VNU JOURNAL OF SCIENCE, Mathematics - Physics, T.XXII, $N_0$4 - 2006

DETERMINING THE PLANE STRAIN BULK MODULUS OF THE COMPOSITE MATERIAL REINFORCED BY ALIGNED FIBRE

Nguyen Dinh Duc(1), **Nguyen Tien Dac**(2)

(1) *Vietnam National University, Hanoi*

(2) *Department of Mathematics-Mechanics and Informatics, College of Science- VNU*

ABSTRACT. The composite material reinforced by aligned fibre is an important type of material because of its practical applications in many components of structures. This material is a tranversely isotropic medium having five independent elastic constants. Many researchers are involved in determining these elastic constants. By using complex function method for solving the plane strain problem in elasticity theory, [4] has given a relatively precise method for determining the stiffnesses of transversely isotropic medium. [5] has introduced a method for defining the engineering constants of the composite material reinforced by fibre basing on physical experiments. In this paper, we would give a derivation method to determine the plane strain bulk modulus of the composite material reinforced by aligned fibre basing on a geometric approximation of representative volume element. Our result is identical to some other authors'. This is a new method to obtain the plane strain bulk modulus of the composite material reinforced by aligned fibre.

1. The problem

Assuming a cyclic structure of the composite material reinforced by aligned fibre, its representative volume element is in form of a rectangular parallelepiped containing a cylinder. By using a geometric approximation of outer rectangular parallelepiped, we obtain a new model for representative volume element called composite cylinders model. Specifically, The fibre phase is taken to compose of a number of long circular cylinders embedded by a continuous matrix phase. With each individual fibre of radius a, there is an association with an annulus of matrix material of radius b. Each individual cylinder combination of this type is referred to a composite cylinder [6]. As a result, our problem can be posed as below.

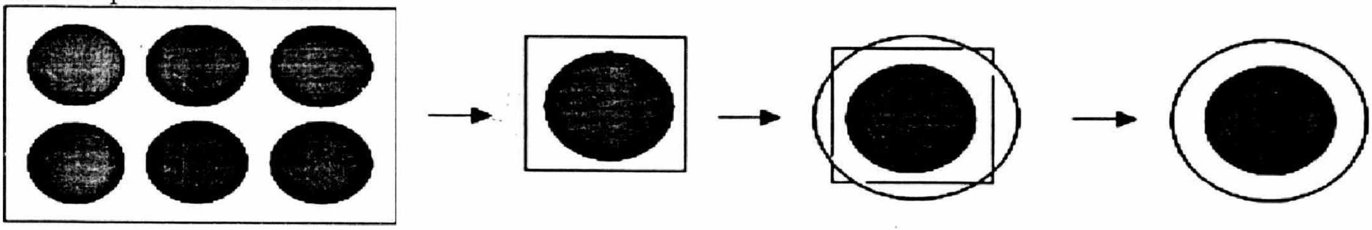

Figure 1. The representative volume element of the composite material reinforced by fibre and composite cylinder model

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

Let us consider a heterogeneous cylinder composed of two isotropic elastic materials. Inner solid cylinder ($0 \preceq r \preceq a$) and outer cylindrical shell ($a \preceq r \preceq b$) made of elastic materials having properties (λ_1, μ_1) and (λ_2, μ_2), respectively. A heterogeneous cylinder is subjected to a hydrostatic stress p_2 on its outer boundary $r = b$. Our objective is to obtain the plane strain bulk modulus of two phase composite as a function of elastic properties and volume concentrations of each phase.

2. Governing relationships

By setting the above problem, the root of the problem is found in form of

$$u_r = u_r(r) \ , \ u_\theta = u_z = 0. \tag{1}$$

We have the equilibrium as follow

$$\frac{d\sigma_{rr}}{dr} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0. \tag{2}$$

Taking Eq.(1) into Eq.(2) we have

$$\frac{d^2 u_r}{dr^2} + \frac{1}{r}\frac{du_r}{dr} - \frac{1}{r^2}u_r = 0. \tag{3}$$

The solution of Eq.(3) is given by

$$u_r = Ar + \frac{B}{r}. \tag{4}$$

The solution Eq.(4) implies specific forms in the separate fibre and matrix phases.

3. Solving method

The problem is solved by applying elasticity theory for the plane strain case. Specifically, the problem about cylinderical shell is subjected to the hydrostatic stresses on boundaries. We will consider the displacement and stress states in the separate fibre and matrix phases of composite cylinder before examining equivalent homogeneous cylinder.

3.1. The matrix phase of composite cylinder

In the area of the matrix phase, the displacement and stress states in forms

$$u_r = A_2 r + \frac{B_2}{r}, \tag{5}$$

$$\sigma_{rr}^{(2)} = 2(\lambda_2 + \mu_2)A_2 - 2\mu_2\frac{B_2}{r^2}. \tag{6}$$

Introducting $\sigma_{rr}^{(2)}$ into the boundary and interface conditions

$$\sigma_{rr}^{(2)}\Big|_{r=b} = p_2 \ , \ \sigma_{rr}^{(2)}\Big|_{r=a} = p \tag{7}$$

(where p is the interaction stress between the fiber and matrix phases), we define integration constants as well as the displacement and stress fields in the matrix phase as follows.

