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The dependence of the nonlinear absorption coefficient of strong electromagnetic waves caused by electrons confined in rectangular quantum wires on the temperature of the system

Hoang Dinh Trien*, Bui Thi Thu Giang, Nguyen Quang Bau

Faculty of Physics, Hanoi University of Science, Vietnam National University
334 Nguyen Trai, Thanh Xuan, Hanoi, Vietnam

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Abstract. The nonlinear absorption of a strong electromagnetic wave caused by confined electrons in cylindrical quantum wires is theoretically studied by using the quantum kinetic equation for electrons. The problem is considered in the case electron-acoustic phonon scattering. Analytic expressions for the dependence of the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in rectangular quantum wires on the temperature T are obtained. The analytic expressions are numerically calculated and discussed for GaAs/GaAsAl rectangular quantum wires.

Keywords: rectangular quantum wire, nonlinear absorption, electron- phonon scattering.

1. Introduction

It is well known that in one dimensional systems, the motion of electrons is restricted in two dimensions, so that they can flow freely in one dimension. The confinement of electron in these systems has changed the electron mobility remarkably. This has resulted in a number of new phenomena, which concern a reduction of sample dimensions. These effects differ from those in bulk semiconductors, for example, electron-phonon interaction and scattering rates [1, 2] and the linear and nonlinear (dc) electrical conductivity [3, 4]. The problem of optical properties in bulk semiconductors, as well as low dimensional systems has also been investigated [5-10]. However, in those articles, the linear absorption of a weak electromagnetic wave has been considered in normal bulk semiconductors [5], in two dimensional systems [6-7] and in quantum wire [8]; the nonlinear absorption of a strong electromagnetic wave (EMW) has been considered in the normal bulk semiconductors [9], in quantum wells [10] and in cylindrical quantum wire [11], but in rectangular quantum wire (RQW), the nonlinear absorption of a strong EMW is still open for studying. In this paper, we use the quantum kinetic quation for electrons to theoretically study the dependence of the nonlinear absorption coefficient of a strong EMW by confined electrons in RQW on the temperature T of the system. The problem is considered in two cases: electron-optical phonon scattering and electron-acoustic phonon scattering. Numerical calculations are carried out with a specific GaAs/GaAsAl quantum wires to

* Corresponding author. Tel.: +84913005279
E-mail: hoangtrien@gmail.com

show the dependence of the nonlinear absorption coefficient of a strong EMW by confined electrons in RQW on the temperature T of the system.

2. The dependence of the nonlinear absorption coefficient of a strong EMW in a WQW on the temperature T of the system

In our model, we consider a wire of GaAs with rectangular cross section ($Lx \times Ly$) and length Lz, embedded in GaAlAs. The carriers (electron gas) are assumed to be confined by an infinite potential in the (x,y) plane and are free in the z direction in Cartesian coordinates (x,y,z). The laser field propagates along the x direction. In this case, the state and the electron energy spectra have the form [12]

$$|n,\ell,\vec{p}\rangle = \frac{2e^{ip_z z}}{\sqrt{L_z L_x L_y}} sin(\frac{\pi n x}{L_x}) sin(\frac{\pi \ell y}{L_y}); \quad \varepsilon_{n,\ell}(\vec{p}) = \frac{p_z^2}{2m} + \frac{\pi^2}{2m}(\frac{n^2}{L_x^2} + \frac{\ell^2}{L_y^2}) \tag{1}$$

where n and ℓ (n, ℓ=1, 2, 3, ...) denote the quantization of the energy spectrum in the x and y direction, $\vec{p} = (0,0,p_z)$ is the electron wave vector (along the wire's z axis), m is the effective mass of electron (in this paper, we select $\hbar$=1).

