THE CJT EFFECTIVE ACTION APPROACH APPLIED TO THE SU(3) GENERALIZED NJL MODEL

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Abstract: *The SU(3) generalized Nambu - Jona - Lasinio (NJL) model is considered by means of the Cornwall - Jackiw - Tomboulis (CJT) effective action. This method provides a very general framework for investigating many important non- pertubative effects: quark condensate and mesons, thermodynamical quantities at finite temperature. It is shown that the mixing of flavors of u, d and s quarks exists in this formalism. The gap equations, which are directly obtained from the effective potential involved the quark condensate.*

1. INTRODUCTION

Description of quark matter within the framework of non - pertubative theory turns out to be more crucial for relativistic quantum theoretical study of condensable matter. At low energy (about 1GeV) the non - pertubative effects concern with the confinement of quarks and the dynamical breaking of chiral symmetry. In this respect, many authors constructed different symmetry conserving approximation schemes: the mean - field approximation [1]-[2], the "ϕ - derivable" method [3], the (second) random phase approximation (RPA) [4], an expansion in powers of the inverse number of colors [5], the one - loop approximation of the effective action [6]... In these works, it is worth to mention that the CJT effective action method [7], which obviously includes the Schwinger - Dyson (SD) equation approach [8], may hopefully provide a promised approximation beyond two - loop calculations. Its priority is expressed by the fact that the vacuum expectation values of field operators and propagators are treated on the same footing; therefore it takes into account all the possible correlation effects. In addition to the preceding trend, one has made great attempts to investigate the role of chiral symmetry in condensate matter [1], [10].

The Nambu - Jona - Lasinio (NJL) model originally was a model contained nucleons [9]. Nowadays this model is used to study the properties of quarks instead of nucleons. The SU(2) version of NJL has been applied by many authors to study the restoration of chiral symmetry at critical temperature and nonzero density [1], [11]. However, the recent consideration [12], [13] indicate that the strange quark matter could be the absolute ground state of matter. This leads to the SU(3) version of NJL model, which includes in addition to up and down quarks also strange quarks.

Our main aim is to present in detail the CJT effective action approach, which is applied to study systematically the SU(3) generalized NJL model. In this connection, it is possible to consider our work as being complementary to [1].

This paper is organized as follows. In section 2, the chiral symmetry in SU(3) version of NJL model and CJT effective action formalism are presented. Section 3 is devoted to loop expansion of effective potential. Hence SD equations and gap equations are directly derived. In section 4 the CJT effective potential is evaluated at $T \neq 0$. The conclusion and discussion are given in section 5.

2. FORMALISM

1.1. Chiral symmetry

Let us consider the SU(3) generalized NJL model whose Lagrangian reads

$$\begin{aligned}\pounds =&\bar{\Psi}(i\gamma^{\mu}\partial_{\mu} - m_q)\Psi + \frac{G_S}{2}\sum_{a=0}^{8}\left[(\bar{\Psi}\lambda^a\Psi)^2 + (\Psi i\gamma^5\lambda^a\Psi)^2\right]\\ &+ G_D det_f\left[\bar{\Psi}(1+\gamma_5)\Psi\right] + G_D det_f\left[\bar{\Psi}(1-\gamma_5)\Psi\right]\end{aligned} \tag{2.1}$$

where $\Psi(x)$ are quark fields $\Psi = u, d, s$ with three colors ($N_c = 3$) and three flavors ($N_f = 3$),$\lambda^a(a = 0 \div 8)$ are the Gell-Mann matrices with $\lambda_0 = \sqrt{\frac{2}{3}}$. There are two chiral invariant coupling constant G_S and G_D of four and six fermions interaction.

The current quark mass

$$m_q = diag(m_u, m_d, m_s) = \sum_{a=0,3,8} m_a\lambda_a \tag{2.2}$$

explicit breaks chiral symmetry, and the six - point vertex

$$\bar{\Psi}(1-\gamma_5)\Psi = \Phi \tag{2.3}$$

leads to the mixing between singlet, octet and triplet.