$$u_r^{(2)} = \frac{pa^2 - p_2 b^2}{2(\lambda_2+\mu_2)(a^2-b^2)} r + \frac{(p-p_2)a^2 b^2}{2\mu_2(a^2-b^2)}\frac{1}{r} \ , \tag{8}$$

$$\sigma_{rr}^{(2)} = \frac{pa^2 - p_2 b^2}{a^2-b^2} - \frac{(p-p_2)a^2 b^2}{a^2-b^2}\frac{1}{r^2}. \tag{9}$$

3.2. The fibre phase of the composite cylinder

In this part ($0 \preceq r \preceq a$), the displacement and stress fields have forms

$$u_r^{(1)} = A_1 r \ , \tag{10}$$

$$\sigma_{rr}^{(1)} = 2(\lambda_1+\mu_1)A_1. \tag{11}$$

By the continuity conditions of displacement and stress on interface $r = a$

$$\sigma_{rr}^{(1)}\Big|_{r=a} = \sigma_{rr}^{(2)}\Big|_{r=a} \ ; \ u_r^{(1)}\Big|_{r=a} = u_r^{(2)}\Big|_{r=a} \tag{12}$$

we obtain the following relations of $u_r^{(1)}$ and interaction stress p.

$$u_r^{(1)} = \frac{pr}{2(\lambda_1+\mu_1)} \ , \tag{13}$$

$$p = \frac{(\lambda_1+\mu_1)(\lambda_2+2\mu_2)p_2 b^2}{(\lambda_2+\mu_2)\big[(\lambda_1+\mu_1)+\mu_2\big]b^2 + \mu_2\big[(\lambda_1+\mu_1)-(\lambda_2+\mu_2)\big]a^2} \tag{14}$$

3.3. The equivalent homogeneous cylinder

We consider equivalent homogeneous solid cylinder having radius $r = b$ made of isotropic elastic material of properties (λ, μ). The homogeneous cylinder is subjected to the hydrostatic stress p_2 on the boundary $r = b$.

It is clear that solved forms are similar to those of the fibre phase

$$u_r = Ar, \tag{15}$$

$$\sigma_{rr} = 2(\lambda+\mu)A. \tag{16}$$

By defining integration constant from boundary condition, we obtain as follow

$$u_r = \frac{p_2}{2(\lambda+\mu)} r. \tag{17}$$

In fact, the displacements at the outer boundaries of the composite cylinder and the equivalent homogeneous cylinder are equated to provide the same average state of dilatation within each of them

$$u_r^{(2)}\Big|_{r=b} = u_r\Big|_{r=b}. \tag{18}$$

Introducting the relationships Eq.(8), Eq.(17) into Eq.(18) taking into account Eq.(14) gives us the following relationship.

$$(\lambda+\mu) = (\lambda_2+\mu_2) + \frac{(\lambda_2+2\mu_2)\big[(\lambda_1+\mu_1)-(\lambda_2+\mu_2)\big]\xi}{(\lambda_1+\mu_1+\mu_2)-\xi\big[(\lambda_1+\mu_1)-(\lambda_2+\mu_2)\big]}.$$

Where $\xi = a^2/b^2$ is volume fraction of the fibre phase.

In fact, $K_{23} = K+\mu/3 = \lambda+\mu$ is plane-strain bulk modulus of transversely isotropic medium [7]. We carry out some transformations at Eq.(19) and obtain as follows.

$$K_{23} = K_2 + \mu_2/3 + \frac{(K_2+4\mu_2/3)\big[K_1-K_2+(\mu_1-\mu_2)/3\big]\xi}{(K_2+4\mu_2/3)+(1-\xi)\big[K_1-K_2+(\mu_1-\mu_2)/3\big]}, \tag{19}$$

where K_i , μ_i $(i = 1,2)$ are the bulk and shear moduli of the fibre and matrix isotropic phases, respectively.

Expression Eq.(20) is a formula for determining the plane strain bulk modulus of the composite material reinforced by aligned fibre which is a transversely isotropic medium. It coincides with the relationship derived by Christensen [1].

4. Conclusions

By using a geometric approximation of the representative volume element of the composite material reinforced by aligned fibre, this paper presents an alternative derivation method in order to obtain the plane strain bulk modulus of this material. The governing ideas of this approach base on the composite cylinder model and applying elasticity theory for the plane strain problem. Our result is identical to Christensen's [1]. We believe that this derivation method can be widenned to determine the remaining elastic moduli of the composite material reinforced by aligned fibre, which is a transversely isotropic medium.

The results of researching presented in this paper have been supported by the National Council for National Sciences.

References

1. R. M. Christensen, *Mechanics of Composite Materials*, John Wiley and Sons Inc, New York, 1979.
2. G. A. Vanin, *Micro - mechanics of composite Materials*, " Naukova dumka", Kiev, 1985.
3. Dao Huy Bich, *The theory of elasticity*, The Vietnam National University Publisher, Hanoi, 2001.
4. Nguyen Hoa Thinh, Nguyen Dinh Duc, *The Composite Material - Mechanics and technology*, The Science and Technique Publisher, Hanoi, 2002.
5. Tran Ich Thinh, *Composite material-Mechanics and structural calculations*, The Education Publisher, Hanoi, 1994.
6. Nguyen Dinh Duc, Hoang Van Tung, Do Thanh Hang, A method for determining the module K of composite material with sphere pad seeds, *Journal of Science, Mathemathics-Physics, VNU*, T.XXII, No2, 2006.
7. D. K. Hale, Review the physical properties of composite materials, *Journal of materials science* 11 (1976) pp 2105-2141.