Hamiltonian of the electron-phonon system in a rectangular quantum wire in the presence of a laser field $\vec{E}(t) = \vec{E}_0 sin(\Omega t)$, can be written as

$$H(t) = \sum_{n,l,\vec{p}} \varepsilon_{n,l}(\vec{p} - \frac{e}{c}\vec{A}(t)) a^+_{n,l,\vec{p}} a_{n,l,\vec{p}} + \sum_{\vec{q}} \omega_{\vec{q}} b^+_{\vec{q}} b_{\vec{q}}$$
$$+ \sum_{n,l,n',l',\vec{p},\vec{q}} C_{\vec{q}} I_{n,l,n',l'}(\vec{q}) a^+_{n,l,\vec{p}+\vec{q}} a_{n',l',\vec{p}} (b_{\vec{q}} + b^+_{-\vec{q}}) \tag{2}$$

where e is the electron charge, c is the light velocity, $\vec{A}(t) = \frac{c}{\Omega}\vec{E}_0 cos(\Omega t)$ is the vector potential, $\vec{E}_0$ and Ω is the intensity and frequency of EMW, $a^+_{n,\ell,\vec{p}}$ ($a_{n,\ell,\vec{p}}$) is the creation (annihilation) operator of an electron, $b^+_{\vec{q}}$ ($b_{\vec{q}}$) is the creation (annihilation) operator of a phonon for a state having wave vector $\vec{q}$, $C_{\vec{q}}$ is the electron-phonon interaction constants. $I_{n,l,n',l'}(\vec{q})$ is the electron form factor, it is written as [13]

$$I_{n,l,n',l'}(\vec{q}) = \frac{32\pi^4 (q_x L_x n n')^2 (1-(-1)^{n+n'} cos(q_x L_x))}{[(q_x L_x)^4 - 2\pi^2 (q_x L_x)^2 (n^2 + n'^2) + \pi^4 (n^2 - n'^2)^2]^2} \times$$
$$\frac{32\pi^4 (q_y L_y \ell\ell')^2 (1-(-1)^{\ell+\ell'} cos(q_y L_y))}{[(q_y L_y)^4 - 2\pi^2 (q_y L_y)^2 (\ell^2 + \ell'^2) + \pi^4 (\ell^2 - \ell'^2)^2]^2} \tag{3}$$

The carrier current density $\vec{j}(t)$ and the nonlinear absorption coefficient of a strong electromagnetic wave α take the form [6]

$$\vec{j}(t) = \frac{e}{m}\sum_{n,\ell,\vec{p}} (\vec{p} - \frac{e}{c}\vec{A}(t)) n_{n,\ell,\vec{p}}(t); \alpha = \frac{8\pi}{c\sqrt{\chi_\infty} E_0^2} \langle \vec{j}(t) \vec{E}_0 sin\Omega t \rangle_t \tag{4}$$

where $n_{n,\ell,\vec{p}}(t)$ is electron distribution function, $\langle X \rangle_t$ means the usual thermodynamic average of X ($X \equiv \vec{j}(t)\vec{E}_0 sin\Omega t$) at moment t, χ_∞ is the high-frequency dielectric constants.

In order to establish analytical expressions for the nonlinear absorption coefficient of a strong EMW by confined electrons in RQW, we use the quantum kinetic equation for particle number operator of electron $n_{n,\ell,\vec{p}}(t) = \langle a^+_{n,\ell,\vec{p}} a_{n,\ell,\vec{p}} \rangle_t$

$$i\frac{\partial n_{n,\ell,\vec{p}}(t)}{\partial t} = \langle [a^+_{n,\ell,\vec{p}} a_{n,\ell,\vec{p}}, H] \rangle_t \quad (5)$$

From Eq.(5), using Hamiltonian in Eq.(2) and realizing calculations, we obtain quantum kinetic equation for confined electrons in CQW. Using the first order tautology approximation method (This approximation has been applied to a similar exercise in bulk semiconductors [9.14] and quantum wells [10]) to solve this equation, we obtain the expression of electron distribution function $n_{n,\ell,\vec{p}}(t)$.