The chiral transformation is defined by

$$\Psi \rightarrow U(\alpha)U_5(\beta)\Psi = exp\left[i\left(\alpha - \beta\gamma_5\right)\frac{\lambda^a}{2}\right]\Psi \tag{2.4a}$$

$$\bar{\Psi} \rightarrow \bar{\Psi}U_5^+(\beta)U^+(\alpha) = \bar{\Psi}exp\left[-i\left(\alpha + \beta\gamma_5\right)\frac{\lambda^a}{2}\right] \tag{2.4b}$$

where

$$\Psi_{L(R)} \rightarrow U(\alpha)\Psi_{L(R)} = exp\left(i\sum_{a=1}^{8}\alpha_a\frac{\lambda^a}{2}\right)\Psi_{L(R)} \tag{2.5}$$

is the $SU_L(3) \otimes SU_R(3)$ transformation in the three flavors, and the transformation

$$\Psi \rightarrow U_5(\beta)\Psi = exp\left(i\gamma_5\sum_{a=1}^{8}\beta_a\frac{\lambda^a}{2}\right)\Psi \tag{2.6}$$

is the Q_5 transformation, which leads to the anomalous divergence of the flavor singlet axial current [1].

$$J_{\mu}^{5} = -\frac{\delta L}{\delta\partial^{\mu}\beta} = i\bar{\Psi}\gamma_{\mu}\gamma_5\Psi \tag{2.7}$$

$$\partial_{\mu}J_5^{\mu} = -4\mathrm{N_f G_D Im}\left(det_f\Phi\right) + 2im\bar{\Psi}\gamma_5\Psi \tag{2.8}$$

Under $SU_L(3) \otimes SU_R(3)$ transformation, the operators Φ, Φ^+ are transformed as follows

$$\Phi \to U(\alpha)\Phi U^+(\beta) \tag{2.9a}$$
$$\Phi^+ \to U(\beta)\Phi^+ U^+(\alpha) \tag{2.9b}$$

It is evident that $\det_f \Phi$ and $\det_f \Phi^+$ are invariant due $\det U = 1$, but two last terms in (2.1) breaks the $U_A(1)$ symmetry

$$\mathbf{\Psi} \to e^{-i\theta(x)}\mathbf{\Psi}; \qquad \bar{\mathbf{\Psi}} \to \bar{\mathbf{\Psi}} e^{i\theta(x)} \tag{2.10}$$

The composite operators are defined by

$$\mathbf{S}_a = -\frac{g_s}{m^2}\bar{\mathbf{\Psi}}\lambda^a\mathbf{\Psi}; \qquad \mathbf{P}_a = -\frac{g_s}{m^2}\bar{\mathbf{\Psi}} i\gamma^5\lambda^a\mathbf{\Psi} \tag{2.11}$$

with $G_S = g_s^2/m^2$.

The action corresponding to (2.1) now takes the form

$$I\left[\bar{\mathbf{\Psi}}, \mathbf{\Psi}\right] = \int dx \left\{ \bar{\mathbf{\Psi}}(x)\left(i\gamma^\mu\partial_\mu - m_q\right)\mathbf{\Psi}(x) - \frac{1}{2}m^2 \sum_a \left(\mathbf{S}_a^2 + \mathbf{P}_a^2\right) \right\}$$
$$- g_s\bar{\mathbf{\Psi}}\lambda^a\left(\mathbf{S}_a - i\gamma_5\lambda^a\mathbf{P}_a\right)\mathbf{\Psi} + \mathrm{G_D}\det{}_f\left[\mathbf{\Phi} + h.c\right] \tag{2.12}$$

In the chiral limit ($m_q = 0$), the Lagrangian (2.1) is invariant under the chiral transformation (2.4a-b).

1.2. Finite temperature CJT effective action

In order to consider high-temperature contributions in loop approximation of the Cornwall - Jackiw - Tomboulis (CJT) effective action for composite operator corresponding to (2.12), we start from the CJT finite temperature generating functional for the connected Green's function

$$Z_\beta^{CJT} = \exp iW_\beta[\eta, \bar{\eta}, J_S, J_P, K, K_S, K_P] = \frac{1}{Z[0]}\int [D\bar{\mathbf{\Psi}}][D\mathbf{\Psi}][D\mathbf{S}][D\mathbf{P}]$$
$$\exp i\Big\{\int dx\Big[L + \mu\bar{\mathbf{\Psi}}\gamma_0\mathbf{\Psi} + \bar{\eta}(x)\mathbf{\Psi}(x) + \bar{\mathbf{\Psi}}(x)\eta(x) + J_s^a(x)\mathbf{S}_a(x) + J_p^a(x)\mathbf{P}_a(x)\Big]$$
$$+ \frac{1}{2}\int dxdy\Big[2\bar{\mathbf{\Psi}}(x)K(x,y)\mathbf{\Psi}(y) + \mathbf{S}_a(x)K_s^{ab}(x,y)\mathbf{S}_b(y) + \mathbf{P}_a(x)K_p^{ab}(x,y)\mathbf{P}_b(y)\Big]\Big\} \tag{2.13}$$

