QUANTUM THEORY OF THE ABSORPTION OF A WEAK ELETROMAGNETIC WAVE BY THE FREE CARRIERS IN TWO DIMENSIONAL ELECTRON SYSTEM

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ABSTRACT. Analytic expressions for the obsorption coefficient of a weak Electromagnetic Wave (EMW) by free carriers for the case electron-optical phonon scattering in 2 dimensional system (quantum wells and doped superlattices) are calculated by the Kubo-Mori method in two cases: the absence of a magnetic field and the presence of a magnetic field applied perpendicular to its barriers. A different dependence of the absorption coefficient on the temperature T of system, the electromagnetic wave frequency ω, the cyclotron frequency Ω (when a magnetic field is present), and characteristic parameters of a 2 dimensional system in comparision with normal semiconductors are obtained. The analytic expressions are numerically evaluated, plotted and discussed for a specific 2 dimensional system (AlAs/GaAs/AlAs quantum well and n-GaAs/p-GaAs superlattice)

1. Introduction

Recently, there has been considerable interest in the behaviour of low dimensional system, in particular, of 2 dimensional systems, such as doped superlattices and quantum wells. The confinment of electrons in these systems considerably enhances the electron mobility and leads to their unusual behaviours under external stimuli. Many papers have appeared dealing with these behaviours: electron-phonon interaction and scattering rates [1-3], dc electrical conductivity [4-5]...The problems of absorption coefficient of a EMW in semiconductor superlattices have been investigated in considerable details [6-7]. In this paper, we study the absorption coefficient of a weak EMW by free carriers confined in a 2 dimensional system (quantum wells and doped superlattices) in the case of the absence of a magnetic field and the presence of a magnetic field applied perpendicular to its barriers. The electron-optical phonon scattering mechanism is assumed to be dominant. We shall asume that the weak EMW is plane-polarized and has high frequency in the range $\omega\tau \gg 1$ (τ is the characteristic momentum relaxation time and ω is the frequency of the weak EMW $\vec{E} = \vec{E}_o cos(\omega t)$).

It starts from Kubo's formula for the conductivity tensor [8]:

$$\sigma_{\mu\nu}(\omega) = \lim_{\delta\to+0}\int_0^\infty dte^{i\omega t-\delta t}(J_\mu, J_\nu(t)), \tag{1}$$

where, J_μ is the μ-component of current density operator (μ = x, y, z) and $J_\mu(t)$ is operator J_μ in Heisenberg picture, the quantity δ is infinitesimal and appears by the assumption of adiabatic interaction of external electromagnetic wave. The time correlation function used in (1) is defined by the formula:

$$(A,B) = \int_0^\beta \left\langle e^{\lambda H} A e^{-\lambda H} B \right\rangle d\lambda, \tag{2}$$

where, $\beta = 1/k_B T$(k_B-the Boltzmann constanst, T-the temperature of system), the symbol $\langle ... \rangle$ means the averaging of operators with Hamiltonian H of the system.

In ref. 9 Mori pointed out that the Laplaces's transformation of the time correlation function (2) can be represented in the form of an infinite continued fraction. One of advantages of this representation is that the function will converge faster than that represented in a power series.

Using Mori's method, in the second order approximation of interaction, we obtain the following formula for the components of the conductivity tensor [7,10,11]:

$$\sigma_{\mu\nu}(\omega) = \lim_{\delta\to+0} (J_\mu, J_\nu)\Big[\delta - i(\omega+\eta) + (\frac{i}{\hbar})^2 (J_\mu, J_\nu)^{-1} \int_0^\infty dt e^{i\omega t - \delta t}([U, J_\mu], [U, J_\nu]_{int})\Big]^{-1} \tag{3}$$

with

$$\hbar\eta = \langle [J_\mu, J_\nu] \rangle (J_\mu, J_\nu)^{-1}, \tag{4}$$

here G_{int} is operator G in interaction picture, [A,B]=AB-BA, u is the energy of electron-photon interaction. The averaging of operators in eqs. (3) and (4) is implemented with non-interaction Hamiltonian H_0 of the electron-photon systems.

The structure of quantum wells and doped superlattices also modifies the dispersion relation of optical phonons, which leads to interface modes and confined modes [1]. However, the calculation on electron scattering rates [2] showed that for large width of the well, the contribution from these two modes can be well approximated by calculations with bulk phonons. So in this paper, we will deal with bulk (3 dimensional) phonons with the assumption that the well width is larger than 100 A and consider compensated n-p DSL with equal thicknesses $d_n = d_p = d/2$ of the n-doping and p-doping layer and equal constant doping concentrations $n_D = n_A$ in the respective layers.

2. The absorption coefficient of a weak electromagnetic wave by free carriers in quantum wells

2.1. In the case of the absence of a magnetic field

It is well known that the motion of an electron in a quantum well is confined and its energy spectrum is quantized into discrete levels. We assume that the quantization direction is the z direction. The Hamiltonian of the electron-optical phonon system in a quantum well in second quantization representation can be written as

$$H = H_0 + U, \tag{5}$$

$$H_0 = \sum_{k_\perp,n} \varepsilon_{k_\perp,n} a^+_{k_\perp,n} a_{k_\perp,n} + \sum_q \hbar\omega_0 b^+_q b_q, \tag{6}$$

$$U = \sum_{n,n',k_\perp,q} C_q I_{n'n}(q_z) a^+_{k_\perp+q_\perp,n'} a_{k_\perp,n}(b_q + b_{-q}+), \tag{7}$$

where n denotes quantization of the energy spectrum in the z direction (n=1,2,...), ($k_\perp$,n) and ($k_\perp+q_\perp$,n') are electron states before and after scattering, $k_\perp(q_\perp)$-the in plane (x,y) wave vector of electron (phonon), $a^+_{k_\perp,n}$ and $a_{k_\perp,n}(b^+_q andb_q)$ the creation and annihilation operators of electron (phonon) respectively, $q = (q_\perp, q_z)$, $\hbar\omega_0$ is the energy of optical phonon ; C_q is a constant, in the case of electron-optical phonon interaction it is:[3,5]

$$|C_q|^2 = \frac{1}{LS}\frac{2\pi e^2\hbar\omega_0}{(q_\perp^2 + q_z^2)}\frac{1}{2k_0}\left(\frac{1}{\kappa_\infty} - \frac{1}{\kappa_0}\right) \tag{8}$$

here L is the thickness of the well, $L.S$ is the normalization volume; κ_0 and κ_∞ are the static and the high-frequency dielectric constant, respectively; k_0 is the electrical constant;

$$I_{n',n}(q_z) = \frac{2}{L}\int_0^L dz sin(k_z^{n'}z) sin(k_z^n z) e^{iq_z z} \tag{9}$$

