INFRARED SINGULARITIES OF FERMION GREEN'S FUNCTION AND THE WILSON LOOP

Nguyen Suan Han
Faculty of Physics, College of Natural Sciences, VNUH

Abstract: *Gauge-invariant and path-dependent objects being infrared asymptotics of a gauge-invariant spinor Green function in the QED have studied. It is proved that the infrared singularities of fermion Green's function can be factorized as the Wilson loop that contains the primary Path and the straight-line contour and accumulates all the dependence on the form of the path of the initial Green function.*

The present report is devoted to the study of infrared asymptotics of the gauge-invariant spinor Green function. The interest in this problem comes from the hope the problem of quark confinement in the framework of QCD can be solved in this way [1-4]. Usually the standard fermion propagator $< 0|T\Psi(x)\bar{\Psi}(y)|0 >$ is studied that. as it is well known is a gauge-dependent quantity. At the same time it is known that the infrared behavior of the complete fermion propagator essentially depends on the gauge choice/ 4.5/. Thus, for example, in the Abelian case in the class of covariant α-gauges the fermion propagator has a branch point at $p^2 = m^2$ which only at $\alpha- = 3$ leads to a pole singularity of the propagator [4]. That is why one can conclude that a consistent study of the gauge-theories should be done on the basis of the gauge-independent quantities. In particular. instead of the standard spinor propagator one can consider a gauge-invariant Green function [6]

$$G^{inv}(x, y|C) = - < 0|T \left\{ \Psi(x) P \exp \left[-ie \int_x^y dZ_\mu A_\mu(z) \right] \bar{\Psi}(y) \right\} |0 > . \tag{1}$$

In contradistinction with the standard propagator the Green function (1) contains the exponential with the path integral taken over the gauge field along an arbitrary path C that connects the points x and y. Our aim consists in studying the infrared behavior of the path-dependent gauge-invariant propagator (1). Let us use the representation (1) in a form of the functional integral over the spinor and vector fields

$$G(x, y|C) = - \int D\Psi D\bar{\Psi} DA\delta[f(A)]\Psi(x) P \exp \left[-ie \int_x^y dz_\mu A_\mu(z) \right] \bar{\Psi}(y). \tag{2}$$

Here. the integration measure over the gauge field DA includes some gauge condition

$$DA = \bar{D}A.\delta[f(A)], \tag{3}$$

the explicit form of which due to the gauge independence of (1) is not essential. Performing in (2) the integration over the fermionic fields we get (with $f(A) = \partial_\mu A_\mu(x)$)

$$G(x, y|C) = \int DA\delta(\partial_\mu A_\mu).\frac{\det[\gamma_\mu D_\mu - m]}{\det[\gamma_\mu \partial_\mu - m]} \exp[-S_0(A)]G(x, y|A) \exp[-ie \int_x^y dz_\mu A_\mu(z)] \tag{4}$$

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX where $S_0[A]$ is the Euclidean action of the free electromagnetic field $S_0[A] = \frac{1}{4} d^4x F^2_{\mu\nu}(x)$, $D_\mu = \partial_\mu + ieA_\mu$, i.e., is covariant derivative, and $G(x,y|A)$ if the Green function of the electron in an external field A_μ, which satisfies the equation

$$[\gamma_\mu D_\mu - m]G(x,y|A) = \partial(x-y).$$

In what follows we shall represent the field in accordance with as a sum of slowly and rapidly varying components $A^{(0)}$ and $A^{(1)}$. For the Green function of fermion in an electromagnetic field $A_\mu$we use the approximate formula [7]

$$G(x,y|A^{(0)} + A^{(1)}) \exp(ie \int_x^y dz_\mu A_\mu^{(0)}(z)), \tag{5}$$

that is valid in an infrared limit. The integration in (5) is performed along the piece of the straight line П that connects the points x and y:

$$\Pi \qquad z_\mu = x_\mu + s(y-x)_\mu, \qquad 0 \leq x \leq 1. \tag{6}$$

With account of formula (5) and the approximate relation

$$\frac{\det[\gamma_\mu D_\mu - m]}{\det[\gamma_\mu \partial_\mu - m]} = \frac{\det[\gamma_\mu(\partial_\mu + ieA_\mu) - m]}{\det[\gamma_\mu \partial_\mu - m]} = \det[1 + \frac{1}{\gamma_\mu \partial_\mu - m} ie\gamma_\mu A_\mu] \approx 1,$$

