THE CJT EFFECTIVE ACTION APPLIED TO CRITICAL PHENOMENA FOR THE ABELIAN HIGGS MODEL

Phan Hong Lien
Institute of Military Engineering

Abstract: *The Abelian Higgs Model is considered by means of the Cornwall - Jackiw - Tomboulis (CJT) effective action at finite temperature. The calculations in Hartree - Fock approximation is presented and it is shown that the symmetry is restored at the critical temperature which is derived directly from the gap equation.*

1. INTRODUCTION

The CJT effective action [1] and its thermal effective potential is known as a method which provides approximation beyond two - loop and higher, in particular, in the non-perturbative sector [2]. Weinberg [3], Doland - Jackiw [4], Kirznhits and Linde [5] showed that the renormalizable field theories which is spontaneously broken symmetry can restore the symmetries at critical temperature T_c.

Our main aim is to present in detail a general formalism of thermal CJT effective action because it concerns the nature of the phase transition. It is important to study the high temperature symmetry restoration model. We have used dimensional regularization at finite temperature to calculate the CJT effective potential up to the second order and the critical temperature in Higgs model. Up to now, Higgs mechanism is recognized as an optimized generation of masses via spontaneous symmetry breaking.

The paper is organized as follows. In the section II the Abelian Higgs model and the formalism of CJT effective action are presented. Section III is devoted to considering the thermal effective potential and the SD equations. In the section IV the critical temperature at which symmetry is restored is directly derived from the gap equation. The discussion and conclusion are given in section V.

2. THE HIGGS MECHANISM AND THE CJT EFFECTIVE ACTION FOR THE HIGGS MODEL

Let us apply the formalism of CJT effective action to investigating the Higgs model, which is described by the Lagrangian

$$\begin{aligned}\mathcal{L} = &- \frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + [(\partial_\mu + ie\mathbf{A}_\mu)\mathbf{\Phi}^*][(\partial_\mu - ie\mathbf{A}_\mu)\mathbf{\Phi}] - m^2\mathbf{\Phi}^*\mathbf{\Phi} \\ &- \lambda(\mathbf{\Phi}^*\mathbf{\Phi})^2 + \boldsymbol{\eta}^+\partial_\mu\partial^\mu\boldsymbol{\eta} + \mathcal{L}_{GF}\end{aligned} \tag{2.1}$$

where $\mathbf{\Phi}, \mathbf{A}_\mu$ and $\boldsymbol{\eta}$ are the complex scalar, the gauge and the ghost fields, respectively and $\mathbf{F}_{\mu\nu} = \partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu$ is the invariant field tensor, $\mathcal{L}_{GF}$ is the term gauge - fixing.

It is well known, the Lagrangian (2.1) is invariant under the local Abelian gauge transformation

$$\mathbf{\Phi}(x) \to \mathbf{\Phi}'(x) = e^{-i\Theta(x)}\mathbf{\Phi}(x) \tag{2.2}$$

$$\mathbf{A}_\mu(x) \to \mathbf{A}'_\mu(x) = \mathbf{A}_\mu(x) - \frac{1}{g}\partial_\mu\Theta(x) \tag{2.3}$$

If $m^2 > 0$ there is a symmetric ground state $\mathbf{\Phi} = \mathbf{\Phi}^* = 0$. If $m^2 < 0$ there is again a ring of degenerate ground states, whose expectation value (thermal average) is

$$\langle 0|\mathbf{\Phi}|0\rangle_\beta = \nu \tag{2.4}$$

Let us choose

$$\mathbf{\Phi}'(x) = \frac{1}{\sqrt{2}}\left[\chi'_1(x) + i\chi'_2(x)\right] \tag{2.5}$$

so that $\langle 0|\chi'_1|0\rangle_\beta = \nu$, $\langle 0|\chi'_2|0\rangle_\beta = 0$ and define the physical field $\chi_1 = \chi'_1 - \nu, \chi_2 = \chi'_2$.

