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Composite cylinder under unsteady, axisymmetric, plane temperature field

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Abstract. With advantages such as high strength, high stiffness, high chemical resistance, light weight...composite tubes are widely applied in urban construction and petroleum industry. In this report, the authors used the displacement method to study the mechanical behavior (stress, strain...) of an infinite hollow cylinder made of composite material under unsteady, axisymmetric plane temperature field. In the numerical calculations, we mainly studied the influence of time and volume ratio of the particle on the displacement and thermoelastic stress of a cylinder made of Titanium /PVC composite.

1. Introduction

Nowadays, composite materials are increasingly promoting their preeminences (such as high shock capacity, high thermal-machanical load capacity...) when applied in real structures. The study of thermal-mechanical behavior of composite cylinder has attracted the attention of many authors and series of articles have been published on this field. The transient thermal stress problems of multi-layered cylinder as well as hollor composite cylinder are studied in [1-4] by different methods. Iyengar et al. [5] investigated thermal stresses in a finite hollow cylinder due to an axisymmetric temperature field at the end surface. Soldatos et al. [6] presented the three dimensional static, dynamic, thermoelastic and buckling analysis of homogeneous and lamilated composite cylinders. Bhattacharyya et al. [7] obtained the exact solution of elastoplastic response of an infinitely long composite cylinder during cyclic radial loading. Ahmed et al. [8] studied thermal stresses problem in non-homogeneous transversely isotropic infinite circular cylinder subjected to certain boundary conditions by the finite difference method. Jiann-Quo Tarn [9] obtained the exact solution for functionally graded (FGM) anisotropic cylinders subjected to thermal and mechanical loads. Chao et al. [10] investigated thermal stresses in a vis-coelastic three-phase composite cylinder. The thermal stresses and thermal-mechanical stresses of FGM circular hollow cylinder subjected to certain boundary conditions presented in [11-15]. By using the finite integral transform, Kong et al. [16] obtained the exact solution of thermal-magneto-dynamic and perturbation of magnetic field vector in a non-homogeneous hollow cylinder. Recently, the nonlinear thermoelastic problems of FGM cylinder has also been con-cerned to resolve in [17, 18]. In the articles above, some authors supposed that the material properties depend on both temperature and radius, some other authors assumed that they are independent from the temperature and only depend on the radius r.

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In this paper, based on the governing equations of the theory of elasticity, the authors use the displacement method to find the analytical solution for displacement, strain, and thermoelastic stress of an an infinite hollow cylinder made of particle filled composite material subjected to an unsteady, axisymmetric plane temperature field. We assumed that the composite material is elastic, homogeneous and isotropic. We also ignored the interaction between matrix phase and particle phase. The material's thermo-mechanical properties are independent from temperature. There is no heat source inside the cylinder. Since the heat flows generated by deformation and the dynamic effects by unsteady heat are minimal, they are also ignored.

2. Governing equations

Consider an infinite hollow cylinder made of spherical particle filled composite mateiral. The cylinder having internal radius a, external radius b is sujected to an unsteady, axisymmetric plane temperature field T(r, t).

The composite's physical and mechanical constants are calculated as below [19, 21, 23]:

$$G = G_m\left(1 - \frac{15(1-\nu_m)\left(1-\dfrac{G_c}{G_m}\right)\xi}{7-5\nu_m + 2(4-5\nu_m)\dfrac{G_c}{G_m}}\right), \quad K = K_m + \frac{(K_c-K_m)\xi}{1+(K_c-K_m)\left(K_m+\dfrac{4}{3}G_m\right)^{-1}};$$

$$\alpha = \alpha_m + (\alpha_c-\alpha_m)\frac{K_c(3K_m+4G_m)}{K_m(3K_c+4G_m)+4(K_c-K_m)G_m\xi}; \tag{1}$$

$$E = \frac{9KG}{3K+G}, \; \nu = \frac{\lambda}{2(\lambda+\mu)} = \frac{3K-2G}{6K+2G}, K_i = \frac{E_i}{3(1-2\nu_i)}, G_i = \frac{E_i}{2(1+\nu_i)}, (i=m,c).$$

Here ξ is the particle's volume ratio; (λ, μ), G, K, E, ν, α are Lame's constants, shear modulus, bulk modulus, Young's modulus, Poission's ratio, thermal expansion coefficient, respectively; the subcripts m and c respectively belong to the matrix phase and particle phase.