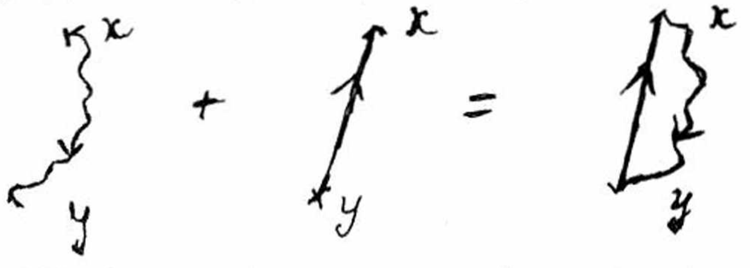
that holds for the infrared limit, it is not difficult to show that propagator (4) can be represented as a product of two factors

$$G^{inp}(x,y|C) = J^{UV}.J^{IR}. \tag{7}$$

Here the first J^{UV} (UV-ultraviolet) is the quantum Green function obtained only with account of the rapidity varying field $A_\mu^{(1)}$. The second factor J^{IR} (IR-infrared) is obtained with account of slowly varying component $A_\mu^{(0)}$ only. It reflects the interaction with the soft photon and thus contains the infrared singularities. This factor has the form

$$J^{IR} = \int DA^{(0)}. \exp\left\{-ie \oint_L dz_\mu A_\mu^{(0)}\right\}, \tag{8}$$

where, $\oint_L dz_\mu A_\mu^{(0)}(z)$ is the integral over the closed path $L = C + \Pi$ of the form $L = C + \Pi =$



It is not difficult to see that expression (8) is nothing but the Wilson loop $J^{IR} = W(L)$ where,

$$W(L) = <0|T \exp\left[-ie \oint_L dz_\mu A_\mu^{(0)}(z)\right] |0>. \tag{9}$$

Thus, we have arrived to an interesting conclusion that all the properties of the infrared behavior of the gauge-invariant propagator (1) are accumulated in the Wilson loop (9).

Let us consider, as an example, a particular choice of the path C and the price of the straight line from the point x up to the point y. With such a choice of the path the propagator (1) takes the form.

$$G^{inv}(x,y|C=\Pi) = - < 0|T\Psi(x)\exp\left[ie\int_0^1 d\alpha(y-x)_\nu A_\nu(x+\alpha(y-x))\right]\bar{\Psi}(y)|0>, \tag{10}$$

where the integration in the exponential is performed along the piece of the straight line $C=\Pi_{xy}$ of form (6). It is clear that in this case

$$\oint_L dz_\mu A_\mu(z) = \int_x^y dz_\mu A_\mu(z) = 0, \tag{11}$$

so $J^{IR}=1$. Thus, additional infrared singularities (like a branch point) that appear due to the interaction with the "soft" photons do not appear here, and Fourier transformation of the propagator in the infrared limit has a simple pole $G^{inv}(p|C=\Pi)\approx(1/\hat{p}+m)$. This result exactly agrees with that of the calculation of the infrared asymptotics of the propagator (1) done in [6].

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CÁC KỲ DỊ HỒNG NGOẠI CỦA HÀM GREEN FECMION VÀ CHU TUYẾN WILSON

Nguyễn Xuân Hãn

Khoa Lý, ĐH Khoa học Tự nhiên, ĐHQGHN

Các đối tượng bất biến chuẩn và phụ thuộc vào quỹ đạo là các tiệm cận hồng ngoại của hàm Green spinor bất biến chuẩn trong điện động lực học lượng tử (QED) đã được nghiên cứu. Đã chứng minh rằng: các kỳ dị hồng ngoại của hàm Green fecmion có thể được giai thừa hoá như là chu tuyến Wilson, mà nó chứa quỹ đạo ban đầu và đường thẳng viên quanh và tập trung mọi sự phụ thuộc vào dạng quỹ đạo của hàm Green ban đầu.