The Lagrangian (2.1) becomes

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + \frac{1}{2}(\partial_\mu\chi_1)^2 + \frac{1}{2}(\partial_\mu\chi_2)^2 - \frac{1}{2}\left(m^2 + 3\lambda\nu^2\right)\chi_1{}^2 \\ &- \frac{1}{2}\left(m^2 + \lambda\nu^2\right)\chi_2{}^2 - \lambda\nu\chi_1\left(\chi_1^2 + \chi_2^2\right) - \frac{\lambda}{4}\left(\chi_1^2 + \chi_2^2\right) \\ &- e\nu\mathbf{A}_\mu\partial^\mu\chi_2 - e\mathbf{A}^\mu\left(\chi_1\partial_\mu\chi_2 - \chi_2\partial_\mu\chi_1\right) + \frac{1}{2}e^2\mathbf{A}_\mu\mathbf{A}^\mu\left(\chi_1^2 + \chi_2^2\right) \\ &+ \frac{1}{2}e^2\mathbf{A}_\mu\mathbf{A}^\mu\left(\nu^2 + 2\nu\chi_1\right) + \eta^+\partial_\mu\partial^\mu\eta + \mathcal{L}_{GF}\end{aligned} \tag{2.6}$$

The masses of χ_1,χ_2 and $\mathbf{A}_\mu$ bosons, respectively, are

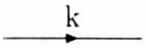
$$\begin{aligned}\mu_1 &= -(m^2 + 3\lambda\nu^2) \\ \mu_2 &= -(m^2 + \lambda\nu^2) \\ M &= e\nu\end{aligned}$$

At $T = 0$, as its well known, $\nu \to \nu_0 = \sqrt{-\frac{m^2}{\lambda}}$ and χ_2 is massless and does not represent an observable particle in scattering experiments (Abbers and Lee, 1973).

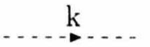
Take the gauge - fixing term in R gauge as

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}\left(\partial^\mu\mathbf{A}_\mu + \xi e\nu\chi_2\right)^2 = -\frac{1}{2\xi}\left(\partial^\mu\mathbf{A}_\mu\right)^2 - \frac{1}{2}\xi e^2\nu^2\chi_2^2 + e\nu\mathbf{A}_\mu\partial^\mu\chi_2 \tag{2.7}$$

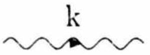
So, the mixing term $e\nu\mathbf{A}_\mu\partial^\mu\chi_2$, which corresponds to the coupling ∿∿∿►---- , got rid of the Lagrangian. The free propagators in Euclide momentum space of χ_1, χ_2 and $\mathbf{A}_\mu$ in the R_ξ gauge, respectively, are



$$D_o^1(k) = \frac{1}{k^2 + \mu^2 + i\epsilon} \tag{2.8}$$



$$D_o^2(k) = \frac{1}{k^2 + \xi\mu^2 + i\epsilon} \tag{2.9}$$



$$\begin{aligned}D_{o\mu\nu}(k) &= -\frac{1}{k^2 + \xi\mu^2 + i\epsilon}\left[g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2 - \xi M^2}\right] \\ &= -\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2}}{k^2 + M^2 + i\epsilon} - \frac{\frac{k_\mu k_\nu}{M^2}}{k^2 + \xi M^2}\end{aligned} \tag{2.10}$$

where $\mu^2 = -(m^2 + 3\lambda\nu^2)$, $M = e\nu$ and ξM^2 are the bare masses of the χ_1, χ_2 bosons and A_μ gauge boson, respectively, in the R_ξ gauge.

In the limit $\xi = 0$ one gets the Landau gauge and the vector field satisfies the Lorentz condition $\partial_\mu A^\mu = 0$, the theory is "manifestly renormalizable". For $\xi = 1$ there is so - called t'Hooft - Feynman gauge. The unitary gauge is recovered in the limit $\xi \to \infty$.

The shifted full Lagrangian is being

$$\begin{aligned}\mathcal{L} = &- \frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + \frac{1}{2}(\partial_\mu\chi_1)^2 + \frac{1}{2}(\partial_\mu\chi_2)^2 + \frac{1}{2}\mu_1^2\chi_1^2 + \frac{1}{2}\mu_2^2\chi_2^2 \\ &- \lambda\nu\chi_1(\chi_1^2+\chi_2^2) - \frac{\lambda}{4}(\chi_1^2+\chi_2^2) + \frac{1}{2}M^2\mathbf{A}_\mu\mathbf{A}^\mu \\ &+ \frac{1}{2}e^2\mathbf{A}_\mu\mathbf{A}^\mu(\chi_1^2+\chi_2^2) - \frac{1}{2\xi}(\partial^\mu\mathbf{A}_\mu)^2 + \eta^+\partial_\mu\partial^\mu\eta \end{aligned} \tag{2.11}$$

The generation of mass for the vector field via spontaneous symmetry breaking is known as the Higgs mechanism. It is a central concept in modern gauge theories.