where $\int dx = \int_0^\beta d\tau \int d\vec{x}$, the summation here over repeat variable (indices) is assumed and external sources are time - independent; $\int dx\bar{\Psi}\gamma_0\Psi = N$ is the number operator for u, d and s quarks. The integration has to be performed over antiperiodic Grassman fields

$$\mathbf{\Psi}(0, \vec{x}) = -\mathbf{\Psi}(\beta, \vec{x})$$

and periodic bosonic fields

$$\mathbf{B}(0, \vec{x}) = \mathbf{B}(\beta, \vec{x})$$

where $\mathbf{B} = (\Phi, \mathbf{S}_a, \mathbf{P}_a)$.

The propagators of quarks, scalars and pseudoscalar meson are determined from

$$\frac{\delta^2 W_\beta}{\delta\bar{\eta}(x)\delta\eta(y)} = G(x,y); \frac{\delta^2 W_\beta}{\delta J_s^a(x)\delta J_s^a(y)} = D_{ab}(x,y); \frac{\delta^2 W_\beta}{\delta J_p^a(x)\delta J_p^a(y)} = \Delta_{ab}(x,y) \tag{2.14}$$

We defined the mean values of field operators as follows

$$\begin{aligned}
\frac{\delta W_\beta}{\delta\eta(x)} &= \left\langle \bar{\mathbf{\Psi}}(x) \right\rangle = \bar{\varphi}(x) \\
\frac{\delta W_\beta}{\delta\bar{\eta}(x)} &= \left\langle \mathbf{\Psi}(x) \right\rangle = \varphi(x) \\
\frac{\delta W_\beta}{\delta J_s^a(x)} &= \left\langle \mathbf{S}_a(x) \right\rangle = S_a(x) \\
\frac{\delta W_\beta}{\delta J_p^a(x)} &= \left\langle \mathbf{P}_a(x) \right\rangle = P_a(x)
\end{aligned} \tag{2.15}$$

as it's well known

$$\begin{aligned}
\frac{\delta W_\beta}{\delta K} &= 2\left\langle \bar{\mathbf{\Psi}}\mathbf{\Psi} \right\rangle = \phi = \left(\bar{\varphi}\varphi + G\right) \\
\frac{\delta W_\beta}{\delta K_s^{ab}} &= \left\langle \mathbf{S}_a\mathbf{S}_b \right\rangle = \frac{1}{2}\left(S_aS_b + D_{ab}\right) \\
\frac{\delta W_\beta}{\delta K_{ps}^{ab}} &= \left\langle \mathbf{P}_a\mathbf{P}_b \right\rangle = \frac{1}{2}\left(P_aP_b + \Delta_{ab}\right)
\end{aligned} \tag{2.16}$$

The CJT effective action $\Gamma_\beta\left[\phi, S, P, G, D, \Delta\right]$ is defined as the double Legendre transform of W_β

$$\begin{aligned}
\Gamma_\beta\left[\phi, S, P, G, D, \Delta\right] &= W_\beta\left[\eta, \bar{\eta}, J_s, J_p, K, K_s, K_p\right] \\
&- \int dx \left[\bar{\varphi}(x)\eta(x) + \bar{\eta}(x)\varphi(x) + J_s^a(x)S_a(x) + J_p^a(x)P_a(x)\right] \\
&- \frac{1}{2}\int dxdy \left[2\psi(x)K(x,y)\psi(y) + S^a(x)D_{ab}(x,y)S^b(y) + P^a(x)\Delta_{ab}(x,y)P^b(y)\right] \\
&- \frac{1}{2}\int dxdy \left[2G(x,y)K(y,x) + D_{ab}(x,y)K_s^{ab}(y,x) + \Delta_{ab}(x,y)K_p^{ab}(y,x)\right]
\end{aligned} \tag{2.17}$$

