i2} R^q_{j2}$.

In case of CP conserving MSSM (say $\phi_q = 0$), R^q_{ij} is real and we have

$$\widetilde{\delta}_{ij} = \delta_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Therefore, only interaction modes with $i = j$ exist, (for example: $\widetilde{t}_2^\alpha \to \widetilde{t}_2^\beta + g$, $\widetilde{b}_1^\alpha \to \widetilde{b}_1^\beta + \gamma$). If CP tion is taken into account ($\phi_q \neq 0$), using (6) we have

$$\widetilde{\delta}_{ij} = \begin{pmatrix} e^{i\phi_q}\cos^2\theta_q + e^{-i\phi_q}\sin^2\theta_q & (e^{-i\phi_q} - e^{i\phi_q})\sin\theta_q\cos\theta_q \\ (e^{-i\phi_q} - e^{i\phi_q})\sin\theta_q\cos\theta_q & e^{-i\phi_q}\cos^2\theta_q + e^{i\phi_q}\sin^2\theta_q \end{pmatrix}. \tag{9}$$

Thus, in this case, new interaction vertices with $i \neq j$ appear, *e.g.* $\widetilde{t}_2^\alpha \widetilde{t}_1^\beta \gamma$, $\widetilde{b}_2^\alpha \widetilde{b}_1^\beta g$. These new couplings are nonzero and depend on $\phi_{\tilde{q}}$, hence giving contributions to the decay width and production cross section of squark as we will see below.

3. Numerical results and discussions

Our terminologies and notations are as in Ref. [12, 13] Firstly, let us consider the processes $\ell^+\ell^- \to \tilde{q}_i\bar{\tilde{q}}_j$ ($\ell = e, \mu$) in which γ is one of the exchanged bosons.

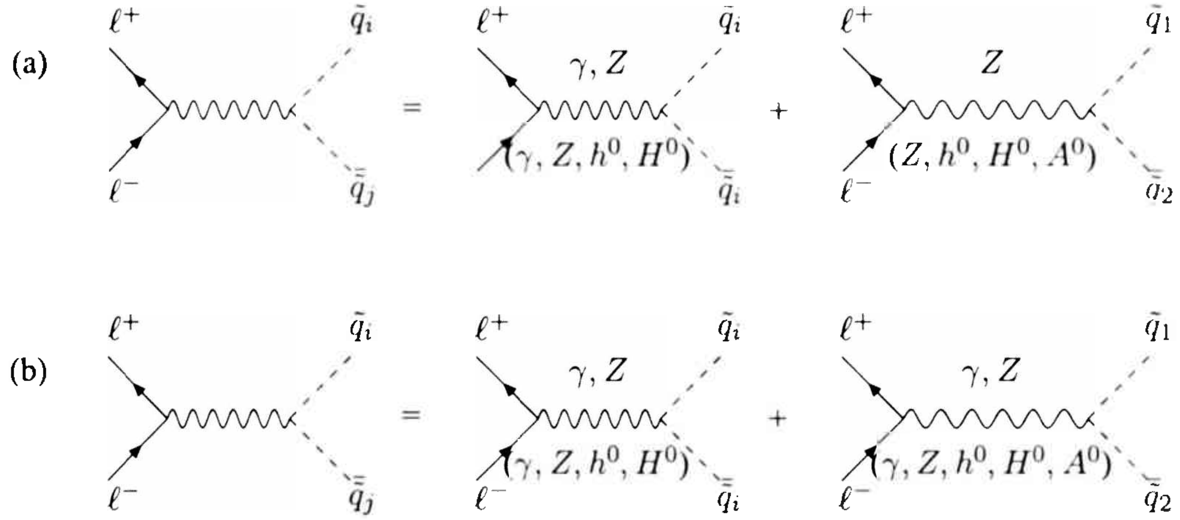

Fig. 1. Feyman diagrams for the process $\ell^+\ell^- \to \widetilde{q_i}\bar{\tilde{q}}_j$ ($\ell^+\ell^- = e^+e^-(\mu^+\mu^-)$), (a) CP conserving case, (b) CP violating case.