Hence, the quadratic Lagrangian for the Abelian Higgs model takes the form

$$\begin{aligned}\mathcal{L} = &- \frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} + \frac{1}{2}(\partial_\mu\chi_a)^2 + \frac{1}{2}\chi_a M_{ab}\chi_b + \frac{1}{2}M^2\mathbf{A}_\mu \\ &- \frac{\lambda}{4}\chi^4 + \frac{1}{2}\mathbf{A}_\mu^2\chi^2 - \frac{1}{2\xi}(\partial^\mu\mathbf{A}_\mu)^2 + \eta^+\partial_\mu\partial^\mu\eta \end{aligned} \tag{2.12}$$

where $\chi^2 = \chi^a\chi_a$, $\chi^4 = (\chi^2)^2$; $a = 1,2$ and $M = e\nu$ is mass of vector boson, M_{ab} is diagonal mass matrices. At $T = 0$ it takes the form:

$$M_{ab}^0 = -\begin{pmatrix} m^2 + 3\lambda\nu_o^2 & 0 \\ 0 & m^2 + \lambda\nu_o^2 \end{pmatrix} \tag{2.13}$$

The classical action is given by

$$\begin{aligned} I[\chi, A_\mu, \eta] &= \int dx\mathcal{L}(x) \\ &= \iint dxdy\Big[\chi_a(x)D_{oab}^{-1}(x-y)\chi_b(y) + \mathbf{A}^\mu(x)\Delta_{o\mu\nu}^{-1}(x-y)\mathbf{A}^\nu(y) \\ &\quad + \eta^+(x)S_o^{-1}(x-y)\eta(y)\Big] + \int dx\mathcal{L}_{int}(x) \end{aligned} \tag{2.14}$$

The CJT generating functional for connected Green's function is defined by

$$\begin{aligned} Z_\beta^{CJT}[J,K] = \exp iW_\beta^{CJT}[J,K] = \frac{1}{Z[0,0]}\int\Big[D\chi_a\Big]\Big[DA_\mu\Big]\Big[D\eta^+\Big]\Big[D\eta\Big] \\ \exp i\Big\{\int dx\Big[\mathcal{L}(x)J_a\chi_a(x) + J_\mu(x)A^\mu(x) + j^+(x)\eta(x) + \eta^+(x)j(x)\Big] \\ + \frac{1}{2}\iint dxdy\Big[\chi_a(x)K_{ab}(x,y)\chi_b(y) \\ + A^\mu(x)K_{\mu\nu}(x,y)A^\nu(y) + \eta^+(x)K(x,y)\eta(y)\Big]\Big\} \end{aligned} \tag{2.15}$$

where the physical fields satisfy the periodic condition

$$\mathbf{\Phi}\left(\frac{\beta}{2},\mathbf{x}\right) = \mathbf{\Phi}\left(-\frac{\beta}{2},\mathbf{x}\right) \tag{2.16}$$

with $\mathbf{\Phi} = (\boldsymbol{\chi}_a, \boldsymbol{A}_\mu, \boldsymbol{\eta})$. We have then

$$\frac{\delta W_\beta^{CJT}}{\delta J_a(x)} = \chi_a(x); \qquad \frac{\delta W_\beta^{CJT}}{\delta J_\mu(x)} = A_\mu(x) \tag{2.17}$$

$$\frac{\delta W_\beta^{CJT}}{\delta j(x)} = \eta^+(x); \qquad \frac{\delta W_\beta^{CJT}}{\delta j^+(x)} = \eta(x) \tag{2.18}$$

and

$$\frac{\delta W_\beta^{CJT}}{\delta K_{ab}(x,y)} = \frac{1}{2}\left[\boldsymbol{\chi}_a(x)\boldsymbol{\chi}_b(y) + D_{ab}(x,y)\right] \tag{2.19}$$

$$\frac{\delta W_\beta^{CJT}}{\delta K_{\mu\nu}(x,y)} = \frac{1}{2}\left[\boldsymbol{A}^\mu(x)\boldsymbol{A}^\nu(y) + \Delta^{\mu\nu}(x,y)\right] \tag{2.20}$$

$$\frac{\delta W_\beta^{CJT}}{\delta K(x,y)} = \frac{1}{2}\left[\boldsymbol{\eta}^+(x)\boldsymbol{\eta}(y) + S(x,y)\right] \tag{2.21}$$

