A NEW METHOD FOR SEPARATION OF RANDOM NOISE FROM CAPACITANCE SIGNAL IN DLTS MEASUREMENT

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Abstract. *We introduce a new statistical method for separation of random noise from capacitance signal in DLTS measurement. For the interference of a white random noise ξ with capacitance signals $C(t)$ of general exponential form $C_0e^{-\epsilon t}$, we show that noise ξ and emission factor ϵ are statistically different and can be well separated each from other. Theoretical formalism for reconstruction of noise-free capacitance signals based on determination of emission factor is presented. The method has been tested for various signal-to-noise ratios from 1000 down to 10. Simulation and examples are given.*

Abbreviations

T temperature

t time

$C_n(t)$ normalized capacitance at certain fixed T

$L(t)$ $Ln(C_n)$ e.g. natural logarithm of normalized capacitance at fixed T

$p(\xi)$ density probability of random variable ξ

$P(\xi)$ cumulative probability of random variable ξ

ϵ_i emission factor of a deep center i

E_i activation energy of a deep center i

ω_i ratio E_i/k between E_i and Boltzmann constant k for a deep center i

Definition of terms

1. We will work with a so-called *normalized capacitance* C_n at certain temperature T defined as $C_n(t) = C_0^{-1} \times [C(t) - C_1]$, where C_0 is $C(t)$ at $t = 0$ and C_1 is $C(t)$ at $t = \infty$. For $0 < t < \infty, C_n(t)$ always specifies relation $0 < C_n(t) < 1$, this means that $Ln(C_n)$ has definite and negative value within this range. Taking Ln on $Ln(C_n)$ is not possible but $Ln[-Ln(C_n)]$ has definite values.

2. The average value of a variable χ defined on the probability distribution $p(\xi)$ of a random variable ξ will be denoted by $<\chi>_\xi$. Practically we will consider the average values of $X = \text{Exp}(-E/kT)$ and $Ln(X)$ according to probability distribution of emission factor $p(\epsilon)$. Generally, the small $p - s$ denotes density function where the capital $P - s$ means cumulative probability.

3. A random noise with uniform probability distribution in whole range of frequencies is called *white random noise*. White random noise in restricted area of frequencies will be called *white gaussian noise* if possessing Gaussian distribution.

I. Introduction

The occurrence of noise always disturbs the signals and lowers the quality of measurement or even makes it impossible. In a fine-tuned measurement system like DLTS the occurrence of noise is extremely critical for many important cases. Doolittle & Rohatgi have tested the functionality of various techniques when noise interferes and there have observed that all techniques failed except for lock-in [1]. In general there are two kinds of noise resource: i) equipment precision threshold ability which produces noise in form of either temperature or frequency micro-fluctuation and ii) white random noise which produces constant additive outputs to the signal at all temperatures and frequencies. While the first kind of noise always disturbs signal exponentially, i.e. the measure of disturbance grows exponentially with increased time or temperature variable, the white random noise is statistically independent to the signal. In this paper we will focus on this kind of noise. There are many techniques how to filter the random noise, probably the most popular one is lock-in. In general these techniques may be considered as the correlation averaging techniques which rely on the correlation between input and output and/or the averaging of signal over preset time period [2]. They major disadvantage is that the smooth local structure of signal within the preset time is usually removed together with the averaging process so no information is then available for examination of close-spaced states. Obviously, the peak structure of any correlation integral of signal is more widened and more smoothened than of the signal itself. Thus when the close-spaced deep levels occur (and their DLTS finger prints overlap) the correlation averaging techniques usually lead to the average value, not to the real ones. In this paper we discuss a new method for recovering signal from noise while preserving the signal original structure. The method is based on the differences in statistical behaviors of signal and noise and is able to separate them in heavy noisy environment due to their characteristic signatures. The mathematical concept is discussed in section II and in section III we introduce the full automated computer-based procedure for reconstructing emission factor and thus the capacitance signal. The applicability of this procedure is illustrated by simulation for sample with two preset close-spaced deep levels and then tested in measurement with SiAu sample.