$$n_{n,\ell,\vec{p}}(t) = -\sum_{\vec{q},n',\ell'} |C_{\vec{q}}|^2 |I_{n,\ell,n',\ell'}|^2 \sum_{k,l=-\infty}^{\infty} J_k\left(\frac{e\vec{E}_0,\vec{q}}{m\Omega^2}\right) J_{k+l}\left(\frac{e\vec{E}_0,\vec{q}}{m\Omega^2}\right) \frac{1}{l\Omega} e^{-il\Omega t} \times$$
$$\times\left\{-\frac{\bar{n}_{n,\ell,\vec{p}}(N_{\vec{q}}+1) - \bar{n}_{n',\ell',\vec{p}+\vec{q}} N_{\vec{q}}}{\varepsilon_{n',\ell',\vec{p}+\vec{q}} - \varepsilon_{n,\ell,\vec{p}} + \omega_{\vec{q}} - k\Omega + i\delta} - \frac{\bar{n}_{n,\ell,\vec{p}} N_{\vec{q}} - \bar{n}_{n',\ell',\vec{p}+\vec{q}}(N_{\vec{q}}+1)}{\varepsilon_{n',\ell',\vec{p}+\vec{q}} - \varepsilon_{n,\ell,\vec{p}} - \omega_{\vec{q}} - k\Omega + i\delta} + \right.$$
$$\left. + \frac{\bar{n}_{n',\ell',\vec{p}-\vec{q}}(N_{\vec{q}}+1) - \bar{n}_{n,\ell,\vec{p}} N_{\vec{q}}}{\varepsilon_{n,\ell,\vec{p}} - \varepsilon_{n',\ell',\vec{p}-\vec{q}} + \omega_{\vec{q}} - k\Omega + i\delta} + \frac{\bar{n}_{n',\ell',\vec{p}-\vec{q}} N_{\vec{q}} - \bar{n}_{n,\ell,\vec{p}}(N_{\vec{q}}+1)}{\varepsilon_{n,\ell,\vec{p}} - \varepsilon_{n',\ell',\vec{p}-\vec{q}} - \omega_{\vec{q}} - k\Omega + i\delta}\right\} \quad (6)$$

where $N_{\vec{q}}(\bar{n}_{n,\vec{p}})$ is the time independent component of the phonon (electron) distribution function, $J_k(x)$ is Bessel function, the quantity δ is infinitesimal and appears due to the assumption of an adiabatic interaction of the electromagnetic wave. We insert the expression of $n_{n,\ell,\vec{p}}(t)$ into the expression of $\vec{j}(t)$ and then insert the expression of $\vec{j}(t)$ into the expression of α in Eq.(4). Using properties of Bessel function and realizing calculations, we obtain the nonlinear absorption coefficient of a strong EMW by confined electrons in RQW

$$\alpha = \frac{8\pi^2\Omega}{c\sqrt{\chi_\infty} E_0^2} \sum_{n,\ell,n',\ell'} |I_{n,\ell,n',\ell'}|^2 \sum_{\vec{q},\vec{p}} |C_{\vec{q}}|^2 N_{\vec{q}} \sum_{k=-\infty}^{\infty} [\bar{n}_{n,\ell,\vec{p}} - \bar{n}_{n',\ell',\vec{p}+\vec{q}}] \times$$
$$\times k J_k^2\left(\frac{eE_0\vec{q}}{m\Omega^2}\right) \delta(\varepsilon_{n',\ell',\vec{p}+\vec{q}} - \varepsilon_{n,\ell,\vec{p}} + \omega_{\vec{q}} - k\Omega) + [\omega_{\vec{q}} \to -\omega_{\vec{q}}] \quad (7)$$

where $\delta(x)$ is Dirac delta function.

In the following, we study the problem with different electron-phonon scattering mechanisms. We only consider the absorption close to its threshold because in the rest case (the absorption far away from its threshold) α is very smaller. In the case, the condition $|k\Omega - \omega_0| \ll \bar{\varepsilon}$ must be satisfied. We restrict the problem to the case of absorbing a photon and consider the electron gas to be non-degenerate:

$$\bar{n}_{n,\ell,\vec{p}} = n_0^* exp\left(-\frac{\varepsilon_{n,\ell,\vec{p}}}{k_b T}\right), \text{ with } n_0^* = \frac{n_0(e\pi)^{\frac{3}{2}}}{V(m_0 k_b T)^{\frac{3}{2}}} \quad (8)$$

where, V is the normalization volume, n_0 is the electron density in RQW, m_0 is the mass of free electron, k_b is Boltzmann constant.

2.1 Electron- optical Phonon Scattering

In this case, $\omega_{\vec{q}} \equiv \omega_0$ is the frequency of the optical phonon in the equilibrium state. The electron-optical phonon interaction constants can be taken as [6-8] $|C_{\vec{q}}|^2 \equiv |C_{\vec{q}}^{op}|^2 = e^2\omega_0\left(1/\chi_\infty - 1/\chi_0\right)/2\varepsilon_0 q^2 V$, here V is the volume, ε_0 is the permittivity of free space, χ_∞ and χ_0 are the high and low-frequency dielectric constants, respectively. Inserting $C_{\vec{q}}$ into Eq.(7) and using Bessel function, Fermi-Dirac distribution function for electron and energy spectrum of electron in RQW, we obtain the explicit expression of α in RQW for the case electron-optical phonon scattering