In the cylindrical coordinate system (r, θ, z) [19]: From the symmetric property, every point is only displaced in the radial direction, so the displacement filed has the form:

$$u_r = u(r,t), \quad u_z = u_\theta = 0. \tag{2}$$

The Cauchy relation for strain and displacement are:

$$e_{rr} = \frac{\partial u}{\partial r}, \quad e_{\theta\theta} = \frac{u}{r}, \quad e_{zz} = e_{rz} = e_{\theta z} = e_{r\theta} = 0. \tag{3}$$

The stress strain relations according to the linear thermoelastic theory are given by

$$\begin{aligned} \sigma_{rr} &= \lambda\theta + 2\mu e_{rr} - (3\lambda+2\mu)\alpha(T-T_0), \\ \sigma_{\theta\theta} &= \lambda\theta + 2\mu e_{\theta\theta} - (3\lambda+2\mu)\alpha(T-T_0), \\ \sigma_{zz} &= \lambda\theta - (3\lambda+2\mu)\alpha(T-T_0), \\ \tau_{r\theta} &= \tau_{rz} = \tau_{z\theta}, \end{aligned} \tag{4}$$

where T_0 is the initial temperature of the cylinder; $\theta = e_{rr} + e_{\theta\theta}$.

When there is no heat source inside the cylinder and the thermal deformation caused of volume change is ignored, the heat conduction equation is expressed in the form

$$k\Delta T = \rho C \frac{\partial T}{\partial t}, \tag{5}$$

Here $\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$ is the Laplace operator; k, ρ, C are respectively the coefficients of thermal conductivity, mass density, heat capacity. They are determined as follow

$$k = (1-\xi)\, k_m + \xi\, k_c; \;\; \rho = (1-\xi)\rho_m + \xi\rho_c; \;\; C = \frac{(1-\xi)C_m\rho_m + \xi C_c\rho_c}{(1-\xi)\rho_m + \xi\rho_c}. \tag{6}$$

Since the inertia term is ignored, the equilibrium equation is given by

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0. \tag{7}$$

Subtitute Eq. (3) and Eq. (4) into Eq. (7) we get

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} = \frac{3\lambda + 2\mu}{\lambda + 2\mu}\alpha\frac{\partial T}{\partial r}. \tag{8}$$

Introduce the following notations

$$E_1 = \frac{E}{1-\nu^2}, \quad \nu_1 = \frac{\nu}{1-\nu}, \quad \alpha_1 = \alpha(1+\nu). \tag{9}$$

Eq. (8) can be rewritten as

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru)\right) = (1+\nu_1)\alpha_1\frac{\partial T}{\partial r}. \tag{10}$$

The initial and boundary conditions of the temperature field are [23]

$$\begin{aligned} &T(r,0) = [T(r,t)]_{t=0} = T_0, \\ &\left[\frac{\partial T}{\partial r} - \frac{\beta_1}{k}(T - \vartheta_1)\right]_{r=a} = 0, \\ &\left[\frac{\partial T}{\partial r} + \frac{\beta_2}{k}(T - \vartheta_2)\right]_{r=b} = 0, \end{aligned} \tag{11}$$

Here ϑ_1, β_1 are the temperature of the surrounding environment and the surface heat transfer coefficient on the inner edge r = a; ϑ_2, β_2 are the corresponding values on the outer edge r = b (T_0, ϑ_1, ϑ_2 considered as constants).