The electron energy takes the simple form:

$$\varepsilon_{k_\perp,n} = \frac{\hbar^2}{2m^*}(k_\perp^2 + k_z^{n2}) \tag{10}$$

here ε and m^* are the effective charge and mass of electron, respectively; k_z^n takes discrete values: $k_z^n = n\pi/L$.

Using the Kubo-Mori method, we obtain the following formula for the transverse component of the high-frequency conductivity tensor $\sigma_{xx}(\omega)$:

$$\sigma_{xx}(\omega) = C_\perp[-i\omega + F(\omega)]^{-1} \tag{11}$$

with $C_\perp \equiv (J_x, J_x)$, and

$$F(\omega) = \lim_{\delta\to+0}\left(\frac{i}{\hbar}\right)^2 C_\perp^{-1}\int_0^\infty dt e^{i\omega t-\delta t}([U, J_x], [U, J_x]_{int}). \tag{12}$$

Knowing the hig-frequency conductivity tensor, the absorption coefficient can be found by the common relation

$$\alpha_{xx}(\omega) = (4\pi/cN^*)Re\sigma_{xx}(\omega) \tag{13}$$

here N^* is the refraction index, c is the light velocity.

Since the weak EMW has a high-frequency, using formulae (3)-(13), we obtain:

$$\alpha_{xx}(\omega) = \frac{4\pi}{cN^*}\frac{C_\perp \Gamma(\omega)}{\omega^2} \tag{14}$$

where

$$C_\perp = e^{\beta\zeta}\Big(\frac{e}{\hbar}\Big)^2 \frac{S}{2\pi\beta}\sum(\beta\hbar\omega_0/L^{*2}) \equiv \frac{e^2 n_e}{m^*} \tag{15}$$

$$\Gamma(\omega) = ReF(\omega) = \Gamma^+(\omega) + \Gamma^-(\omega) \tag{16}$$

$$\Gamma^\pm(\omega) = \Gamma^\pm_{intra} + \Gamma^\pm_{inter} \tag{17}$$

$$\Gamma_{inter} = 3\Gamma_0\Big(N_0 + \frac{1}{2} \pm \frac{1}{2}\Big)\frac{e^{\beta\hbar\omega} - 1}{\omega L^*}\exp\Big[-\frac{\beta\hbar\omega_0}{2}\Big(\Big|\frac{\omega}{\omega_0} \pm 1\Big| + \Big(\frac{\omega}{\omega_0} \pm 1\Big)\Big)\Big] \tag{18}$$

$$\Gamma_{inter} = 3\Gamma_0\Big(N_0 + \frac{1}{2} \pm \frac{1}{2}\Big)\frac{e^{\beta\hbar\omega} - 1}{\omega L^*}\sum{}^{-1}\Big(\frac{\beta\hbar\omega_0}{L^{*2}}\Big)$$
$$\times \sum_{n\neq n'} \exp\Big[-\frac{\beta\hbar\omega_0}{2}\Big(\Big|\frac{n^2 - n'^2}{L^{*2}} + \frac{\omega}{\omega_0} \pm 1\Big| + \Big(\frac{n'^2 + n^2}{L^{*2}} + \frac{\omega}{\omega_0} \pm 1\Big)\Big)\Big] \tag{19}$$

$$\Gamma_0 = \frac{\hbar\omega_0}{4L_{opt}}\Big(\frac{e}{\hbar}\Big)^2 \frac{1}{4k_0}\Big(\frac{1}{\kappa_\infty} - \frac{1}{\kappa_0}\Big) \tag{20}$$

L^*- is the dimensionless well width, $L^* = L/L_{opt}$, with $L^2_{opt} = \hbar^2\pi^2/(2m * \hbar\omega_0)$; N_0 and n_e, respectively, are the phonon and electron concentration; ζ is the chemical potential; $\Gamma^\pm_{intra}$ denotes the contribution from intrasubband transitions ($n = n'$), $\Gamma^\pm_{inter}$ denotes the contribution from intersubband transitions ($n \neq n'$), the symbol $\sum(\alpha)$ denotes the convergent series $\sum(\alpha) = \sum_{n=1}^{+\infty} e^{-\alpha n^2}$; The sign ($\pm$) in the superscript of the operators $\Gamma^\pm, \Gamma^\pm_{intra}$ and $\Gamma^\pm_{inter}$ corresponds to the sign ($\pm$) in eqs. (17) - (19). The upper sign (+) corresponds to phonon absorption and the lower sign (-) to phonon emission in the absorption process.

From eqs. (11) and (14) we can easily see that $F(\omega)$ play the role of the well-known mass operator of electron in Born approximation in the case of the absence of a magnetic field.