II. Statistical theory of interference of random white noise and exponential signal

a) Statistics of emission factor $p(\epsilon)$ in absence of noise

At each time t and fixed temperature T, the average value of emission factor ϵ is given by: $<\epsilon>=\sum_i p_i(\epsilon)\epsilon_i$ where $p_i(\epsilon)$ is a statistical weight for emission factor i. To determine the density probability function $p(\epsilon)$ we perform the calculation for all measured t:

$$\{Ln[C_n(t)^{-1/t}]\}_t = \epsilon_t = <\epsilon> . \qquad \text{(a.1)}$$

With respect to this distribution C_n reads:

$$C_n = \mathrm{Exp}[-<\epsilon>t] = \mathrm{Exp}[-t\sum_i p_i(\epsilon)\epsilon_i] = \Pi_i \mathrm{Exp}[-tp_i(\epsilon)\epsilon_i].$$

Denote $C_i = \text{Exp}[-tp_i(\epsilon)\epsilon_i]$ we have the emission law for the close-spaced deep centers:

$$C_n = \Pi_i C_i \tag{a.2}$$

C_i may be refereed to as the *partial capacitance* of deep center i in statistical distribution $p(\epsilon)$.

b) Statistics of activation energy $p(E)$ in absence of noise

Define $X_i = \text{Exp}(-E_i/kT)$ with E_i is activation energy of deep center i. We have $Ln(X_i) = -E_i/kT$. Giving any probability distribution $p(\eta)$, the averages $< Ln(X) >_\eta$ and $- < E >_\eta /kT$ must be identical. To determine the density probability function $p(\eta)$ we perform the calculation for all measured t (with respect to that $\epsilon = \rho T^2\text{Exp}(-E/kT)$ where ρ is a constant):

$$\{\eta = Ln[-t^{-1}T^{-2}LnC_n(t)]\}_t = \{Ln(\rho) - E/kT\}_t. \tag{b.1}$$

As seen, $p(\eta)$ does not reveal $< E >_\eta$ directly but $< Ln(\rho) - E/kT >_\eta$ in case $Ln(\rho)$ holds fixed we may suppose that:

$$< Ln(\rho) - E/kT >_\eta = Ln(\rho) - < E/kT >_\eta = Ln(\rho) - < E >_\eta /kT. \tag{b.2}$$

As consequence $p(E) = p(\eta)$. However, statistics (b.1) always produces $< Ln(\rho) - E/kT >_\eta$ not $< E >_\eta$ in general.

c) Relation between $p(\epsilon)$ and $p(E)$

Suppose that (b.2) holds e.g. $p(E) = p(\eta)$. In term of $< E >_\eta$, the average $< LnX >_\eta$ reads:

$$< LnX >_\eta = - < E >_\eta /kT = -\sum_i p_i(E)E_i/kT. \tag{c.1}$$

Emission factor becomes $< \epsilon >_\eta = \rho T^2\text{Exp}(< LnX >_\eta)$. While in term of $< X >_\epsilon$, $< \epsilon > = \sum_i p_i(\epsilon)\epsilon_i = \rho T^2 \sum_i pi(\epsilon)X_i = \rho T^2 < X >_\epsilon$. Comparing these two relations leads to:

$$Ln < X >_E = < LnX >_\eta . \tag{c.2}$$

We use this relation to check how much $p(\epsilon)$ and $p(E)$ differ each from other. If they differ too much then the relation (b.2) may not hold for the case under investigation. The physical meaning of (b.2) is that the noise effecting activation energy does not influence level concentration and capture cross-section, that is to say, E and $Ln(\rho)$ are statistically independent.

d) Statistics of emission factor $p(\epsilon)$ in occurrence of white random noise

With existence of a random white noise, capacitance signal has the form:

$$C_n = \text{Noise} + \text{Exp}[- < \epsilon > t]. \tag{d.1}$$

Re-write C_n to:

$$C_n = \text{Exp}[- < \epsilon > t](1 + \text{Noise}/\text{Exp}[- < \epsilon > t])$$

and put:

$$\text{Noise} = \kappa \text{Exp}[- < \epsilon > t]\text{Exp}[-\xi t], \tag{d.2}$$

where κ is constant and ξ is a random variable. We have C_n as:

$$C_n = \text{Exp}[- < \epsilon > t](1 + \kappa \text{Exp}[-\xi t]).$$

Denote $C_\epsilon = \text{Exp}[- < \epsilon > t]$, $C_\xi = (1 + \kappa \text{Exp}[-\xi t])$ and $C_{\xi n} = (C_\xi - 1)/\kappa$:

$$C_n = C_\epsilon C_\xi \text{ or} C + n = C_\epsilon(1 + \kappa C_{\xi n}). \tag{d.3}$$

This means that the capacitance transient in occurrence of noise follows relation (a.2) for close-spaced deep centers, e.g. random noise behaves as if it is a deep center. This would not be true if ξ does not have density probability similar to C_n . Fortunately, for arbitrary positive noise level [Noise] equation (d.2) always has solution $\xi = Ln(\text{Noise}/\kappa)^{-1/t} - < \epsilon >$. If [Noise] is a random noise with uniform density, than ξ has density probability of $Ln(\text{Noise}/\kappa)^{-1/t} - < \epsilon >$ which is practically the same as C_n . (See Fig.1)