It is evident that

$$\begin{aligned}
\frac{\delta\Gamma_\beta}{\delta\varphi(x)} &= -\bar{\eta}(x) - \int dx K(x,y)\bar{\varphi}(y) \\
\frac{\delta\Gamma_\beta}{\delta\bar{\varphi}(x)} &= -\eta(x) - \int dx \varphi(x)K(y,x) \\
\frac{\delta\Gamma_\beta}{\delta J_a(x)} &= -J_s^a(x) - \int dx K_s^{ab}(x,y)\sigma_b(y) \\
\frac{\delta\Gamma_\beta}{\delta P_a(x)} &= -J_p^a(x) - \int dx K_p^{ab}(x,y)P_b(y)
\end{aligned} \tag{2.18}$$

and

$$\begin{aligned}\frac{\delta\Gamma_\beta}{\delta G(x,y)} &= -K(y,x)\\ \frac{\delta\Gamma_\beta}{\delta D_{ab}(x,y)} &= -\frac{1}{2}K_s^{ab}(y,x)\\ \frac{\delta\Gamma_\beta}{\delta\Delta_{ab}(x,y)} &= -\frac{1}{2}K_p^{ab}(y,x)\end{aligned} \tag{2.19}$$

To proceed further let us emphasize that when all external sources vanish, ones gets

$$\bar{\varphi} = \varphi = 0$$

and ϕ, σ_a, P_a tend to condensate quarks and meson's vacuum expectation values, respectively

$$\begin{aligned}\left\langle \bar{\Psi}\Psi \right\rangle_0 \rightarrow \phi &= diag\left(\phi_u, \phi_d, \phi_s\right)\\ S_a &= \langle 0|\mathbf{S}_a|0\rangle = \sigma\\ P_a &= \langle 0|\mathbf{P}_a|0\rangle\end{aligned} \tag{2.20}$$

where P_a, it's well known are eight pseudoscalar meson in SU(3) $K^0, K^\pm, \pi^0, \pi^\pm, \delta$ and η.

The stationary condition for physical processes which correspond to vanishing of external sources require

$$\left(\frac{\delta\Gamma}{\delta\phi}\right)_0 = 0; \qquad \left(\frac{\delta\Gamma}{\delta S_a}\right)_0 = 0; \qquad \left(\frac{\delta\Gamma}{\delta P_a}\right)_0 = 0 \tag{2.21}$$

and

$$\frac{\delta\Gamma}{\delta G} = 0; \qquad \frac{\delta\Gamma}{\delta D} = 0; \qquad \frac{\delta\Gamma}{\delta\Delta_{ab}} = 0 \tag{2.22}$$

The system of equations (2.22) is just Schwinger - Dyson equations for the propagators of quarks, scalar and pseudoscalar mesons, respectively.

The expression for Γ can be derived directly basing on [7]

$$\begin{aligned}\Gamma_\beta = I\left[\phi,\sigma\right] &+ iTr\left[\ln G_0G^{-1} - G_0^{-1}(\phi,k)G + 1\right]\\ &- \frac{i}{2}Tr\left[\ln D_{0,ab}D_{ab}^{-1} - \mathfrak{D}_{0,ab}^{-1}(\sigma,k)D_{ab} + 1\right]\\ &- \frac{i}{2}Tr\left[\ln \Delta_{0,ab}\Delta_{ab}^{-1} - \Delta_{0,ab}^{-1}(k)\Delta_{ab} + 1\right] + \Gamma_\beta^2\end{aligned} \tag{2.23}$$

where the trace, the logarithm and product $G_0G^{-1}, D_0D^{-1}, \Delta_0\Delta^{-1}$... are taken in the functional sense. $G_0, D_0, \Delta_{0,ab}$ are, respectively, the propagators of quarks, scalar and pseudoscalar mesons, their momentum representation reads

$$\begin{aligned}iG_0^{-1}(k) &= \hat{k} - m_q + \mu\gamma_0\\ iD_{0,ab}^{-1}(k) &= -m^2\delta_{ab}\\ i\Delta_{0,ab}^{-1}(k) &= -m^2\delta_{ab}\end{aligned} \tag{2.24}$$

and the momentum representation of $G_0^{-1}(\phi)$, $\mathfrak{D}_0^{-1}(\phi)$ and $\Delta_{0,ab}^{-1}(\phi)$ are determined from