The static boundary conditions are

$$\begin{aligned} &\sigma_{rr}\big|_{r=a} = 0, \\ &\sigma_{rr}\big|_{r=b} = 0. \end{aligned} \tag{12}$$

3. Solution method

By using the Laplace transform and the Bessel functions, A.D. Kovalenko [23] found out the general analytical solution of Eq. (5) with the conditions (11) as below

$$T = \vartheta_2 + (\vartheta_1 - \vartheta_2)\frac{\gamma_1 R_1 (1-\gamma_2 \ln R)}{\gamma_2 + \gamma_1 R_1 (1-\gamma_2 \ln R_1)} - 2\sum_{n=1}^{\infty} A_n u_0(\omega_n R) e^{-\omega_n^2 \tau}, \tag{13}$$

Here:
$$R_1 = \frac{a}{b},\ R = \frac{r}{b},\ \tau = \frac{\eta t}{b^2},\ \gamma_1 = \frac{\beta_1 b}{k},\ \gamma_2 = \frac{\beta_2 b}{k},\ \eta = \frac{k}{\rho C_V}, \tag{14}$$

$$A_n = \frac{\gamma_2(\vartheta_2 - T_0)\, u_0(\omega_n) + \gamma_1 R_1 (\vartheta_1 - T_0)\, u_0(\omega_n R_1)}{(\gamma_2^2 + \omega_n^2)\, u_0^2(\omega_n) - R_1^2(\gamma_1^2 + \omega_n^2)\, u_0^2(\omega_n R_1)},\ (n = 1, 2, ...), \tag{15}$$

$$u_m(x) = \left[Y_1(\omega R_1) + \frac{\gamma_1}{\omega} Y_0(\omega R_1)\right] J_m(x) - \left[J_1(\omega R_1) + \frac{\gamma_1}{\omega} J_0(\omega R_1)\right] Y_m(x),\ (m = 0, 1), \tag{16}$$

$J_m(x)$, $Y_m(x)$ $(m = 0, 1)$ are the Bessel functions of order m of the first and second kinds [20], respectively; ω_n $(n = 1, 2, \ldots)$ are the roots of the transcendental equation

$$\frac{\omega u_1(\omega)}{u_0(\omega)} - \gamma_2 = 0. \tag{17}$$

The general solution of Eq. (10) may be expressed in the form

$$u = D_1 r + \frac{D_2}{r} + \frac{(1+\nu_1)\alpha_1}{r}\int_a^r \left[T - T(a,t)\right] r dr, \tag{18}$$

Where D_1, D_2 are the constants of integration determined from the conditions (12).

Substituting Eq. (18) into Eq. (3) and the first expression of Eqs (4), we have

$$\sigma_{rr} = \frac{E_1}{1-\nu_1}\left[D_1 + \alpha_1 (T_0 - T(a,t))\right] - \frac{E_1}{1+\nu_1}\frac{D_2}{r^2} + \frac{E_1 \alpha_1}{r^2}\int_a^r [T - T(a,t)] r dr, \tag{19}$$

Substituting Eq. (19) into Eq. (12), we find out the constants of integration D_1, D_2

$$\begin{aligned} D_1 &= \frac{(1-\nu_1)\alpha_1}{b^2 - a^2}\int_a^b [T - T(a,t)] r dr - \alpha_1 [T_0 - T(a,t)]; \\ D_2 &= \frac{(1+\nu_1)\alpha_1 a^2}{b^2 - a^2}\int_a^b [T - T(a,t)] r dr. \end{aligned} \tag{20}$$

Substituting Eq. (20) and Eq. (13) into Eq. (18), we obtain the expresstion for the radical displacement

$$\begin{aligned} u = \frac{\alpha_1}{r}\Bigg\{ &\frac{(1-\nu_1) r^2 + (1+\nu_1) a^2}{b^2 - a^2}\left(Q_1 + \sum_{n=1}^{\infty} A_n M_n e^{-\omega_n^2 \tau}\right) \\ &+ (1+\nu_1)\left(Q_2 + \sum_{n=1}^{\infty} A_n L_n e^{-\omega_n^2 \tau}\right) - [T_0 - T(a,t)]\, r^2\Bigg\}. \end{aligned} \tag{21}$$