2.2 In the case of the presence of a magnetic field

We consider a quantum well with a magnetic field B applied perpendicular to its barriers (z direction). The Hamiltonian of the electron-optical phonon system in second quantization representation can be written:[4,5,12,13,14]

$$H = H_0 + U, \tag{21}$$

$$H_0 = \sum_{N,k_\perp,n} \varepsilon^H_{N,k_\perp,n} a^+_{N,k_\perp,n} a_{N,k_\perp,n} + \sum_q \hbar\omega_0 b^+_q b_q, \tag{22}$$

$$U = \sum_{n,n',N,N',k_\perp,q} C_q I_{n'n}(q_z) J_{N'N}(u) a^+_{N',k_\perp+q_\perp,n'} a_{N,k+\perp,n}(b_q + b^+_{-q}) \tag{23}$$

where N is the Landau level index (N=0,1,2,...), $(N, k_\perp, n)$ and $(N', k_\perp + q_\perp, n')$ are the set of quantum numbers characterizing electron's states befer and after scattering; $a^+_{N,k_\perp,n}$ and $a_{N,k_\perp,n}$ are the creation and annihilation operators of electron, respectively, and $\varepsilon^H_{N,k_\perp,n} = (N+1/2)\hbar\Omega + (\hbar^2\pi^2/2m^*L^2)n^2$ is the energy of electron in quantum wells in the presence of a magnetic field applied in the z direction; Ω is the cyclotron frequency ($\Omega = eB/cm^*$); C_q and $I_{n',n}(q_z)$ are defined by eqs. (8) and (9), respectively, and $J_{N',N}(u)$ takes the form

$$J_{N',N}(u) = \int_{-\infty}^{+\infty} dx\phi_{N'}(r_\perp - a_c^2 k_\perp - a_c^2 q_\perp) e^{iq_\perp r_\perp} \phi_N(r_\perp - a_c^2 k_\perp) \quad (24)$$

where $r_\perp$ is the position of electron and a_c is the radius of the orbit in the (x,y) plane $a_c^2 = c\hbar/eB, u = a_c^2 q_\perp^2/2, \phi_N$ represents harmonic oscillator wave functions.

When a magnetic field is present, for using Kubo-Mori method, [7,10,11] instead of J_x and J_y we use operators J_+ and J_- with $J_\pm = j_x \pm iJ_y$, The transverse components of the conductivity tensor are defined by the formulae

$$\sigma_{xx}(\omega,\Omega) = \sigma_{yy}(\omega,\Omega) = \lim_{\delta\to+0} \frac{1}{4}\Big\{ \int_0^\infty e^{i\omega t - \delta t}(J_-, J_+(t))dt + \int_0^\infty e^{i\omega t-\delta t}(J_+, J_-(t))dt \Big\} \quad (25)$$

Instead of eqs. (3) and (4), in the second order approximation of interaction, we obtain

$$\int_0^\infty e^{i\omega t-\delta t}(J_-, J_+(t))dt$$
$$= (J_-, J_+)\Big[\delta - i(\omega - \Omega) + \Big(\frac{i}{\hbar}\Big)^2 (J_-, J_+)^{-1} \int_0^\infty e^{i\omega t - \delta t}([U, J_-], [U, J_+]_{int})dt\Big]^{-1} \quad (26)$$

$$\int_0^\infty e^{i\omega t-\delta t}(J_+, J_-(t))dt$$
$$= (J_+, J_-)\Big[\delta - i(\omega - \Omega) + \Big(\frac{i}{\hbar}\Big)^2 (J_+, J_-)^{-1} \int_0^\infty e^{i\omega t - \delta t}([U, J_+], [U, J_-]_{int})dt\Big]^{-1} \quad (27)$$

From eqs. (25) - (27) we obtain the following expression for transverse components of the high-frequency conductivity tensor

$$\sigma_{xx}(\omega,\Omega) = \sigma_{yy}(\omega,\Omega)\frac{1}{4}\Big\{\frac{C^H_{-+}}{-i(\omega-\Omega) + F_{-+}(\Omega)} + \frac{C^H_{+-}}{-i(\omega+\Omega) + F_{+-}(\Omega)} \quad (28)$$

with $C^H_{-+} \equiv (J_-, J_+), C^H_{+-} \equiv (J_+, J_-)$, and

$$F_{-+}(\omega) = \lim_{\delta\to+0} \Big(\frac{i}{\hbar}\Big)^2 (C^H_{-+})^{-1} \int_0^\infty e^{i\omega t-\delta t}([U, J_-], [U, J_+]_{int})dt \quad (29)$$

$$F_{+-}(\omega) = \lim_{\delta\to+0} \Big(\frac{i}{\hbar}\Big)^2 (C^H_{+-})^{-1} \int_0^\infty e^{i\omega t-\delta t}([U, J_+], [U, J_-]_{int})dt \quad (30)$$

Knowing the hig-frequency conductivity tensor, the absorption coefficient can be found by eq. (13). The transverse components of the absorption coefficient of a weak EMW in quantum well in the presence of a magnetic field take the form

$$\alpha_{xx}(\omega,\Omega)=\alpha_{yy}(\omega,\Omega)=\frac{\pi C_{\perp}^{H}}{cN^{*}}\Big\{\frac{\Gamma_{-+}(\Omega)}{(\omega-\Omega)^2}+\frac{\Gamma_{+-}(\Omega)}{(\omega+\Omega)^2}\Big\} \tag{31}$$

are

$$C_{-+}^{H}=C_{+-}^{H}\equiv .C_{\perp}^{H}=\frac{M^{*}e^2\Omega^2a_c^2}{2\pi\hbar^2}e^{\beta\varsigma}\sum(\frac{\beta\hbar\omega_0}{L^{*2}})sh^{-1}(\frac{\beta\hbar\Omega}{2}), \tag{32}$$

$$\Gamma_{-+}(\Omega)=ReF_{-+}(\Omega)=\Gamma_{-+}^{+}+\Gamma_{-+}^{-} \tag{33}$$

$$\Gamma_{+-}(\Omega)=ReF_{+-}(\Omega)=\Gamma_{+-}^{+}+\Gamma_{+-}^{-} \tag{34}$$

$$\Gamma_{-+}^{\pm}(\Omega)=\Gamma_0\Big(N_0+\frac{1}{2}\pm\frac{1}{2}\Big)\frac{e^{\beta\hbar\omega}-1}{\hbar\omega L^{*}}\sum^{-1}(\frac{\beta\hbar\omega_0}{L^{*2}})\sum_{n,n'}(2+\delta_{n,n'})exp(\frac{-\beta\hbar\omega_0 n'^2}{L^{*2}})$$
$$\times\sum_{M=-\infty}^{+\infty}exp[-(\frac{\beta\hbar\Omega}{2})(|M|-M)]\frac{cth(\beta\hbar\Omega/2)+|M|}{|M-1|}\delta\Big(\frac{\omega\mp\omega_0}{\Omega}+\frac{\Delta_{nn'}}{\hbar\Omega}+M\Big) \tag{35}$$