The CJT effective action is a double Legendre transformation of W_β^{CJT}

$$\begin{aligned}
\Gamma_\beta^{CJT}\Big[\chi_a, A^\mu, \eta, D, \Delta_{\mu\nu}, S\Big] &= W_\beta^{CJT}\Big[J_a, J_{\mu\nu}, j^+, j, K_{ab}, K_{\mu\nu}, K\Big] \\
&- \int dx\Big[J_a(x)\chi_a(x) + J_\mu(x)A^\mu(x) + \eta^+(x)j(x) + j^+(x)\eta(x)\Big] \\
&-\frac{1}{2}\iint dxdy\Big[\chi_a(x)K_{ab}(x,y)\chi_b(y) + D_{ab}(x,y)K_{ba}(y,x) \\
&\quad + A^\mu(x)K_{\mu\nu}(x,y)A^\nu(y) + \Delta^{\mu\nu}(x,y)K_{\nu\mu}(y,x) \\
&\quad + \eta^+(x)K(x,y)\eta(y) + S(x,y)K(y,x)\Big]
\end{aligned} \tag{2.22}$$

Of course, the physical state corresponds to vanishing external source. Physical solution require

$$\frac{\delta\Gamma_\beta^{CJT}}{\delta\phi(x)} = 0 \qquad (\phi = \chi_a, A_\mu, \eta) \tag{2.23}$$

$$\frac{\delta\Gamma_\beta^{CJT}}{\delta G(x)\big|_{G=G_o}} = 0 \qquad (G = D_{ab}, \Delta_{\mu\nu}, S) \tag{2.24}$$

In order to obtain the loop expansion of Γ_β^{CJT}, we define the functional operator as

$$G^{-1}[\phi; x, y] = \frac{\delta^2 I[\chi_a, A_\mu, \eta]}{\delta\phi(x)\delta\phi(y)} = G_o^{-1}(x-y) + \frac{\delta^2 I_{int}}{\delta\phi(x)\delta\phi(y)} \tag{2.25}$$

where the action $I[\chi_a, A_\mu, \eta]$ is obtained from the classical action $I[\boldsymbol{\chi}_a, \mathbf{A}_\mu, \boldsymbol{\eta}]$ in (2.11) by shifting the fields $\boldsymbol{\chi}_a, \boldsymbol{A}_\mu, \boldsymbol{\eta}$ by χ_a, A_μ and η.

If we define the Fourier transformations of G(x-y) as

$$G(k) = \int dx G(x-y)e^{ik(x-y)} \tag{2.26}$$

then we have got the functional operators

$$\mathfrak{D}^{-1}_{oab}(\chi, k) = k^2 + M^2_{ab} \tag{2.27}$$

$$\Delta^{-1}_{o\mu\nu}(\chi, k) = (k^2 + e^2\chi^2)g_{\mu\nu} + k_\mu k_\nu \tag{2.28}$$

where $M^2_{ab} = -(m^2 + \lambda\chi^2)\delta_{ab} + 2\lambda\chi_a\chi_b$ is the mass matrices of Higgs bosons. $(M^2)_{\mu\nu} = e^2\chi^2 g_{\mu\nu}$ is the mass of vector boson.

The expression for Γ^{CJT}_β can be derived directly basing on [1]

$$\begin{aligned}\Gamma^{CJT}_\beta =& I\left[\chi_a, A_\mu, \eta\right] + \frac{1}{2}Tr\left[ln D_{oab}D^{-1}_{ab} + \mathfrak{D}^{-1}_{oab}(\chi, k)D_{ab}\right] \\ & \frac{1}{2}Tr\left[ln\Delta_{o\mu\nu}\Delta^{-1}_{\mu\nu} + \Delta^{-1}_{o\mu\nu}(\chi, k)\Delta_{\mu\nu} - 1\right] \\ & + \frac{1}{2}Tr\left[lnS_oS^{-1} + S^{-1}(k)S - 1\right] + \Gamma^{(2)}_\beta\left[\chi_a, A_\mu, \eta, D_{ab}, \Delta_{\mu\nu}, S\right]\end{aligned} \tag{2.29}$$

where the trace, the logarithm and the product $\mathfrak{D}^{-1}_{oab}D_{ab}$ and $\Delta_{o\mu\nu}\Delta^{-1}_{\mu\nu}\ldots$ are taken in the functional sense.