From Eq. (3) and Eq. (21), the deformation components of the cylinder can be written as

$$\begin{aligned} e_{rr} = \frac{\alpha_1}{r^2}\Bigg\{ &\frac{(1-\nu_1) r^2 - (1+\nu_1) a^2}{b^2 - a^2}\left(Q_1 + \sum_{n=1}^{\infty} A_n M_n e^{-\omega_n^2 \tau}\right) \\ &-(1+\nu_1)\left(Q_2 + \sum_{n=1}^{\infty} A_n L_n e^{-\omega_n^2 \tau}\right) + \left[\nu_1 (T - T(a,t)) + (T - T_0)\right] r^2\Bigg\}, \end{aligned} \tag{22a}$$

$$e_{\theta\theta}=\frac{\alpha_1}{r^2}\left\{\frac{(1-\nu_1)r^2+(1+\nu_1)a^2}{b^2-a^2}\left(Q_1+\sum_{n=1}^{\infty}A_nM_ne^{-\omega_n^2\tau}\right)\right. \tag{22b}$$
$$\left.+(1+\nu_1)\left(Q_2+\sum_{n=1}^{\infty}A_nL_ne^{-\omega_n^2\tau}\right)-[T_0-T(a,t)]r^2\right\}.$$

Substitute Eqs. (22a) and (22b) into Eq. (4), we obtain the expressions of thermal stresses in the cylinder

$$\sigma_{rr}=\frac{E_1\alpha_1}{r^2}\left\{\frac{r^2-a^2}{b^2-a^2}\left(Q_1+\sum_{n=1}^{\infty}A_nM_ne^{-\omega_n^2\tau}\right)-\left(Q_2+\sum_{n=1}^{\infty}A_nL_ne^{-\omega_n^2\tau}\right)\right\}, \tag{23a}$$

$$\sigma_{\theta\theta}=\frac{E_1\alpha_1}{r^2}\left\{\frac{r^2+a^2}{b^2-a^2}\left(Q_1+\sum_{n=1}^{\infty}A_nM_ne^{-\omega_n^2\tau}\right)+\left(Q_2+\sum_{n=1}^{\infty}A_nL_ne^{-\omega_n^2\tau}\right)-[T-T(a,t)]r^2\right\}, \tag{23b}$$

$$\sigma_{zz}=\frac{E_1\alpha_1}{1+\nu_1}\left\{\frac{2\nu_1}{b^2-a^2}\left(Q_1+\sum_{n=1}^{\infty}A_nM_ne^{-\omega_n^2\tau}\right)-\nu_1[T-T(a,t)]-(T-T_0)\right\}. \tag{23c}$$

Where

$$Q_1=\frac{1}{2}\left\{\gamma_1\gamma_2R_1(\vartheta_1-\vartheta_2)\frac{\frac{b^2-a^2}{2}+b^2\ln R_1}{\gamma_2+\gamma_1R_1(1-\gamma_2\ln R_1)}\right\},$$

$$Q_2=\frac{1}{2}\left\{\gamma_1\gamma_1R_1(\vartheta_1-\vartheta_2)\frac{\frac{r^2-a^2}{2}+r^2(\ln R_1-\ln R)}{\gamma_2+\gamma_1R_1(1-\gamma_2\ln R_1)}\right\}, \tag{24}$$

$$M_n=(b^2-a^2)u_0(\omega_nR_1)-\frac{2b}{\omega_n}[bu_1(\omega_n)-au_1(\omega_nR_1)],(n=1,2,...),$$

$$L_n=(r^2-a^2)u_0(\omega_nR_1)-\frac{2b}{\omega_n}[ru_1(\omega_nR)-au_1(\omega_nR_1)],(n=1,2,...).$$

4. Numerical results and discussion

Consider an infinite hollow cylinder made of spherical particle filled composite material. The cylinder has the physical, mechanical and geometrical properties as follows: a = 10 cm; b = 10.5 cm; $T_0=290^0K$.

Properties of PVC matrix: $E_m=3$ GPa, $\nu_m=0.2$, $\alpha_m=8\times10^{-5}K^{-1}$, $k_m=0.16$ W/m.K, $C_m=900$ J/kg.K, $\rho_m=1380$ kg/m^3.

Properties of Titanium: $E_c=100$ GPa, $\nu_c=0.34$, $\alpha_c=4.8\times10^{-6}K^{-1}$, $k_c=22.1$ W/m.K, $C_c=523$ J/kg.K, $\rho_c=4500$ kg/m^3.