$$\Gamma_{+-}^{\pm}(\Omega)=\Gamma_0\Big(N_0+\frac{1}{2}\pm\frac{1}{2}\Big)\frac{e^{\beta\hbar\omega}-1}{\hbar\omega L^{*}}\sum^{-1}(\frac{\beta\hbar\omega_0}{L^{*2}})\sum_{n,n'}(2+\delta_{n,n'})exp(\frac{-\beta\hbar\omega_0 n'^2}{L^{*2}})$$
$$\times\sum_{M=-\infty}^{+\infty}exp[-(\frac{\beta\hbar\Omega}{2})(|M|-M)]\frac{cth(\beta\hbar\Omega/2)+|M|}{|M+1|}\delta\Big(\frac{\omega\mp\omega_0}{\Omega}+\frac{\Delta_{nn'}}{\hbar\Omega}+M\Big) \tag{36}$$

$$\Gamma_0^H=\frac{1}{8\pi\hbar}\frac{e^2\hbar\omega_0}{4k_0L_{opt}}\Big(\frac{1}{\kappa_\infty}-\frac{1}{\kappa_0}\Big) \tag{37}$$

$$\Delta_{nn'}=(n^2-n'^2)\frac{\hbar^2\pi^2}{2m^{*}L^2}=(n^2-n'^2)\frac{\hbar\omega_0}{L^{*2}} \tag{38}$$

where $M=N-N', \delta_{n,n'}$ is the Kronecker delta symbol. The sign ($\pm$) in the superscript of operators $\Gamma_{-+}^{\pm}(\Omega)$ and $\Gamma_{+-}^{\pm}(\Omega)$ corresponds ot the sign ($\pm$) in the quantity $(N_0+\frac{1}{2}\pm\frac{1}{2})$ and to the sign ($\mp$) in the argument of the Dirac delta function. The signs (-+) and (+-) in the subscript of operators $\Gamma_{-+}^{\pm}(\Omega)$ and $\Gamma_{+-}^{\pm}(\Omega)$ correspond to $|M-1|$ in eq. (35) and $|M+1|$ in eq. (36), respectively.

From eqs. (28) and (31) we can see that $F_{-+}(\Omega)$ and $F_{+-}(\Omega)$ play the role of the well-known mass operators of electron in Born approximation in the case of the presence of a magnetic field.

3. Numerical calculation and discussions in the case of quantum wells

In order to clarify the different behaviour of quasi-two-dimensional electron gas confined in a quantum well with respect to bulk electron gas, in this section, we numerically evaluate the analytic formulae (16)-(20) and (33)-(38) for a specific quantum well the AlAs/GaAs/AlAs quantum well. Charateristic parameters of GaAs layer of this quamtum well are $\kappa_\infty = 10.9, \kappa_0 = 12.9, e = 2.07e_0, m* = 0.067m_0, \hbar\omega_0 = 36.1 \times 10^{-3}eV$ (e_0 and m_0 is the charge and the mass of free carrier). The syste is assumed at room temperature ($T = 293^O K$)

3.1. In the case of the absence of a magnetic field

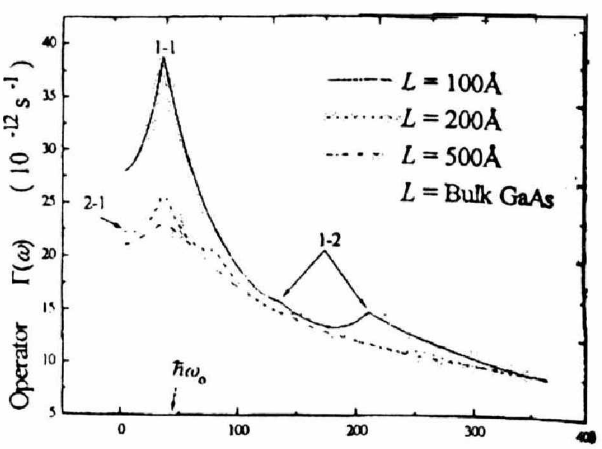

Fig. 1. The dependence of $\Gamma(\omega)$ on ω for difference values of L

Plotted in fig. 1 is the operator $\Gamma(\omega)$ as a function of ω- the frequency of the electromagnetic wave. Different values of the well width L have been used. Corresponding values for bulk GaAs are also plotted for comparison. From this graph, we can see that the confinementof electrons in a quantum well creates new features in the absorption spectra in comparison with that of normal semiconductors.

The well-known peak for optical phonon at $\omega = \omega_0$ is readily obtained, but here, the peak has different physical meaning. It corresponds to intrasubband transitions in which the main contribution comes from $1 \to 1$ transition (fig. 2). It is the confinement of electrons that sharpens the peak in comparison to normal semiconductors. The stronger the confinement (or in other words, the smaller the well width), the sharper the peak. In the right side of this peak lies several other peaks. these peaks appear in pairs, each pair corresponds to the resonance condition: $\varepsilon_n - \varepsilon_{n'} + \hbar\omega \pm \hbar\omega_0$.

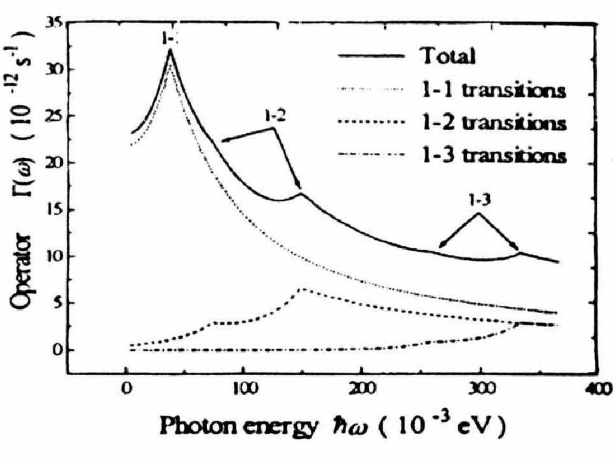

Fig. 2. Contribution to $\Gamma(\omega)$ from different transitions. The main contribution comes from transition between lowest lying levels. The graph is plotted for L=125 $\dot{A}$

When L is small, the distance between levels ε_n is large, electrons can be excited to a few lowest lying levels, so the main contribution to $\Gamma(\omega)$ comes from $1 \to 1$, $1 \to$

2 transitions, as in the graph we see only peaks correspond to these transitions. When L becomes larger, the energy levels ε_n come closer to each other. these additional peaks move closer to the limit value $\omega = \omega_0$. The transitions between higher levels can take place and make comparable contributions to $\Gamma(\omega)$. Therefore, we can see more peaks in the graph. Besides, as L increases the graph becomes smoother and approaches the line for bulk GaAs as asymptote at infinite L.