The momentum representation in the Euclide space of the thermal propagators in the Landau gauge are as follows

$$D_{oab}(k) = \frac{1}{(2\pi nT)^2 + \mathbf{k}^2 + M^{02}_{ab}} \tag{2.30a}$$

$$\Delta_{o\mu\nu}(k) = \frac{\left[\frac{k_\mu k_\nu}{(2\pi nT)^2 + \mathbf{k}^2} - g_{\mu\nu}\right]}{(2\pi nT)^2 + \mathbf{k}^2} \tag{2.30b}$$

$$\Delta^{-1}_{o\mu\nu}(\chi, k) = (k^2 + M^2)g_{\mu\nu} + k_\mu k_\nu \tag{2.30c}$$

$\Gamma^{(2)}_\beta$ is given by all those two - particle irreducible vacuum graphs which, upon cutting off one line, yield proper self - energy graphs. It is easily verified that, corresponding to Lagrangian (2.10), only the diagram of order and e^2, which is shown in Fig. 1 are under discussion.

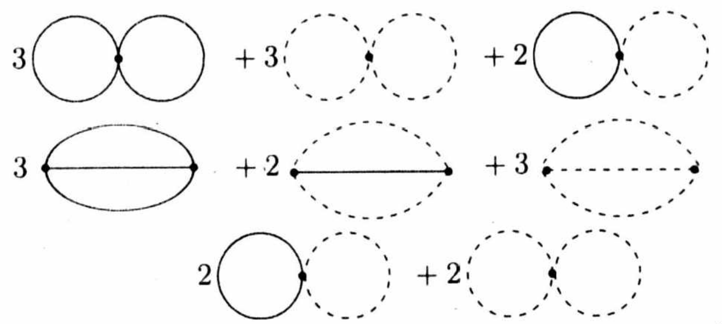


Fig 1. *The two - loop graphs of order λ and e^2 for $\Gamma^{(2)}_\beta$*

$$\begin{aligned}\Gamma^2_\beta =& \frac{3\lambda}{4}\left[3D_{aa}(p)D_{bb}(q) + 2D_{ab}(p)D_{ba}(q)\right] \\ & + \frac{3e^2 g_{\mu\nu}}{4}\left[D_{aa}(p)G_{\mu\nu}(q) + D_{ab}(p)D_{\mu\nu}(q)\right] \\ & + \frac{3e^2 g_{\mu\nu}}{4}D_{ab}(p)G_{\mu\nu}(q)D_{ba}(p+q)\end{aligned} \tag{2.31}$$

3. THERMAL EFFECTIVE POTENTIAL AND THE SCHWINGER DYSON EQUATIONS

The symmetry is spontaneously broken if the equations (2.23) has a non - vanishing solution $\chi(x) \neq 0$. For $\langle\chi\rangle_\beta = const$, the CJT effective potential is defined by

$$\Gamma_\beta^{CJT}[\chi] = -V_\beta^{CJT}[\chi] \int dx \qquad (3.1)$$

In field theory at finite temperature, we use Euclide time τ, which is restricted to the interval $0 \leq \tau \leq \beta (\beta = \frac{1}{kT})$. The Feynman rules are the same as those at zero temperature except that the momentum space integral over the time component k_4 is replace by a sum over Matsubasa frequencies for boson $\omega_n = (2\pi n)/\beta = 2\pi n T$ (we set Boltzmann constant k = 1)

$$\int \frac{d^4 k_E}{(2\pi)^4} = \int \frac{d^3 k}{(2\pi)^3} \int \frac{dk_4}{2\pi} = T \sum_n \int \frac{d^3 k}{(2\pi)^3} = \not{\sum_k} \qquad (3.2)$$

Starting from (2.27) - (2.28) and (3.1),(3.2) we arrive at the expression in Hatree - Fock approximation for the thermal CJT effective potential in Euclide momentum space.