Suppose that the the surrounding medium on the inner edge of the cylinder is water with the heat transfer coefficient $\beta_1=400$ W/m^2.K and the surrounding medium on the outter edge of the cylinder is air with the heat transfer coefficient $\beta_2 = 25\ \mathrm{W/m^2.K}$. In order to simplify the problem, in this paper, we ignore the water pressure on the cylinder wall.

In the following, we will investigate the distribution of the radial displacement and the stresses at different radius and particle's volume ratio when the temperatures of the surrounding mediums on the inner and outer edges of the cylinder are changed.

Case 1: The temperature of the surrounding medium on the inner edge of the cylinder is greater than the corresponding value on the outer edge of the cylinder ($\vartheta_1 = 330^o K$, $\vartheta_2 = 300^o K$). The results are presented in Fig. 1.

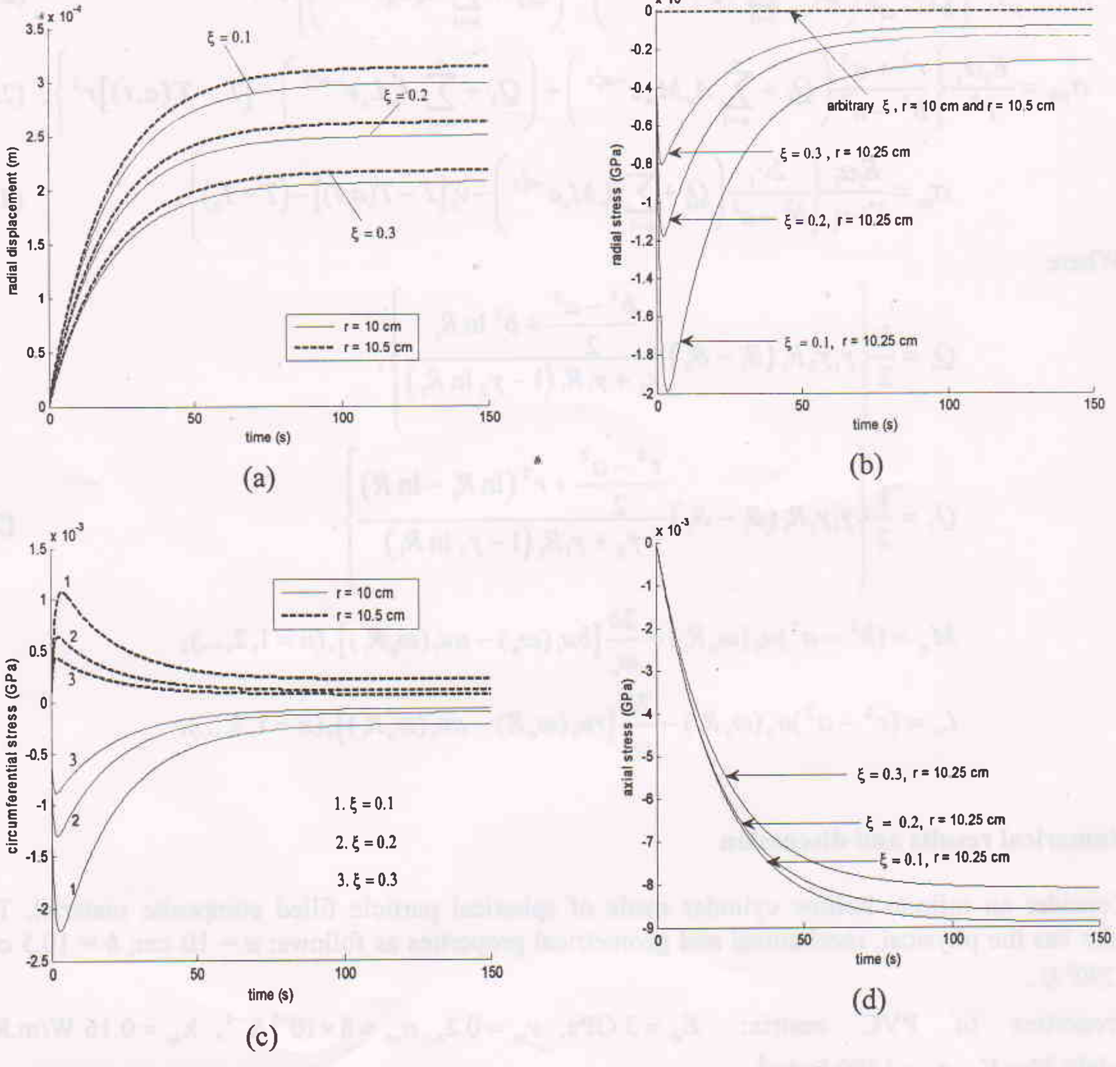

Fig. 1. Distributions of radial displacement and stress components
$T_0 = 290^0 K,\ \ \vartheta_1 = 330^0 K,\ \ \vartheta_2 = 300^0 K.$

Case 2: The temperatures of the surrounding medium on the inner and outer edges of the cylinder are equal ($\vartheta_1 = \vartheta_2 = 320^0 K$). The results are presented in Fig. 2.