The cross sections for these processes read [12,13]

$$\sigma(e^+e^- \to \widetilde{q}_i^\alpha \widetilde{q}_j^\beta) \approx \left(e_q^2|\widetilde{\delta}_{ij}|^2 - \frac{e_q v_e}{4c_w^2 s_w^2}(c_{ij}\widetilde{\delta}_{ij}^+ + c_{ij}^+\widetilde{\delta}_{ij}).D_{\gamma Z} + \frac{v_e^2 + a_e^2}{16c_w^4 s_w^4}|c_{ij}|^2 D_{ZZ} \right), \tag{10}$$

and

$$\sigma(\mu^+\mu^- \to \widetilde{q}_i^\alpha \widetilde{q}_j^\beta) \approx \left(\frac{2k_{ij}^2}{3s^2}.T_{VV} + T_{HH} + \frac{m_{q_i}^2 - m_{q_j}^2}{s} T_{VH} \right), \tag{11}$$

where

$$+ \quad T_{VV} = e_q^2|\widetilde{\delta}_{ij}|^2 - \frac{e_q v_\mu}{4c_w^2 s_w^2}(c_{ij} d_Z \widetilde{\delta}_{ij}^+ + c_{ij}^+ d_Z^+ \widetilde{\delta}_{ij}).s + \frac{(v_\mu^2 + a_\mu^2)|c_{ij}|^2 s^2 |d_Z|^2}{16c_w^4 s_w^4},$$

$$+ \quad T_{HH} = \frac{h_\mu^2 s}{2e^4}\left(|(G_1^q)_{ij}.sin\alpha.d_h - (G_2^q)_{ij}.cos\alpha.d_H|^2 + |(G_3^q)_{ij}.sin\beta.d_A|^2 \right),$$

$$+ \quad T_{VH} = -\frac{m_\mu a_\mu h_\mu sin\beta|(G_3^q)_{ij}|(c_{ij}^+ d_Z^+ d_A + d_A^+ c_{ij} d_Z).s}{2e^2 c_w^2 s_w^2}.\frac{s - M_Z^2}{M_Z^2}$$

In case of CP conserving, there is only Z exchange in the interaction mode with $i \neq j$. Whereas, in case of CP violating, there are both Z and γ contributions (see Fig.1). For the range of $\phi = [0, 0.1]$ we find that the term propotional to $\tilde{\delta}_{ij}$ (with $i \neq j$) which arises from new vertices $\tilde{q}_i\bar{\tilde{q}}_j\gamma$ ($i \neq j$) can contribute from (- 1%) to (+ 1%) to the cross sections of these processes, namely $e^+e^- \to \widetilde{t_i}\widetilde{\bar{t}_j}$, $e^+e^- \to \widetilde{b_i}\widetilde{\bar{b}_j}$, $\mu^+\mu^- \to \widetilde{t_i}\widetilde{\bar{t}_j}$, $\mu^+\mu^- \to \widetilde{b_i}\widetilde{\bar{b}_j}$ ($i \neq j$). For a wider range of $\phi = [0, 1]$ we find that these terms can contribute up to from (- 3.5%) to (+ 3%) to the cross sections of these above processes.

In addition, new vertices $\tilde{t}_2^{\alpha}\tilde{t}_1^{\beta}\gamma$, $\tilde{b}_2^{\alpha}\tilde{b}_1^{\beta}g$ also allow for the decays of squarks to photon and gluon $\tilde{t}_2^{\alpha} \to \tilde{t}_1^{\beta}\gamma$, $\tilde{b}_2^{\alpha} \to \tilde{b}_1^{\beta}g$ (see Fig. 2) which are absent at tree-level nor induced by $O(\alpha_s)$ SUSY-QCD correction in case of CP conserving MSSM [14]. At tree level, the amplitude of the above decay has the general form

$$M^0(\tilde{q}_i^{\alpha} \to \tilde{q}_j^{\beta} + V) = -igC_{ijV}(k_1 + k_2)^{\mu}\epsilon_{\mu}^*(k_3), \tag{12}$$

with: k_1, k_2 and k_3 are the four - momenta of $\tilde{q}_i^{\alpha}$, $\tilde{q}_j^{\beta}$ and V ($V \equiv \gamma, g$),

$$C_{ij\gamma} = Sin\theta_w.e_q.\tilde{\delta}_{ij}, \tag{13}$$

$$C_{ijg} = T_{rs}^a.\tilde{\delta}_{ij}g_s/g. \tag{14}$$

These interesting results will affect the decay pattern of squarks and can not be neglected in the detailed study of squarks.

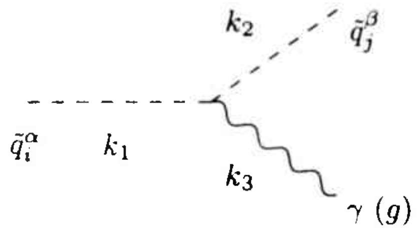

Fig. 2. Feyman diagrams for the decay $\tilde{q}_i^{\alpha} \to \tilde{q}_j^{\beta} + \gamma(g)$.