$$\begin{aligned}
V_\beta^{CJT}[\chi_a, \mathfrak{M}_{ab}, \mathfrak{M}] =& V^0|_{T=0} + V^\beta_{THERMAL}(\chi_a) \\
=& \frac{\mu^2}{2}\chi^2 + \frac{\lambda}{4}\chi^4 + \frac{1}{2}\not{\sum_k}\Big[ln(k^2 + \mathfrak{M}^2_{ab}) + ln(k^2 + \mathfrak{M}^2)\Big] \\
&- \frac{1}{2}\Big[\mathfrak{M}^2_{ab} + (m^2 + \lambda\chi^2)\delta_{ab} - 2\lambda\chi_a\chi_b\Big] D_{ab}(k) \\
&- \frac{1}{2}\Big[\mathfrak{M}^2_{\mu\nu} - e^2\chi^2 g_{\mu\nu}\Big] G_{\mu\nu}(k) \\
&- \frac{3\lambda}{4}\not{\sum_k}\not{\sum_p}\Big[3D_{aa}(k)D_{bb}(p) + 2D_{ab}(k)D_{ba}(p)\Big] \\
&- \frac{3e^2 g_{\mu\nu}}{4}\not{\sum_k}\not{\sum_p}\Big[D_{aa}(k)G_{\mu\nu}(p) + D_{ab}(k)G_{\mu\nu}(p)\Big] \\
&- \frac{3e^2 g_{\mu\nu}}{4}\not{\sum_k}\not{\sum_p} D_{ba}(k)G_{\mu\nu}(p)D_{ba}(k+p)
\end{aligned} \qquad (3.3)$$

The stationary condition require

$$\frac{\delta V_\beta^{CJT}}{\delta\chi_a^2} = \frac{\mu^2}{2} + \frac{\lambda}{2}\chi^2 + 2\lambda D_{ab}\delta_{ab} + \frac{e^2}{2} g_{\mu\nu}G_{\mu\nu} = 0 \qquad (3.4)$$

Substituting (3.3) into (2.24) we have got the system of SD equations for the inverse of full propagators

$$\begin{aligned}
D_{ab}^{-1}(k) =& \mathfrak{D}^{-1}_{oab}(\chi, k) - 2\frac{\delta V_{2\beta}^{CJT}}{\delta D_{ab}(k)} \\
=& \mathfrak{D}^{-1}_{oab}(\chi, k) - \Pi_{ab}(k) = k^2 + \mathfrak{M}^2_{ab}
\end{aligned} \qquad (3.5)$$

where $\mathfrak{M}^2_{ab} = M_{ab} + \Pi_{ab} = (m^2 + \lambda\chi^2)\delta_{ab} + 2\lambda\chi_a\chi_b + \Pi_{ab}$

$$\begin{aligned} G^{-1}_{\mu\nu}(k) =& \mathcal{G}^{-1}_{o\mu\nu}(\chi, k) - 2\frac{\delta V^{CJT}_{2\beta}}{\delta G_{\mu\nu}(k)} \\ =& \mathcal{G}^{-1}_{o\mu\nu}(\chi, k) - \Pi_{\mu\nu}(k) = \left(k^2 + \mathfrak{M}^2\right) g_{\mu\nu} + k_\mu k_\nu \end{aligned} \tag{3.6}$$

where $\mathfrak{M}_{\mu\nu} = e^2\chi^2 g_{\mu\nu} + \Pi_{\mu\nu} = M_{\mu\nu} + \Pi_{\mu\nu}$.

The second terms in Eqs (3.5) and (3.6) are represented by the graphs given in Fig 2

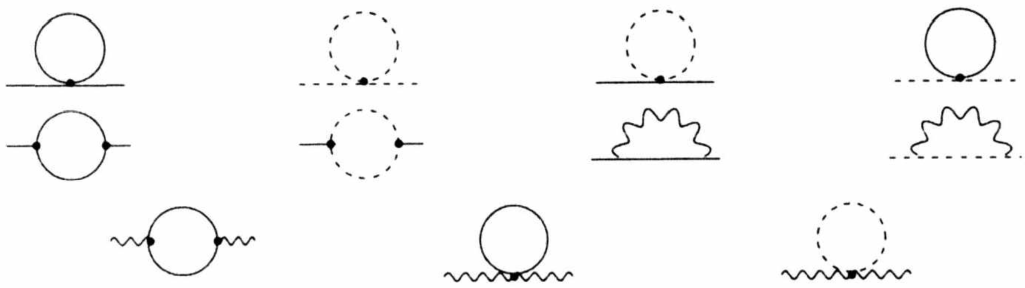


Fig 2. *The graphs corresponds to the thermal proper energy*

$$(a)\ 2\frac{\delta V^{CJT}_{2\beta}}{\delta D_{ab}(k)} = \Pi_{ab}(k), \qquad (b)\ 2\frac{\delta V^{CJT}_{2\beta}}{\delta G_{\mu\nu}(k)} = \Pi_{\mu\nu}(k)$$