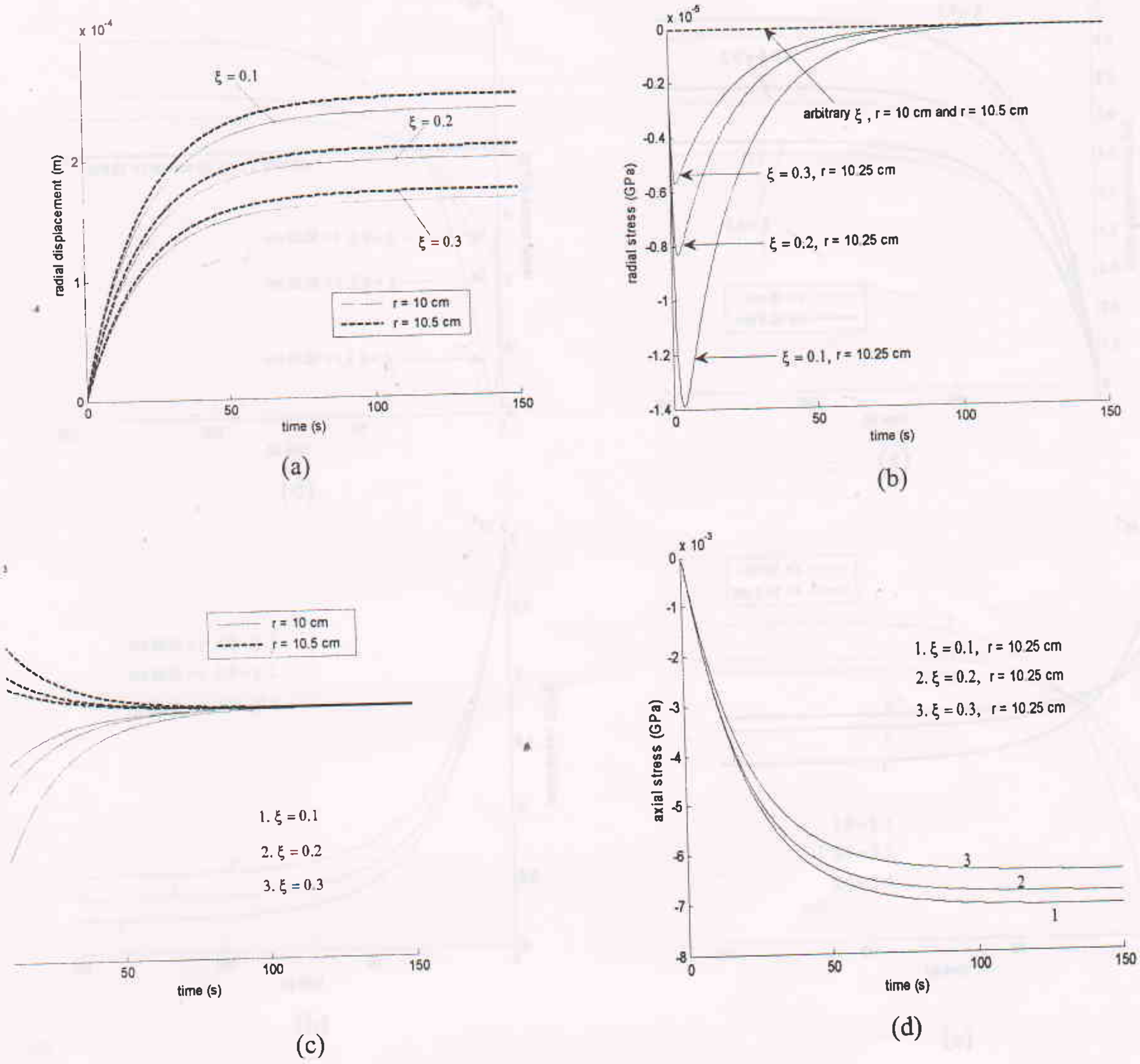

Fig. 2. Distributions of radial displacement and stress components
$T_0 = 290^0 K, \ \vartheta_1 = \vartheta_2 = 320^0 K.$

Case 3: The temperature of the surrounding medium on the inner edge of the cylinder is smaller than the corresponding value on the outer edge of the cylinder ($\vartheta_1 = 300^0 K$, $\vartheta_2 = 320^0 K$). The results are presented in Fig. 3.