$$\alpha = \frac{\sqrt{2\pi}e^4 n_0 (k_b T)^{3/2}}{4c\varepsilon_0\sqrt{m\chi_\infty}\Omega^3 V}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)\sum_{n\ell,n',\ell'} |I_{n\ell,n',\ell'}|^2 \left[exp\{\frac{1}{k_b T}(\omega_0 - \Omega)\} - 1\right]\times$$
$$\times exp\{\frac{1}{k_b T}\frac{\pi^2}{2m}(\frac{n'^2}{L_x^2} + \frac{\ell'^2}{L_y^2})\}\left[1 + \frac{3e^2 E_0^2 k_b T}{8m\Omega^4}(1 + \frac{B}{2k_b T})\right] + [\omega_0 \to -\omega_0] \quad (9)$$

where $B = \pi^2[(n'^2 - n^2)/L_x^2 + (\ell'^2 - \ell^2)/L_y^2]/2m + \omega_0 - \Omega$, n_0 is the electron density in RQW, k_b is Boltzmann constant.

2.2. Electron- acoustic Phonon Scattering

In the case, $\omega_{\vec{q}} \ll \Omega$ ($\omega_{\vec{q}}$ is the frequency of acoustic phonons), so we let it pass. The electron-acoustic phonon interaction constants can be taken as [6-8,10] $|C_{\vec{q}}|^2 \equiv |C_{\vec{q}}^{ac}|^2 = \xi^2 q/2\rho\upsilon_s V$, here V, ρ, υ_s, and ξ are the volume, the density, the acoustic velocity and the deformation potential constant, respectively. In this case, we obtain the explicit expression of α in RQW for the case of electron-acoustic phonon scattering

$$\alpha = \frac{\sqrt{2m\pi}e^2 n_0 \xi^2 (k_b T)^{5/2}}{4c\sqrt{\chi_\infty}\rho\upsilon_s^2\Omega^3 V}\sum_{n,\ell,n',\ell'} |I_{n,\ell,n',\ell'}|^2 exp\{\frac{1}{k_b T}\frac{\pi^2}{2m}(\frac{n'^2}{L_x^2} + \frac{\ell'^2}{L_y^2})\}\times$$
$$\times\left[exp\{\frac{\Omega}{k_b T}\} - 1\right]\left[1 + \frac{D}{2k_b T}\left[1 + \frac{3e^2 E_0^2 (k_b T)^2}{4m\Omega^4 D}\left(\frac{D^2}{4(k_b T)^2} + \frac{3D}{4k_b T} + 3\right)\right]\right] \quad (10)$$

where $D = \pi^2[(n'^2 - n^2)/L_x^2 + (\ell'^2 - \ell^2)/L_y^2] - \Omega$

From analytic expressions of the nonlinear absorption coefficient of a strong EMW by confined electrons in RQWs with infinite potential (Eq.9 and Eq.10), we see that the dependence of the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in rectangular quantum wires on the temperature T is complex and nonlinear. In addition, from the analytic results, we also see that when the term in proportional to quadratic the intensity of the EMW (E_0^2) (in the expressions of the nonlinear absorption coefficient of a strong EMW) tend toward zero, the nonlinear result will turn back to a linear result.

3. Numerical results and discussions

In order to clarify the dependence of the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in rectangular quantum wires on the temperature T, in this section, we numerically calculate the nonlinear absorption coefficient of a strong EMW for a *GaAs/GaAsAl* RQW. The parameters of the CQW. The parameters used in the numerical calculations [6,13] are $\xi = 13.5\, eV$, $\rho = 5.32\, gcm^{-3}$, $\upsilon_s = 5378\, ms^{-1}$, $\varepsilon_0 = 12.5$, $\chi_\infty = 10.9$, $\chi_0 = 13.1$, $m = 0.066 m_0$, m_0 being the mass of free electron, $\hbar\omega = 36.25\, meV$, $k_b = 1.3807\times10^{-23}\, j/K$, $n_0 = 10^{23}\, m^{-3}$, $e = 1.60219\times10^{-19}\, C$, $\hbar = 1.05459\times10^{-34}\, j.s$.