——- a=1

- - - - a=2

The system of equations (3.5) and (3.6) are rewritten, respectively, in the usual form of the gap equations

$$\chi_a^2 = -\frac{\mu^2}{\lambda} - 4D_{ab}\delta_{ab} - \frac{3e^2}{2\lambda} g_{\mu\nu}G_{\mu\nu} \tag{3.7}$$

$$\begin{aligned} \mathfrak{M}^2_{ab} =& -(m^2 + \lambda\chi^2)\delta_{ab} + 2\lambda\chi_a\chi_b + \frac{3\lambda}{2}\sum\!\!\!\!\!\!\int_p \left[D_{ab}(p) + D_{ba}(p)\right] \\ &+ \frac{3e^2 g_{\mu\nu}}{2}\sum\!\!\!\!\!\!\int_p G_{\mu\nu}(p) + \frac{3e^2 g_{\mu\nu}}{4}\sum\!\!\!\!\!\!\int_k\sum\!\!\!\!\!\!\int_p G_{\mu\nu}(p)D_{ba}(k+p) \end{aligned} \tag{3.8}$$

$$\begin{aligned} \mathfrak{M}^2_{\mu\nu} =& e^2\chi^2 g_{\mu\nu} + \frac{3e^2 g_{\mu\nu}}{2}\sum\!\!\!\!\!\!\int_p \left[D_{aa}(p) + D_{ab}(p)\right]\delta_{ab} \\ &+ \frac{3e^2 g_{\mu\nu}}{4}\sum\!\!\!\!\!\!\int_k\sum\!\!\!\!\!\!\int_p \left[D_{ab}(p)D_{ba}(k+p)\right] \end{aligned} \tag{3.9}$$

The thermal effective potential V^β is obtained by evaluating V^{CJT}_β at the values of $\mathfrak{M}^2_{ab}$ and $\mathfrak{M}^2_{\mu\nu}$ given by equations (3.8) and (3.9), which contain divergent thermal loops in the forms

$$I_1 = \sum_n \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\mathbf{k}^2 + M^2(T)} \tag{3.10}$$

where $k^2 = k_E^2 = k_4^2 + \mathbf{k}^2$, and $M^2(T)$ is called "thermal mass"

$$I_2 = \sum_n \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{k^2 + M_1^2(T)} \frac{1}{(k+p)^2 + M_2^2(T)} \tag{3.11}$$

The proper way is to use dimensional regularization and perform $\epsilon \to 0$ limit. When M = 0, it is being

$$\begin{aligned} I_1 &= \lim_{\epsilon \to 0} \mu^{2\epsilon} T \sum_n \int \frac{d^{3-2\epsilon}\mathbf{k}}{(2\pi)^{3-2\epsilon}} \frac{1}{(2\pi n T)^2 + \mathbf{k}^2} \\ &= \frac{T^2}{12}\Big[1 + \epsilon\Big(2ln\frac{4\pi\mu^2 e^\gamma}{4\pi T} + 2\frac{\xi'(-1)}{\xi(-1)} + 2\Big) + 0(\epsilon^2)\Big] \end{aligned} \tag{3.12}$$

where μ is renormalization scale.

When $M \neq 0$ (3.10) and (3.11) take the form

$$\begin{aligned} I_1 &= \lim_{\epsilon \to 0} \mu^{2\epsilon} T \sum_n \int \frac{d^{3-2\epsilon}\mathbf{k}}{(2\pi)^{3-2\epsilon}} \frac{1}{(2\pi n T)^2 + \mathbf{k}^2 + M^2} \\ &= \frac{T^2}{12}\Big[1 + \epsilon\Big(2ln\frac{4\pi\mu^2 e^\gamma}{M^2} + 2\frac{\xi'(-1)}{\xi(-1)} + 2\Big) + 0(\epsilon^2)\Big] \end{aligned} \tag{3.13}$$