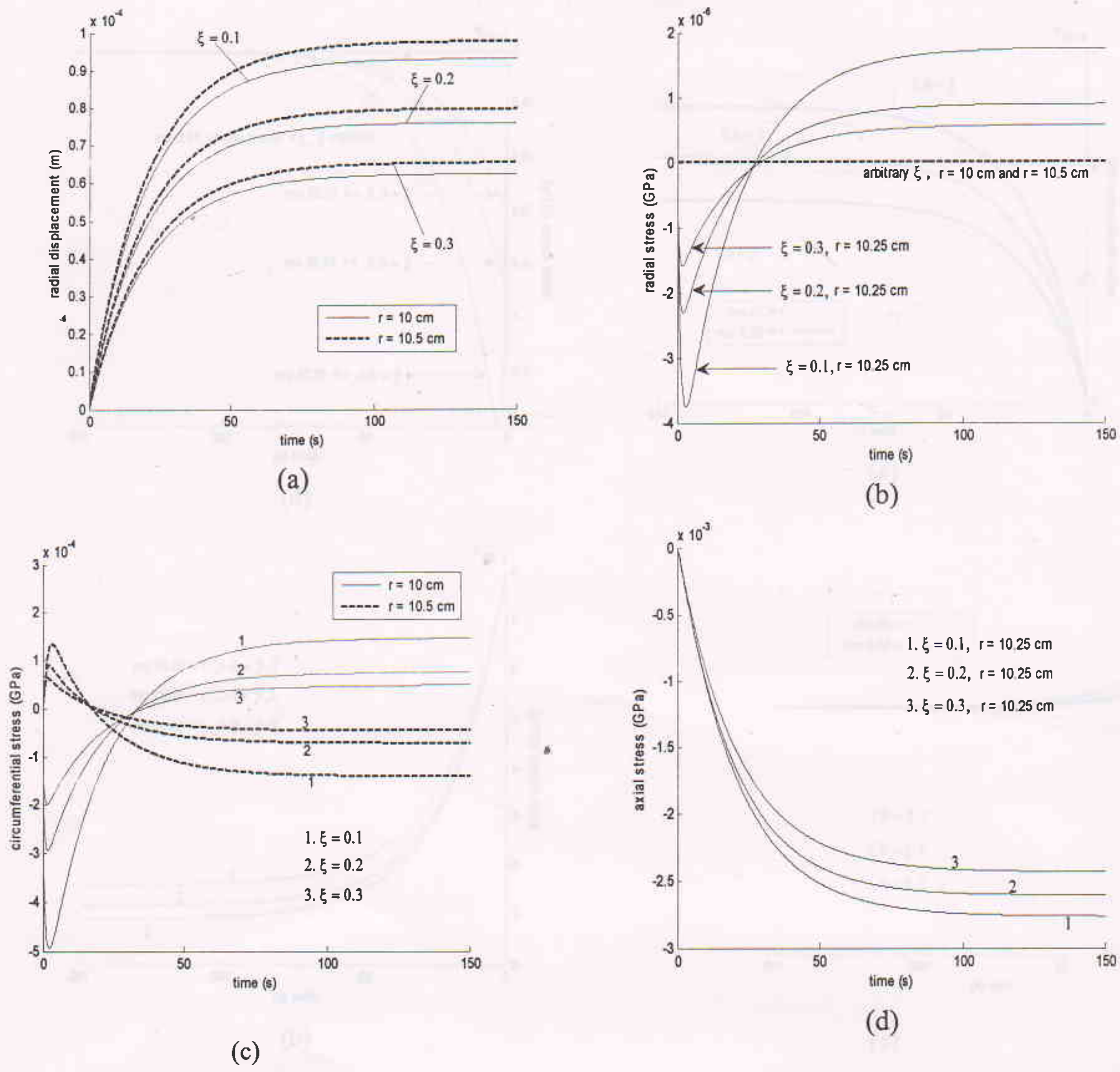

Fig. 3. Distributions of radial displacement and stress components
$T_0 = 290^0 K, \ \vartheta_1 = 300^0 K, \ \vartheta_2 = 320^0 K.$

From figs. 1, 2 and 3, it can be seen that in all three cases, the radial displacement and thermal stresses vary very slowly. The displacement and stresses in the first 50 seconds vary more quickly than in the later time interval. It can be seen from Figs. 1a, 2a and 3a that the radial displacement always has possitive sign and increase slowly with time. From figs. 1d, 2d and 3d it can be seen that the axial stress always has negative sign and its absolute value increases slowly with time. The radial and circumferential stresses in the cases 1 and 2 (figs. 1(b-c) and 2(b-c)) have negative sign and their absolute value increase in the fisrt seconds (from 0s to 3s), then decrease in the later time interval, with the exeption in the case 3, their histories in the fisrt 40 seconds are similar to their histories in two case above (fig. 3b-c) but in the later time interval, they suddenly have possitive sign and increase slowly with time.

In every case, the distribution of the displacement and stresses at different radii are different. The radial stress at inter and outer surfaces of the cylinder (r = 10 cm and r = 10.5 cm) equal zero, which satisfies the given zero boundary conditons.

It can be seen from figs. 1, 2 and 3 that the distributions of the radial displacement and stresses at different particle's volume ratios are different. The absolute values of the radial displacement and stresses at $\xi = 0.3$ are less than theirs at $\xi = 0.1$ and $\xi = 0.2$. Therefore, when the particle's volume ratio is increased, the radial displacement and thermal stresses of the composite cylinder decrease and their histories on the time are slower.