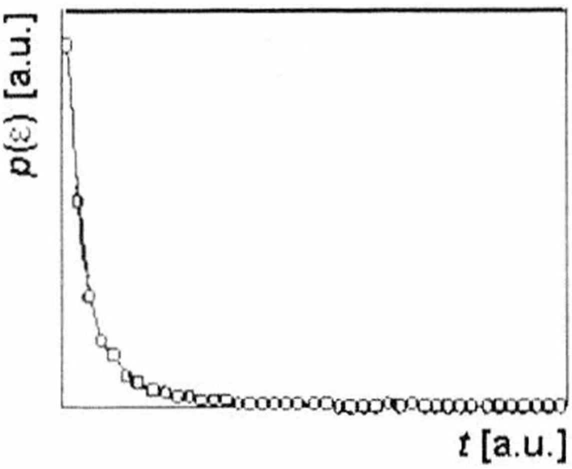

Fig. 1. Density probability p(ξ) of ξ=Ln (Noise/κ)$^{-1/t}$ - <ε>

Clearly, for all measured t the statistics $p(\epsilon)$:

$$\{Ln(Cn)^{-1/t}\}_t = \{- < \epsilon > + \epsilon_\xi\}_t, \tag{d.4}$$

where $\epsilon_\xi = \{Ln(1 + \kappa \text{Exp}[-\xi t])^{-1/t}\}$ will reveal average value of $\{- < \epsilon > + \epsilon_\xi\}$ which differs generally from (a.1). Fig.2 shows $p(\epsilon)$ for 3 different T. As seen, while at the middle T the real ϵ peak is high and proportional to the noise peak ϵ_ξ , at the high T the real ϵ peak is much smaller than the noise peak ϵ_ξ.

The side-effect of ϵ_ξ is that it widdens the width of a delta-like (a.1) peak with the amount proportional to $< \epsilon_\xi >$. One may expect that if $< \epsilon >$ and ϵ_ξ are absolutely additive than the distribution spectrum of (d.4) will contain only one smooth Gaussian peak. However Fig.3 shows two different areas, one corresponds to $< \epsilon >$ and the other to ϵ_ξ.

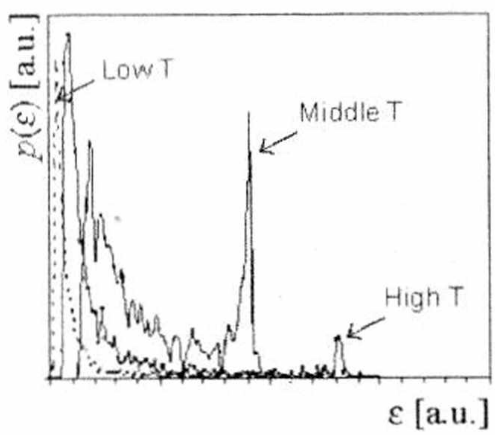

Fig.2. Spectrum $\{Ln(C_n)^{-1/t}\}_t$ at various temperature T. Noise=2% of C_n unit.

This separation is true with two exceptions, the first occurs at low T when C_n is practically equal 1 and the second occurs at high T when C_n is near 0. In both cases, noise becomes so dominating that spectrum $\{Ln(C_n)^{-1/t}\}_t$ contains only values of $\{\epsilon_\xi\}_t$.

e) Temperature dependence of signal-noise separation in $\{Ln(C_n)^{-1/t}\}_t$ spectrum

There exists a threshold temperature T_{crit} where $< \epsilon >$ is small enough and can not be distinguished from ϵ_ξ. Let σ_ϵ^2 and σ_ξ^2 be variance of $< \epsilon >$ and $< \epsilon_\xi >$, the criteria for threshold temperature T_{crit} is that at T_{crit} the displacement $< \epsilon > - < \epsilon_\xi >$ becomes proportional to $(\sigma_\epsilon^2 - \sigma_\xi^2)/2$. This relation is used to filter-off the noise where no signal structure is seen:.