$$\begin{aligned} iG_0^{-1}(\phi) &= \frac{\delta^2 I_{int}}{\delta\phi(x)\delta\phi(y)} = \hat{k} - M + \mu\gamma_0 \\ i\mathfrak{D}_{0,ab}^{-1}(k) &= \frac{\delta^2 I_{int}}{\delta S_a(x)\delta S_b(y)} = -M_S^2\delta_{ab} \\ i\Delta_{0,ab}^{-1}(k) &= \frac{\delta^2 I_{int}}{\delta P_a(x)\delta P_b(y)} = -M_P^2\delta_{ab} \end{aligned} \tag{2.25}$$

Γ_β^2 is given by all those two particle irreducible vacuum graphs which upon cutting of one - line, yield proper self - energy graphs

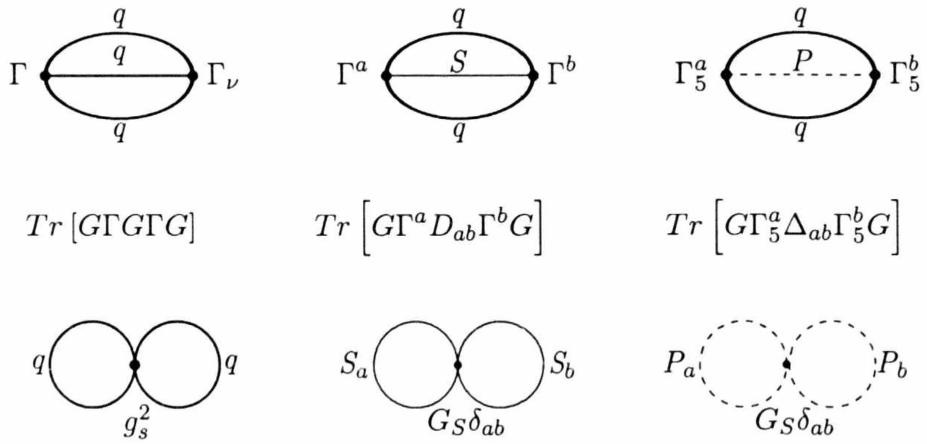

Fig 1: *The 2PI graphs of Γ_β^2 in NJL model*

The bold solid line represents the quarks propagator G, the solid line - scalar meson's propagator D_{ab}, the dashed line - pseudoscalar meson's propagator Δ_{ab} The bold dots $\Gamma, \Gamma^a, \Gamma_5^a$ are the interaction vertices. In the bare vertex approximation $\Gamma = ig_s, \Gamma^a = ig_s\lambda^a, \Gamma_5^b = ig_s\gamma_5\lambda^b$

3. THE LOOP EXPANSION AND THE GAP EQUATIONS

For a translation invariant and constant ground state ϕ, σ, instead of $\Gamma\left[\phi, \sigma, G, D, \Delta\right]$ we consider the finite temperature effective potential

$$V_\beta = -\frac{\Gamma_\beta}{\beta\int d\vec{x}} \tag{3.1}$$

V_β is just the free energy density of quantum by all the thermodynamical parameters of the system can be derived from V_β.

Starting from (2.23) and Fig.1, it isn't difficult to write down the CJT effective potential in momentum space

$$
\begin{aligned}
V^{CJT} =& G_S\left(\phi_u^2+\phi_d^2+\phi_s^2\right)+4G_D\phi_u\phi_d\phi_s \\
&-i\int\frac{d^4p}{(2\pi)^4}Tr\Big[\ln G_0^{-1}(p)G(p)-G^{-1}[\phi]G(p)+1\Big] \\
&+\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}Tr\Big[\ln D_{0,ab}^{-1}(p)D_{ab}(p)-\mathfrak{D}_{0,ab}^{-1}(\sigma,p)D_{ab}(p)+1\Big] \\
&+\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}Tr\Big[\ln \Delta_{0,ab}^{-1}(p)\Delta_{ab}(p)-\Delta_{ab}^{-1}(p)\Delta_{ab}(p)+1\Big] \\
&+\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}Tr\Big[G(p)\Gamma G(p+k)\Gamma G(k)\Big] \\
&+\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}Tr\Big[G(p)\Gamma^a D(p+k)\Gamma^b G(k)\Big] \\
&+\frac{1}{2}\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}Tr\Big[G(p)\Gamma_5^a\Delta_{ab}(p+k)\Gamma_5^b G(k)\Big] \\
&+\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}Tr\Big[G(p)\Gamma(p,k)G(k)\Big] \\
&+iG_S\int\frac{d^4p}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}Tr\Big[D_{ab}(p)D_{ba}(k)+\Delta_{ab}(p)\Delta_{ba}(k)\Big]
\end{aligned} \tag{3.2}
$$