When the temperatures of the surrounding mediums inside and outside the cylinder change, the displacement and stresses of the cylinder change. Their absolute values in the case 1 are maximum, and the corresponding values in the case 3 is minimum. This result satisfies practice, because the coefficients of thermal conductivity and heat transfer coefficient of water are much greater than the corresponding values of air. Hence, the environments inside and outside the cylinder also affect to the thermal-mechanical behavior of the cylinder.

5. Conclusion

Based on the governing equations and the displacement method in the theory of elasticity, the paper determined the analytical solution of stresses, deformations and displacements for an infinite hollow cylinder made of spherical particle filled composite material under an unsteady, axisymmetric, plane temperature field with the assumtion that the composite is elastic, homogeneous, isotropic and the material properties are temperature - independent.

The numerical calculations of the paper clearly analyzed the influence of time, particle's volume ratio and temperature on the states of unsteady thermal stress and displacement in the infinite hollow cylinder made of titanium /PVC composite material.

It can be also confirmed from the numerical results that the particle plays an important role on the states of stress, deformation and displacement of the composite cylinder. Certain volume ratios of particle can decrease the displacementes, strains and stresses of the composite cylinder. Hence, they can increase the crackproof capacity, waterproof capacity as well as heatproof capacity (increase the strenght) for composite. This is the basis to calculate and design the composite cylinder structures which are not only increased in strength, but also decreased in cost.

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