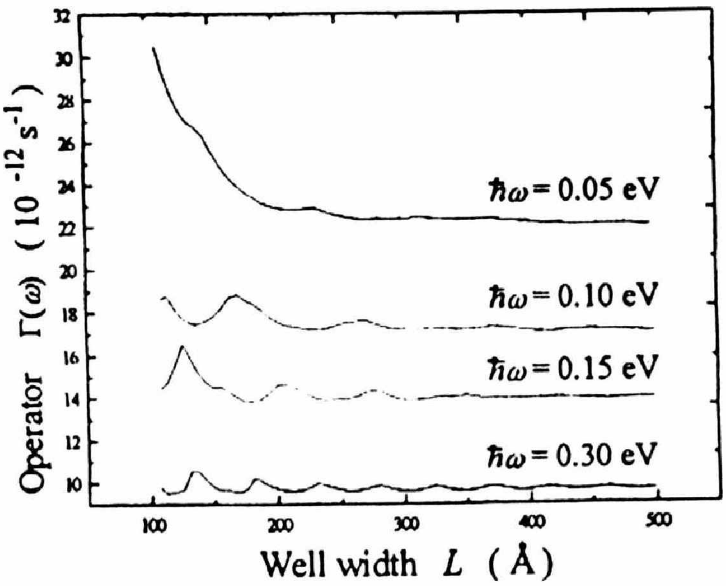

Fig. 3. The dependence of $\Gamma(\omega)$ on the well width for difference values of ω. For ω close to ω_0 this dependence is rather strong. For high ω it may be negligible. It almost disappear when L exceeds 400 $\dot{A}$

When L exceeds a certain value, there appear also some peaks on the left side of the main peak $\omega = \omega_0$. These peaks correspond to "downward" transitions $(n > n')$ and contrary to the peaks on the right side, the left peaks appear individually. That is because for "downward" transitions, $\varepsilon_n - \varepsilon_{n'} > 0$, the resonance condition $\varepsilon_n - \varepsilon_{n'} + \hbar(\omega + \omega_0) = 0$ can not be satisfied, only the resonance condition $\varepsilon_n - \varepsilon_{n'} + \hbar(\omega - \omega_0) = 0$ can be satisfied for $\omega < \omega_0$. It means that for "downward" transitions, electron can not absorb a phonon in the process of absorption. Examples of this kind of peaks can be found in Fig. 2 in the graph for L=200. A, there it correspond to 2 →1 transitions. However, the "downward" peaks are very weak. they soon be flattened and become indistinguishable as L increases.

Another remark is that for all values of L and ω, $\Gamma(\omega)$ is always greater than that of bulk GaAs. this is because, the confinement of electrons in discrete levels leads to more collisions in the system. Consequently, the lifetime of an electron state is shorter, or in other words, $\Gamma(\omega)$ is greater.

Plotted in Fig. 3 is the operator $\Gamma(\omega)$ as a function of L for difference values of ω. We can see that this dependence is rather complicated. For ω near the optical phonon frequency ω_0, this dependence is strong. But as ω increases, it becomes weaker, the line is smoother. For very high frequency, the dependence of $\Gamma(\omega)$ on L may be negligible.

3.2. In the case of the presence of a magnetic field

Plotted in figs. 4(a) and 4(b) are the operators $\Gamma_{-+}(\Omega)$ and $\Gamma_{+-}(\Omega)$ as a function of the cyclotron frequency Ω (for $\hbar\omega = 0.050eV, L = 125\mathring{A}$). Based on the above obtained results we give the following remarks:

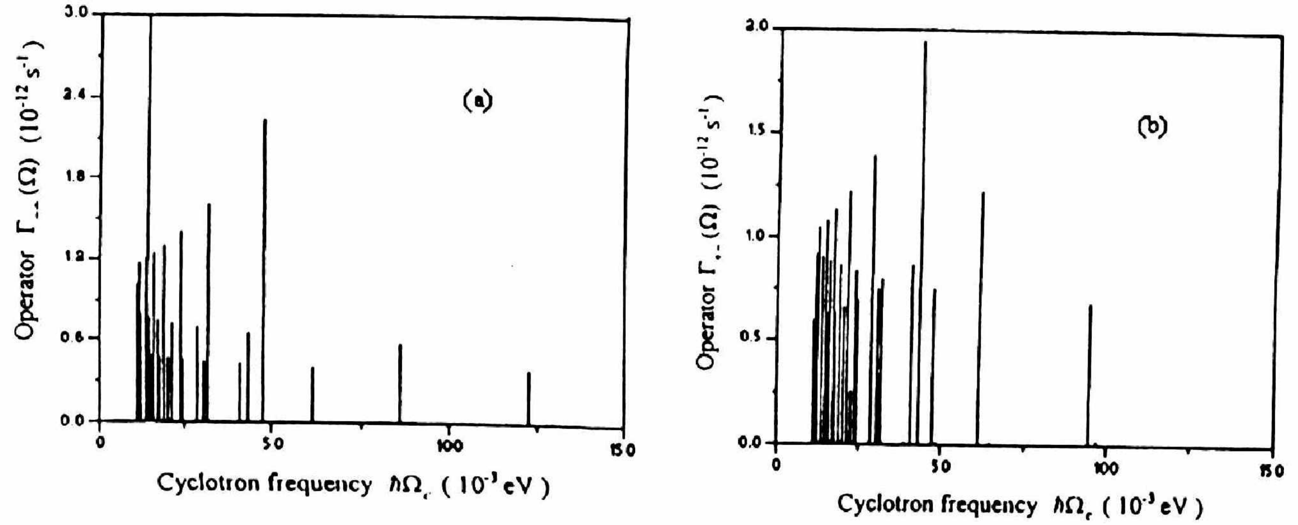

Fig. 4. The dependence of $\Gamma_{-+}(\Omega)$ and $\Gamma_{+-}(\Omega)$ on the Ω-cyclotron frequency for the case of $\hbar\omega = 0.050eV$ and ther width of quantum well L=125 $\mathring{A}$