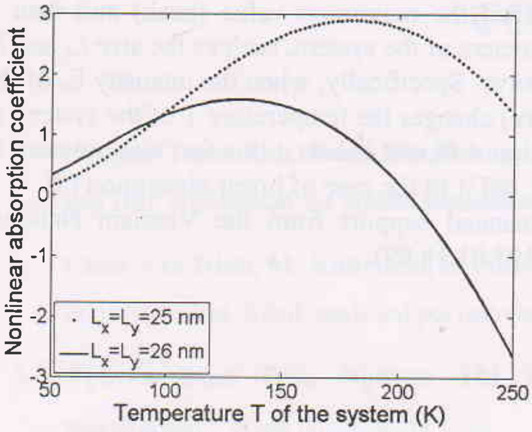

Fig. 1. Dependence of α on T (Electron- optical Phonon Scattering).

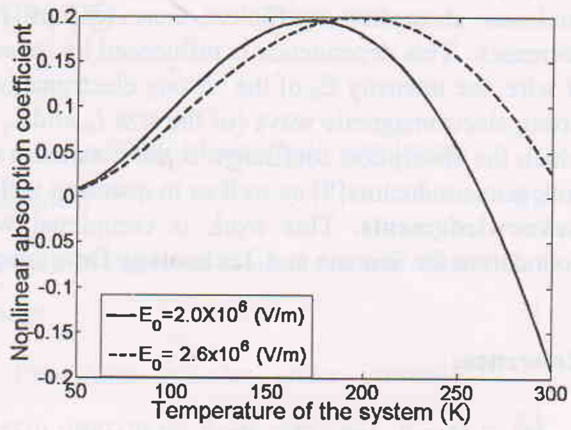

Fig. 2. Dependence of α on T (Electron- acoustic Phonon Scattering).

Figure 1 shows the dependence of the nonlinear absorption coefficient of a strong EMW on the temperature T of the system at different values of size L_x and L_y of wire in the case of electron- optical phonon scattering. It can be seen from this figure that the absorption coefficient depends strongly and nonlinearly on the temperature T of the system. As the temperature increases the nonlinear absorption coefficient increases until it reached the maximum value (peak) and then it decreases. At different values of the size L_x and L_y of wire the temperature T of the system at which the absorption coefficient is the maximum value has different values. For example, at $L_x = L_y = 25nm$ and $L_x = L_y = 26nm$, the peaks correspond to $T \simeq 180K$ and $T \simeq 130K$, respectively

Figure 2 presents the dependence of the nonlinear absorption coefficient α on the temperature T of the system at different values of the intensity E_0 of the external strong electromagnetic wave in the case electron- acoustic phonon scattering. It can be seen from this figure that like the case of electron-optical phonon scattering, the nonlinear absorption coefficient α has the same maximum value but with different values of T. For example, at $E_0 = 2.6\times10^6 V/m$ and $E_0 = 2.0\times10^6 V/m$, the peaks correspond to $T \simeq 170K$ and $T \simeq 190K$, respectively, this fact was not seen in bulk semiconductors[9] as well as in quantum wells[10], but it fit the case of linear absorption [8].

4. Conclusion

In this paper, we have obtained analytical expressions for the nonlinear absorption of a strong EMW by confined electrons in RQW for two cases of electron-optical phonon scattering and electron-acoustic phonon scattering. It can be seen from these expressions that the dependence of the nonlinear absorption coefficient of a strong electromagnetic wave by confined electrons in rectangular quantum wires on the temperature T is complex and nonlinear. In addition, from the analytic results, we also see that when the term in proportional to quadratic the intensity of the EMW (E_0^2) (in the expressions of the nonlinear absorption coefficient of a strong EMW) tend toward zero, the nonlinear result will turn back to a linear result. Numerical results obtained for a *GaAs/GaAsAl* CQW show that α depends strongly and nonlinearly on the temperature T of the system. As the temperature increases the nonlinear absorption coefficient increases until it reached the maximum value (peak) and then it decreases. This dependence is influenced by other parameters of the system, such as the size L_x and L_y of wire, the intensity E_0 of the strong electromagnetic wave. Specifically, when the intensity E_0 of the strong electromagnetic wave (or the size L_x and L_y of wire) changes the temperature T of the system at which the absorption coefficient is the maximum value has different values. , this fact was not seen in bulk semiconductors[9] as well as in quantum wells[10], but it fit the case of linear absorption [8].

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