Two terms on first line of (3.2) correspond to the mean field approximation, three next terms are just one - loop approximation, and the last terms in Γ_β^2 expresses the non - perturbative interaction at two - loop and higher approximation.

The configuration of meson fields is determined from

$$
\frac{dV^{CJT}}{dS_a}=0\rightarrow S_a=\frac{g_s}{m^2}\int\frac{d^4p}{(2\pi)^4}\lambda^a Tr\Big[G(p)\Big]=\sigma=g_s\rho_s \tag{3.3}
$$

It's just the scalar density, which is invariant under Lorentz transformation

$$
\frac{dV^{CJT}}{dP_a}=0\rightarrow P_a=i\frac{g_s}{m^2}\int\frac{d^4p}{(2\pi)^4}\lambda^a Tr\Big[\gamma_5 G(p)\Big]=0 \tag{3.4}
$$

Substituting (3.2) into (2.22), we arrived at the SD equations

$$
G^{-1}(k)=G_0^{-1}\Big[\phi,k\Big]-\Sigma(k) \tag{3.5}
$$

$$
D_{ab}^{-1}(k)=D_{0,ab}^{-1}\Big[1-G_S\Pi^S(k)\Big] \tag{3.6}
$$

$$
\Delta_{ab}^{-1}(k)=\Delta_{0,ab}^{-1}\Big[1-G_S^p\Pi^p(k)\Big] \tag{3.7}
$$

where $\Sigma(k), \Pi^S(k)$ and $\Pi^p_{ab}(k)$ are, respectively, the self - energy of quarks, scalar and pseudoscalar mesons

$$\begin{aligned}\Sigma(k) = &-\int \frac{d^4p}{(2\pi)^4} Tr\Big[\Gamma(k,p)G(p)\Gamma(p,p+k)G(p+k)\Big]\\&+\frac{1}{2}\int \frac{d^4p}{(2\pi)^4} Tr\Big[\Gamma^a(k,p)G(p)\Gamma^b(p,p+k)D_{ab}(p+k)\Big]\\&+\frac{1}{2}\int \frac{d^4p}{(2\pi)^4} Tr\Big[\Gamma^a_5(k,p)G(p)\Gamma^b_5(p,p+k)\Delta_{ab}(p+k)\Big]\\&+\int \frac{d^4p}{(2\pi)^4} Tr\Big[G(p)\Gamma(k,p)\Big]\end{aligned} \tag{3.8}$$

$$\Pi^S(k) = \int \frac{d^4p}{(2\pi)^4} Tr\Big[\Gamma^a(k,p)G(p)\Gamma^b(p,p+k)G(p+k)\Big] \tag{3.9}$$

$$\Pi^p_{ab}(k) = \int \frac{d^4p}{(2\pi)^4} Tr\Big[\Gamma^a_5(k,p)G(p)\Gamma^b_5(p,p+k)G(p+k)\Big] \tag{3.10}$$

The stationary requirement (2.21) takes the form

$$\begin{aligned}\frac{\partial V_\beta}{\partial \phi_i} &= 2G_S\phi_i + 4G_D\phi_j\phi_k + iG_0^{-1} - iG_0^{-1}[\phi]\\&= 2G_S\phi_i + 4G_D\phi_j\phi_k - m_i + M_i + \Sigma = 0\end{aligned} \tag{3.11}$$