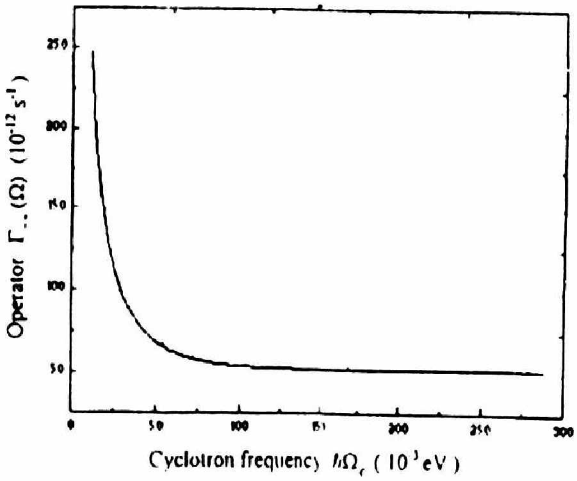

Fig. 5. The dependence of $\Gamma_{-+}\ \Omega)$ and $\Gamma_{+-}(\Omega)$ on the Ω-cyclotron frequency in the specific case of eqs (39) and (40) with n=n', $\hbar\omega = \hbar\omega_0 = 36.1 \times 10^{-3}eV, L = L_{opt} = 125\mathring{A}$. In this case $\Gamma_{-+}(\Omega) \equiv \Gamma_{+-}(\Omega)$

The Dirac delta function in the expressions (32), (33) makes define the index of Landau sub-bands N' which electrons can move to after absorption. It satisfies condidtion

$$\frac{\omega \mp \omega_0}{\Omega} + (n^2 - n'^2)\frac{\omega_0}{\Omega L^{*2}} + (N - N') = 0 \tag{39}$$

We can see that the index N' depends on the frequency of the EMW ω, the width of quantum well L, the limit frequency of optical phonon ω_0 and the cyclotron frequency Ω.

In general, the dependence of the operators $\Gamma)-+(\Omega)$ and $\Gamma)+-(\Omega)$ on the cyclotron frequency Ω is not continuous. It is of line-form (fig. 4). We can see line-density of the graph becomes more and more when $\Omega \approx \omega$ or $\Omega \approx \omega_0$. In the specific case of eq. (39):

$$\omega \mp \omega_0 + (n^2 - n'^2)\omega_0/L^{*2} = 0 \tag{40}$$

the index of Landau sub-bands is constant after absorption (N'=N) and the dependence of the operator $\Gamma)-+(\Omega)$ and $\Gamma)+-(\Omega)$ on the cyclotron frequency Ω is continuous (fig. 5)

4. The absorption coefficient of a weak electromagnetic wave by free carriers in doped superlatices

4.1. In the case of the absence of a magnetic field

Similarly to the case of quantum wells, the motion of an electron is confined in each layer of the DSL and its energy spectrum is also quantized into discrete levels. The Hamiltonian of the electron-optical phonon system in a DSL [15] in the second quantization representation is presented by equations: (5),(6),(7). The electron energy takes the simple form:

$$\varepsilon_{k_\perp,n} = \hbar\Big(\frac{4\pi e^2 n_D}{\kappa_0 m}\Big)^{1/2}\Big(n + \frac{1}{2}\Big) + \frac{\hbar^2}{2m}k_\perp^2 \equiv \varepsilon_0\Big(n + \frac{1}{2}\Big) + \frac{\hbar^2}{2m}k_\perp^2 \tag{41}$$

Here, e and m are the effective charge and mass of electron, respectively; k_0 is the electrical constant; C_q is the electron-phonon interaction, in the case of electron-optical phonon interaction it is:[(3,5)]

$$|C_q|^2 = \frac{1}{V}\frac{2\pi e^2\hbar\omega_0}{(q_\perp^2 + q_z^2)}\frac{1}{2k_0}\Big(\frac{1}{\kappa_\infty} - \frac{1}{\kappa_0}\Big) \tag{42}$$

where V is the normalization volume, κ_0 and κ_∞ are the static and the high-frequency dielectric constant, respectively, and

$$I_{n',n}(q_z) = \sum_{l=1}^{s_0}\int_0^d e^{iq_z z}\Phi_{n'}(z - ld)\Phi_n(z - ld)dz \tag{43}$$

Here, $\Phi_n(z)$ is the eigenfunction for a single potential well(15), and s_0 is the number of period of DSLs. The interaction of the system, which is described by Eqs. (5)-(7) with a weak EMW $\overrightarrow{E} = \overrightarrow{E}_o cos(\omega t)$, is determined by the Hamiltonian.

$$H_t^1 = -e\sum_j (r_j \overrightarrow{E})cos(\omega t)e^{\delta t} \tag{44}$$

where r_j is the radius vector of j-th electron.

Using the Kubo-Mori method, we obtain the following formula for the transverse component of the high-frequency conductivity tensor $\sigma_{xx}(\omega)$:

$$\sigma_{xx}(\omega) = \gamma_0[-i\omega + F(\omega)]^{-1} \tag{45}$$

with $\gamma_0 \equiv (J_x, J_x)$, and

$$F(\omega) = \lim_{\delta\to+0}\left(\frac{i}{\hbar}\right)^2 \gamma_0^{-1}\int_0^\infty dte^{i\omega t-\delta t}([U, J_x], [U, J_x]_{int}) \tag{46}$$

Knowing the hig-frequency conductivity tensor, the absorption coefficient can be found by the common relation

$$R_{xx}(\omega) = (4\pi/c\rho)Re\sigma_{xx}(\omega) \tag{47}$$

Here, ρ is the refraction index and c is the light velocity.