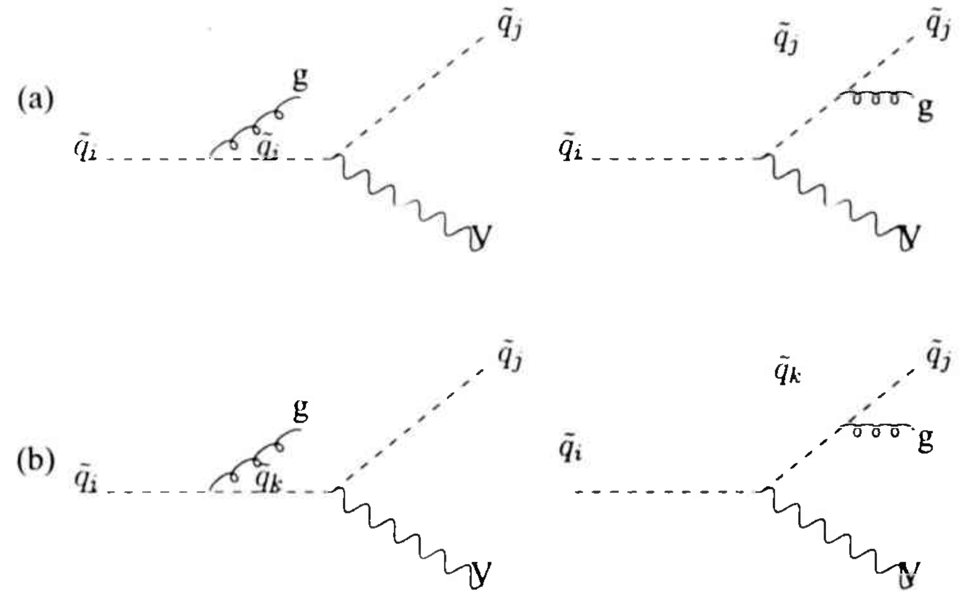

Fig. 3. Example diagrams for real gluon emission in squark decays into vector bosons.
(a) CP conserving case, (b) CP violating case.

Moreover, in other decay modes of squarks $\tilde{q}_i^{\alpha} \to \tilde{q}_j^{\beta} + V$ (V is gauge bosons Z, γ, g, $W^{\pm}$), $\tilde{q}_i^{\alpha} \to \tilde{q}_j^{\beta} + H$ (H is Higg bosons h^0, H^0, A^0, $H^{\pm}$) . . . as well as in the production of squarks at lepton colliders $\ell^+\ell^- \to \tilde{q}_i\overline{\tilde{q}_j}$, the infrared divergences occur when the one loop vertex corrections are included. In oder to cancel these infrared divergences, we need to add real gluon emissions (see

Fig.3), and this will lead to the appearance of the new terms propotional to $\widetilde{\delta}_{ij}$ (with $i \neq j$) in the formulea of decay widths and cross sections . We take some evaluation for the range of $\phi = [0, 0.1]$ and find that these terms can contribute from (- 1%) to (+ 0.5%) to the decay widths of the processes $\widetilde{q}_i^\alpha \rightarrow \widetilde{q}_j^\beta + V$, from (- 1.2%) to (- 0.5%) to the decay width of the processes $\widetilde{q}_i^\alpha \rightarrow \widetilde{q}_j^\beta + H$ and from (- 0.4%) to (+ 0.1%) to the cross section of the processes $e^+e^- \rightarrow \widetilde{t}_i\overline{\widetilde{t}_j}$, $e^+e^- \rightarrow \widetilde{b}_i\overline{\widetilde{b}_j}$, $\mu^+\mu^- \rightarrow \widetilde{t}_i\overline{\widetilde{t}_j}$, $\mu^+\mu^- \rightarrow \widetilde{b}_i\overline{\widetilde{b}_j}$. Particularly, there are some cases with large contribution, from (- 2.2%) to (- 1.5%), for example the decay $\widetilde{t}_2^\alpha \rightarrow \widetilde{t}_1^\beta + A^0$). For a wider range of $\phi = [0, 1]$ we find that these terms can contribute from (-1%) to (+0.5%) to the cross sections of the processes $e^+e^- \rightarrow \widetilde{t}_i\overline{\widetilde{t}_j}$, $e^+e^- \rightarrow \widetilde{b}_i\overline{\widetilde{b}_j}$, $\mu^+\mu^- \rightarrow \widetilde{t}_i\overline{\widetilde{t}_j}$, $\mu^+\mu^- \rightarrow \widetilde{b}_i\overline{\widetilde{b}_j}$.

In conclusion, we have represented the appearance of new vertices concerning with squark interaction. We also deduced the formulea and evaluated analytically the contributions of these new interactions to some of the decay and production of squarks and found that they are typically of (- 3.5%) to (+3 %) depending on particular process. This could have an important imp'ication in the determination of the MSSM at future linear colliders.

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