Here the indices $i, j, k = (u, d, s)$. Similarly, one gets

$$\begin{aligned}\frac{\partial V_\beta}{\partial S_a^2} &= iD^{-1}_{ab} - i\mathfrak{D}^{-1}_{0,ab}\\&= -m^2\left[1 - G_S\Pi^S(k)\right] + M_S^2 = 0\end{aligned} \tag{3.12}$$

$$\begin{aligned}\frac{\partial V_\beta}{\partial P_a^2} &= i\Delta^{-1}_{ab} - i\Delta^{-1}_{0,ab}\\&= -m^2\left[1 - G_S^p\Pi^p(k)\right] + M_p^2 = 0\end{aligned} \tag{3.13}$$

or, equivalently, it is usually written in the form of the gap equations

$$M_i = m_i - 2G_S\phi_i - 4G_D\phi_j\phi_k - \Sigma(k) \tag{3.14}$$

That means the constituent quarks mass M_i is expressed thought both coupling constant G_S, G_D and the flavor mixing of quark condensate $\phi_j\phi_k$ exists in M_i. In the random phase approximation (RPA) the self - energy of quarks is ignored, i.e $\sum(k) = 0$.

Similarly, from (3.12) and (3.13) one gets

$$M_S^2 = m^2\left[1 - G_S\Pi^S(k)\right] \tag{3.15}$$

$$M_p^2 = m^2\left[1 - G_S^p\Pi^p(k)\right] \tag{3.16}$$

From the system of gap equations (3.14) - (3.16), the quark condensate and mesons are systematically considered: It is also shown that the influence of condensate matter on quark masses are really strong.

4. THE CJT EFFECTIVE ACTION AT FINITE TEMPERATURE

As it is well known, the (partial) chiral symmetry is restored at finite temperature and nonzero density. It concerns with the energy density of ground state.

To investigate this system at $T \neq 0$, we can apply the "imagine time" formalism or the "real time" formalism in field theory at finite temperature [15],[16].

In "imagine time" formalism, the Feynman rules as the same as those at zero temperature, except that the momentum space integral over the time component k_0 is replace by sum over Matsubasa frequencies $\omega_n = \pi n T$, and the chemical potential μ should be added to the fermionic frequency in all expression, i.e

$$k_\mu \to (i\omega_n + \mu, \mathbf{k}) \tag{4.1}$$

$$\int \frac{d^4k}{(2\pi)^4} \to iT \sum_n \frac{d^3k}{(2\pi)^3} = \not{\sum_k} \tag{4.2}$$

where n is even (odd) for boson (fermion).

Starting from (3.2) and (3.5) - (3.7) we arrive at the expression of thermal CJT effective potential in Hartree - Fock approximation.

$$\begin{aligned}
V_T[\phi, M, M_s, M_p] =& G_S\left(\phi_u^2+\phi_d^2+\phi_s^2\right) + 4G_D\phi_u\phi_d\phi_s \\
&+ \not{\sum_k} Trln\frac{k - M + \gamma_0\mu}{k - m + \gamma_0\mu_0} + Tr\left[G_0^{-1}G - 1\right] \\
&+ \not{\sum_k} TrlnTr\left[1 - G_S\Pi^S(k)\right] + Tr\left[D_{0,ab}^{-1}D_{ab} - 1\right] \\
&+ \not{\sum_k} TrlnTr\left[1 - G_S\Pi^P(k)\right] + Tr\left[\Delta_{0,ab}^{-1}\Delta_{ab} - 1\right] \\
&- \frac{1}{2}\not{\sum_p}\not{\sum_k} Tr\left[G_(p)\lambda^a D(p+k)\lambda^b G(k)\right] \\
&- \frac{1}{2}\not{\sum_p}\not{\sum_k} Tr\left[G_(p)\gamma_5\lambda^a\Delta_{ab}(p+k)\gamma_5\lambda^b G(k)\right] \\
&- g^2\not{\sum_p}\not{\sum_k} Tr\left[G_(p)G(k)\right] \\
&+ G_S\not{\sum_p}\not{\sum_k}\left[D_{ab}(p)D_{ba}(k) + 3\Delta_{ab}(p)\Delta_{ba}(k)\right]
\end{aligned} \tag{4.3}$$

where

$$\phi_i \equiv \langle\bar{\Psi}_i\Psi_i\rangle = -iN_cTr\not{\sum_k}\frac{1}{\gamma k - M_i} \tag{4.4}$$

and the propagators of boson and fermions are given by

$$D_T(k) = \frac{i}{k^2 - M^2} = \frac{-i}{-k_0^2 + \mathbf{k}^2 + M^2} \tag{4.5}$$

$$G_T(k) = \frac{i}{\not{k} - M} = \frac{-i\left(\not{k} + M\right)}{-k_0^2 + \mathbf{k}^2 + M^2} \tag{4.6}$$