Since the weak EMW has a high-frequency and noting that in compensated n-p DSLs, the bare ionized impurities make the main contribution to the superlattice potential, we obtain

$$R_{xx}(\omega) = \frac{4\pi}{c\rho}\frac{\gamma_0 G(\omega)}{G(\omega)^2+\omega^2} \tag{48}$$

where

$$\gamma_0 = \frac{e^2}{4\pi\beta\hbar^2}exp[\beta(\mu-\varepsilon_0/2)][cosh(\beta\varepsilon_0)+coth(\beta\varepsilon_0)+1] \tag{49}$$

$$G(\omega) = ReF(\omega) = G^+(\omega) + G^-(\omega) \tag{50}$$

$$\begin{aligned} G^\pm(\omega) = {} & \gamma_0^{-1}\frac{\pi^2 e^4\hbar\omega_0}{m\hbar}(\kappa_\infty^{-1}-\kappa_0^{-1})\frac{exp(\beta\mu)[exp(\beta\hbar\omega)-1]}{\hbar\omega}(N_0+\frac{1}{2}\pm\frac{1}{2}) \\ & \times\sum_{i=0}^{+\infty}\sum_{n,n'}[\frac{s_0 d}{\xi}+2^{2i+1}\frac{\Gamma(i)}{\Gamma(2i+1)}](\frac{s_0 d}{\xi})^{2i+1}exp[-2(\frac{s_0 d}{\xi})^2] \\ & \times exp[-\beta\varepsilon_0(n+\frac{1}{2})+\beta\lambda_\pm]|\lambda_\pm|K_1(2\beta|\lambda_\pm|) \end{aligned} \tag{51}$$

$$|\lambda_\pm| = \varepsilon_0(n'-n)-(\hbar\omega\pm\hbar\omega_0) \tag{52}$$

N_0 is the equilibrium distribution of optical phonons, μ is the chemical potential, $\Gamma(x)$ is the Gamma function, $\xi = \hbar(m\varepsilon_0)^{-1/2}$, and $K_1(x)$ is the modified Bessel functions of the second kind. The signs $(\pm)$ in the superscript of $G^{\pm}(\omega)$ and in the lower-script of the function $\lambda_{\pm}$ correspond to the sign $(\pm)$ in Eqs. (d1) and (d2), we can easily see that $G(\omega)$ plays the role of the well-known masses of the electron in the Born approximation in the case of the absence of a magnetic field.

4.2. In the case of the presence of a magnetic field

We consider DSLs with a magnetic field B applied perpendicular to their barriers (z direction). The Hamiltonian of the the electron-optical phonon system in the second representation can be written as (21,22,23).

The energy of the electrons in DSLs in the presence of magnetic field appled in the z direction:

$$\varepsilon^{H}_{N,k_\perp,n} = \hbar\Big(\frac{4\pi e^2 n_D}{\kappa_0 m}\Big)^{1/2}\Big(n+\frac{1}{2}\Big) + \hbar\Omega\Big(N+\frac{1}{2}\Big) \equiv \varepsilon_0\Big(n+\frac{1}{2}\Big) + \hbar\Omega\Big(N+\frac{1}{2}\Big) \quad (53)$$

C_q and $I_{n,n'}(q_z)$, $J_{N',N}(u)$ are defined by Eqs. (2) and (3), (24) respectively.

When a magnetic field is present, we use Kubo-Mori method similarly to the case of quantum wells to obtain the expression for transverse components of the HF conductivity tensor :

$$\sigma_{xx}(\omega,\Omega) = \sigma_{yy}(\omega,\Omega) = \frac{1}{4}\Big[\frac{(J_-,J_+)}{-i(\omega-\Omega)+F_{-+}(\Omega)} + \frac{(J_+,J_-)}{-i(\omega+\Omega)+F_{+-}(\Omega)}\Big] \quad (54)$$

$$F_{-+}(\Omega) = \lim_{\delta\to+0}\Big[\Big(\frac{i}{\hbar}\Big)^2 (J_-,J_+)^{-1}\int_0^{+\infty} e^{i\omega t-\delta t}([U,J_-],[U,J_+])_{int}dt\Big] \quad (55)$$

$$F_{+-}(\Omega) = \lim_{\delta\to+0}\Big[\Big(\frac{i}{\hbar}\Big)^2 (J_+,J_-)^{-1}\int_0^{+\infty} e^{i\omega t-\delta t}([U,J_+],[U,J_-])_{int}dt\Big] \quad (56)$$

The transverse components of the absorption coefficient of an EMW in a DSL in the presence of a magnetic field can be found from Eq. (7) and take the form:

$$R_{xx}(\omega,\Omega) = R_{yy}(\omega,\Omega) = \frac{\pi}{c\rho}\Big[\frac{(J_-,J_+)G_1(\omega,\Omega)}{(\omega-\Omega)^2+[G_1(\omega,\Omega)]^2}\frac{(J_+,J_-)G_2(\omega,\Omega)}{(\omega+\Omega)^2+[G_2(\omega,\Omega)]^2}\Big] \quad (57)$$

where

$$G_1(\omega,\Omega) = ReF_{-+}(\Omega) = G_1^+(\omega,\Omega) + G_1^-(\omega,\Omega) \quad (58)$$

$$G_1^{\mp}(\omega,\Omega) = (J_-,J_+)^{-1}\frac{e^4m^2\Omega^4\hbar\omega_0}{2\pi^2\hbar^3}\left(\frac{1}{\kappa_\infty}-\frac{1}{\kappa_0}\right)\frac{[exp(\beta\hbar\omega)-1]}{\hbar\omega}exp\left[\beta\mu-\frac{1}{2}\beta(\hbar\Omega+\varepsilon_0)\right]$$

$$\times\sum_{i=0}^{+\infty}\sum_{N,N'}\sum_{n,n'}exp[-\beta(\hbar\Omega N+n\varepsilon_0)]\left[\frac{s_0d}{\xi}+2^{2i+1}\frac{\Gamma(i)}{\Gamma(2i+1)}\right]\left(\frac{s_0d}{\xi}\right)^{2i+1}$$