$$\begin{aligned} I_2 &= \lim_{\epsilon \to 0} \mu^{2\epsilon} T \sum_n \int \frac{d^{3-2\epsilon}\mathbf{k}}{(2\pi)^{3-2\epsilon}} \frac{1}{k^2 + M_1^2(T)} \frac{1}{(k+p)^2 + M_2^2(T)} \\ &= \lim_{\epsilon \to 0} \mu^{2\epsilon} T \int \frac{d^{3-2\epsilon}\mathbf{k}}{(2\pi)^{3-2\epsilon}} \frac{1}{\mathbf{k}^2 + M_1^2} \frac{1}{(\mathbf{k}+\mathbf{p})^2 + M_2^2 + p_o^2} \\ &+ 2\mu^{2\epsilon} T \sum_{n=1}^{\infty} \int \frac{d^{3-2\epsilon}\mathbf{k}}{(2\pi)^{3-2\epsilon}} \frac{1}{(2\pi n T)^2 + \mathbf{k}^2 + M_1^2} \frac{1}{(2\pi n T + p_o)^2 + \mathbf{k}^2 + M_2^2} \end{aligned} \tag{3.14}$$

$$I_2 = \frac{T}{8\pi}\Big\{\int_0^1 A^{-1/2} dz - \epsilon\Big[\int_0^1 dz A^{-1/2} ln \frac{A}{4\pi M^2} - (\gamma + 2)\int_0^1 A^{-1/2} dz\Big] + 0(\epsilon^2)\Big\} \tag{3.15}$$

where $A = p^2 z(1-z) - p_o^2 z + M^2$.

So there is not logarithmic UV divergence in I_3, i.e no $1/\epsilon$ term. When $\epsilon \to 0$ the finite part of I_2 is $T(M - \sqrt{M^2 + p_o^2})$. By using (3.11) - (3.15) we can evaluate the masses $\mathfrak{M}_{ab} \to M_{ab} + \left(\frac{\lambda T^2}{6} + \frac{e^2 T^2}{4}\right)^{1/2}$ and $\mathfrak{M} \to M + \left(\frac{e^2 T^2}{3}\right)^{1/2}$

4. RESTORATION OF SPONTANEOUS SYMMETRY IN THE HIGGS MODEL

It's well known that in the case $m^2 < 0$ the symmetry is spontaneously broken if

$$\frac{\partial V_\beta^{CJT}}{\partial \chi^2} = 0 \qquad for \quad \chi \neq 0 \tag{4.1}$$

The symmetry will be restored at high temperature if there exits $\chi = 0$ so that

$$\left.\frac{\partial V_\beta^{CJT}}{\partial \chi^2}\right|_{\chi=0} = 0 \tag{4.2}$$

So, the root non - trivial of equation (2.23) for $\chi^2 = 0$, which is directly derived by minimizing the thermal effective potential $\partial V_\beta^{CJT}/\partial \chi^2 = 0$, leads to the critical temperature.

The gap equations (3.7) for $\chi_a^2 = 0$ takes the form

$$-\frac{\mu^2}{\lambda} - 4D_{ab} - \frac{3e^2}{2\lambda} g^{\mu\nu} G_{\mu\nu} = 0 \tag{4.3}$$

Substituting into (4.3) the part finite of propagators for scalar and vector boson, respectively, one gets

$$-\frac{\mu^2}{\lambda} = 4\frac{T^2}{12} + \frac{3e^2}{2\lambda}\frac{T^2}{6} \tag{4.4}$$

The critical temperature is obtained directly from (4.4)

$$T_C = -\frac{12\mu^2}{(4\lambda + 3e^2)} \tag{4.5}$$

The restoration of symmetry and critical phenomena appear at T_c which depends on two coupling constants e and λ.

5. CONCLUSION AND DISCUSSION

In the preceding section we have calculated the critical temperature T_c, at which the high temperature restoration of the spontaneously broken symmetry takes place. This is a second order phase transition (Weinberg [2], Kapusta [5]).

If $e^2/\lambda \gg 1$ when $\chi > eT_c$ there is the first order phase transition.

In the application of the composite operator method, the next consideration will deal with the critical phenomena in Higgs sector of gauge theory, which provide the (non) restoration of symmetry at high temperature.

The author would like to thank Prof. Tran Huu Phat for suggestion of this problem.

REFERENCES

1. J. Cornwall, R, Jackiw and E. Tomboulis (1974), Phys. Rev. **D10**.
2. Barthmy, M. and Orland, H (1998), Eur. Phys. J. **B6** 537-541.
3. S. Weinberg (1974), Phys. Rev. **D9**, 3357.
4. L. Doland and R. Jackie (1974), Phys. Rev. **D9**, 3320.
5. D.A.Kitznhits and A.D. Linde, Sov (1975). Phys JETP **40**, 628.
6. J.I Kapusta (1989), *Finite- temperature field theory.* Cambridge University Press