Using the well - known results [17]

$$\begin{aligned}\frac{1}{2}Trln\left[k^2-y^2\right] &\equiv \frac{1}{2}\sum\!\!\!\!\!\!\int_k ln\left[-(\pi nT)^2-\mathbf{k}^2-y^2\right]\\ &\approx -\frac{\pi^2T^4}{90}+\frac{y^2T^2}{24}-\frac{y^3T}{12\pi}+\frac{C_\Omega y^4}{32\pi^2}\end{aligned} \tag{4.7}$$

$$\begin{aligned}\Omega(y) &\equiv \sum\!\!\!\!\!\!\int_k \frac{1}{(\pi nT)^2+\mathbf{k}^2+y^2}\\ &= \frac{1}{y}\frac{\partial}{\partial y}\left[\frac{1}{2}Trln\left(k^2-y^2\right)\right] \approx \frac{T^2}{12}-\frac{Ty}{4\pi}+\frac{C_\Omega}{8\pi^2}y^2\end{aligned} \tag{4.8}$$

$$C_\Omega \equiv \frac{1}{2}ln\frac{T^2}{\eta^2}+\frac{1}{2}-ln(4\pi)-\gamma_{Euler} \tag{4.9}$$

where η is renormalization scale, $\gamma_{Euler}=0,577$ and as planned the (zero - temperature) ultraviolet divergent contributions have been omitted, it is easily to evaluate the high - temperature approximation of integrands in (4.3)

$$Tr\left[G_0^{-1}G-1\right]=Tr\left[\left(G_0^{-1}-G^{-1}\right)G\right]=\left(m_i^2-M_i^2\right)\Omega(M_i) \tag{4.10}$$

$$TrlnG_0^{-1}G=Trln\frac{\gamma k-M_i+\gamma_0\mu}{\gamma k-m_i+\gamma_0\mu_0} \tag{4.11}$$

At chiral limit $m_i=0$ and $\mu_0=0$, it is being

$$TrlnG_0^{-1}G=Trln\frac{k^2-\mathfrak{M}_i^2}{k^2}=\frac{T^2}{12}\mathfrak{M}_i^2-\frac{T}{6\pi}\mathfrak{M}_i^3-\frac{C_\Omega}{16\pi^2}\mathfrak{M}_i^4 \tag{4.12}$$

where $\mathfrak{M}_i^2=(M_i-\gamma_0\mu)^2$.

Finally, the part dependent temperature of CJT effective potential is obtained in one loop approximation as a function of quark and meson masses

$$\begin{aligned}V_T\left[\phi,\mu,M_i,M_s,M_p\right]=&G_S\left(\phi_u^2+\phi_d^2+\phi_s^2\right)+4G_D\phi_u\phi_d\phi_s\\ &-\frac{T^2}{8}\mathfrak{M}_i^2+\frac{T^2}{12}\left(M_p^2+M_s^2-2m^2\right)\\ &+\frac{T}{24\pi}\left[\mathfrak{M}_i^3-M_p^2-M_s^3-m^2\left(M_p+M_s-2m\right)\right]\\ &+\frac{3C_\Omega}{32\pi^2}\left[\mathfrak{M}_i^4-M_p^4-M_s^4-2m^2\left(M_p^2+M_s^2-2m^2\right)\right]\end{aligned} \tag{4.13}$$

where M_i, M_p, M_s are the solution of the gap equations (3.14) - (3.16). Note that in (4.12) all term linear in the effective masses have canceled out. The term involve square of quark masses in proportional to $T^2/8$. It is just the part dependent temperature of effective quark masses in QCD at hard thermal one loop [18], [19]. Two last terms are higher contributions of the CJT effective potential evaluated at the values of quark and meson masses.

5. CONCLUSION AND DISCUSSION

In the preceding sections the CJT effective action was used to study systematically SU(3) NJL model, where the condensate matter involve u, d and s quarks automatically concluded in this formalism. The gap equations are directly derived from the effective potential, which is also evaluated at finite temperature. Note that all term linear in the effective masses have canceled out. Due to the fact that our next paper is intended to consider the quark condensate and mesons in this mechanism. For numerical computation purpose the Hartree - Fock HF) approximation will be presented.

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