$$\times exp\left[-2\left(\frac{s_0d}{\xi}\right)^2\right][N'^2+(N+1)^2]\left(N_0+\frac{1}{2}\mp\frac{1}{2}\right)\delta(\Delta\varepsilon-\hbar\omega\pm\hbar\omega_0) \quad (59)$$

$$G_2(\omega,\Omega) = ReF_{+-}(\Omega) = G_2^+(\omega,\Omega)+G_2^-(\omega,\Omega) \quad (60)$$

$$G_2^{\mp}(\omega,\Omega) = (J_+,J_-)^{-1}\frac{e^4m^2\Omega^4\hbar\omega_0}{2\pi^2\hbar^3}\left(\frac{1}{\kappa_\infty}-\frac{1}{\kappa_0}\right)\frac{[exp(\beta\hbar\omega)-1]}{\hbar\omega}exp\left[\beta\mu-\frac{1}{2}\beta(\hbar\Omega+\varepsilon_0)\right]$$

$$\times\sum_{i=0}^{+\infty}\sum_{N,N'}\sum_{n,n'}exp[-\beta(\hbar\Omega N+n\varepsilon_0)]\left[\frac{s_0d}{\xi}+2^{2i+1}\frac{\Gamma(i)}{\Gamma(2i+1)}\right]\left(\frac{s_0d}{\xi}\right)^{2i+1}$$

$$\times exp\left[-2\left(\frac{s_0d}{\xi}\right)^2\right][N^2+(N'+1)^2]\left(N_0+\frac{1}{2}\mp\frac{1}{2}\right)\delta(\Delta\varepsilon-\hbar\omega\pm\hbar\omega_0) \quad (61)$$

$$(J_-,J_+) = \frac{(sqrt(2)e\Omega a_c)^2m}{2\pi\hbar^2}[1-exp(-\beta\hbar\omega)]exp\left[\beta\mu-\frac{1}{2}\beta(\hbar\Omega+\varepsilon_0)\right]$$

$$\times\sum_{n,N}(N+1)exp[-\beta(\hbar\Omega N+n\varepsilon_0)] \quad (62)$$

$$(J_+,J_-) = \frac{(sqrt(2)e\Omega a_c)^2m}{2\pi\hbar^2}[exp(-\beta\hbar\omega)-1]exp\left[\beta\mu-\frac{1}{2}\beta(3\hbar\Omega+\varepsilon_0)\right]$$

$$\times\sum_{n,N}(N+1)exp[-\beta(\hbar\Omega N+n\varepsilon_0)] \quad (63)$$

$$\Delta\varepsilon = (N-N')\hbar\Omega+\varepsilon_0(n-n') \quad (64)$$

with μ being the chemical potential and $\delta(x)$ the Dirac-Delta function. The sign ($\pm$) in the superscript of $G_1^{\mp}(\omega,\Omega)$ and $G_2^{\mp}(\omega,\Omega)$ corresponds to the sign ($\pm$) in Eqs. (59) and (61). The upper sign (+) corresponds to phonon absorption and the lower sign (-) to a phonon emission in the absorption process. It is seen easily from Eqs. (54) and (58) that $G_1(\omega,\Omega)$ and $G_1(\omega,\Omega)$ play the roles of the well-known masses of the electron in the Born approximation in the case of the presence of a magnetic field.

5. Numerical Calculation and Discussion in the case of Doped Superlattices

In order to clarify the different behaviors of a quasi-two-dimensional electron gas confined in a DSL with respect to a bulk electron gas, in this section, we numerically evaluate the analytic formulae in section 4 for a compensated n-p n-GaAs/p-GaAs DSL. The characteristic parameters of the GaAs layer of the DSL are $\chi_\infty = 10.9, \chi_0 = 12.9, n_D = 10^{17}cm^{-3}, d = 2d_n = 2d_p = 80nm, \mu = 0.01meV, m = 0.067m_0$, and $\hbar\omega_0 = 36.1meV$, (m_0 is the mass of free electron). The system is assumed to be at room temperature (T=293K).

the sharper the peak. there are some additional peaks in the left and in the rght side of this main peak. The peaks in the right side correspond to "upward" transitions and appear in pairs. The peaks in the left side are much weaker, correspond to "downward" transitions and appear individually. The dependence of the absorption coefficient on the well width L is complicated. This dependence is rather strong when the electromagnetic wave frequency ω is close to the optical phonon frequency ω_0 but maybe neglgible for high ω. When L$\rightarrow \infty$, we obtain the values for normal semiconductors. As L comes to this limit, the additional peaks move closer to the main peak $\omega = \omega_0$, become weaker and disappear at infinite L.

In the case of the presence of a magnetic field applied perpendicular to the barriers, the analytic expressions indicate a complicated, different dependence of the absorption coefficient on the well width L, the frequency of a weak EMW ω, the cyclotron frequency Ω, and the temperature of system T in comparison with normal semiconductors [14,15] in the presence of a magnetic field and quantum wells in the absense of a magnetic field. The index of Landau sub-bands which electrons can move to after absorption is defined.

The numerical evaluations of these formulae for compensated n-p doped superlattices (n-GaAs/p-GaAs) show that the confinement of electrons in the doping superlattices not only leads to differences on the EMW frequency ω and the temperature of system T in comparison with normal semiconductors and quantum wells but also creates many significant differences in the absorption coefficient.

In the case of the absence of a magnetic field, the resonant regions on the two side of the main resonant peak in the absorption spectra of $G(\omega)$ at $s_0 = 15$ (on the number of the doping-layer axis) is obtained. the results show that the lifetimes for an electron to be smaller than it is for semiconductor superlattices [7] and quantum wells .

In the case of the presence of a magnetic field applied perpendicular to the barriers, the analytical expressions indicate a complicated, but different, dependence of th HF conductivity tensor and the absorption coefficient on the characteristic parameters of the DSL: The frequency of the EMW, ω, the temperature of system, T, and the cyclotron frequency, Ω, than is observed in the case of normal semiconductors [16,17] and quantum wells in the presence of a magnetic field. The absorption spectra of an EMW in doped superlattices depends strongly on the condition in Eq. (37), and the index of the Landau sub-band to which the electrons can move after absorption is defined by this condition.

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