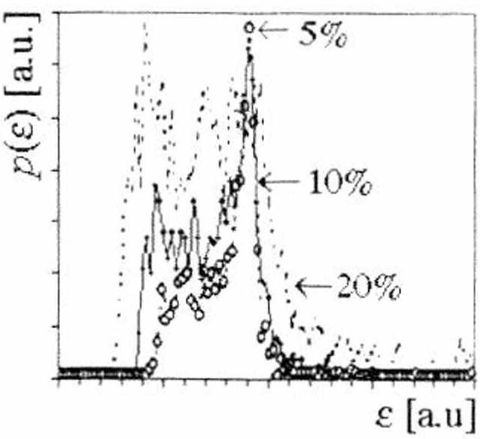

Fig. 3. The exitstence of two different area for <ε> and ε_ξ at noise level 5%, 10% and 20% of C_n unit

$$\frac{(< \epsilon > - < \xi >)_{T_{crit}}}{\sigma_\epsilon^2 - \sigma_\xi^2} \approx \frac{1}{2}. \qquad \text{(e.1)}$$

III. Simulation and measurement

a) Procedure for the reconstruction of noise-free capacitance signal

Data in the capacitance transient measurement are usually collected at preset temperature T when the emission factor ϵ can be considered as constant. To obtain the statistical characteristics of ϵ we should measure $C_n(t)$ as dense as possible. However the number of several hundreds data is adequate and 1000 recorded data provide quite satisfied results on simulation.

At the first step a logic circuit should be available to transform $C_n(t)$ into $Ln[C_n(t)^{-1/t}]$ and then into $Ln[-t^{-1}T^{-2}LnC_n(t)]$. This is easy with computer. The statistics $p(\epsilon)$ is obtained after recording all $Ln[C_n(t)^{-1/t}]$ and similarly $p(\eta)$ by all $Ln[-t^{-1}T^{-2}LnC_n(t)]$. Two statistics are then checked against each other using relation (c.2) to see if $p(E)$ can be set equal to $p(\eta)$. If $p(E) = p(\eta)$ holds we have a simple case of one noise-free center, otherwise overlapped centers occur and noise should be filtered. A numeric calculation of derivation $[dp(\epsilon)/d\epsilon]$ should provide peak value $\epsilon_{\max}$ of $p(\epsilon)$. As noted before, we have two different cases: i) at the extremely low and high end T there is only one ϵ_ξ peak. This noise-driven exponentially-distributed peak should be removed

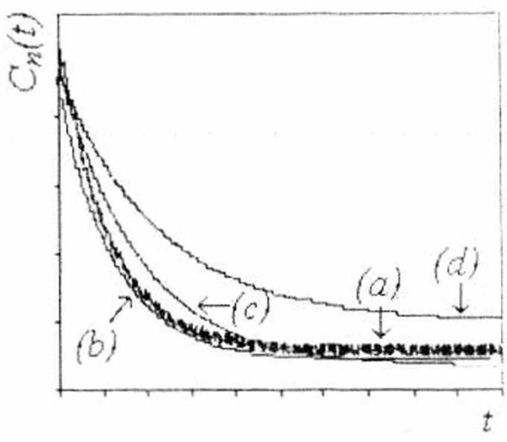

Fig. 4. (a) Un-filtered signal $C_n(t)$; (b) $C_n(t)_\varepsilon$ reconstructed by $\varepsilon_{\max}$; (c) $C_n(t)_L$ obtained using lock-in; (d) $C_n(t)_\xi$ reconstructed by ξ. Noise = 2% of C_n unit

since it does not correspond to signals and contains no information about emission factor; ii) at the middle range T there are two peak values, one corresponds to emission factor ϵ and the second refers to ϵ_ξ. They can be distinguished easily since $p(\epsilon)$ is a delta-like Gaussian symmetrical distribution while $p(\epsilon_\xi)$ is a wide-spread asymmetrical exponential one. Some statistical tests exist to help to automate the selection process.

Normally when measurement is kept in a reasonable T range, the first case should not occur and we should only observe the change in peak height for $\epsilon_{\max}$ and ϵ_ξ when T varies. With T increased peak $\epsilon_{\max}$ also grows and height ratio ϵ/ϵ_ξ reaches maximum at certain T. The height ratio ϵ/ϵ_ξ is proportional to signal-to-noise ratio at preset T. If T grows further, noise-driven ϵ_ξ becomes higher and may grow faster than $\epsilon_{\max}$. At extreme T noise may even dominate over signal. This is due to the fact that at extreme high T the $C_n(t)$ is practically zeroed and we measure only noise. On simulation we have observed that the signal is still separable from noise at noise level 10-times higher than the signal. By averaging technique one would obtain false average at $(\epsilon_{\max} + < \epsilon_\xi >)/2$ instead of real value $\epsilon_{\max}$. Once $\epsilon_{\max}$ is collected for each T, the noise-free capacitance curve $C_n(t)$ can be reconstructed. Fig.4 compares $C_n(t)_\epsilon$ reconstructed by $\epsilon_{\max}$ and the capacitance signal obtained by lock-in. Fig.5 shows DLTS finger-prints obtained using $C_n(t)_\epsilon$ and $C_n(t)_\xi$. Clearly, noise participates as a set of emission centers.

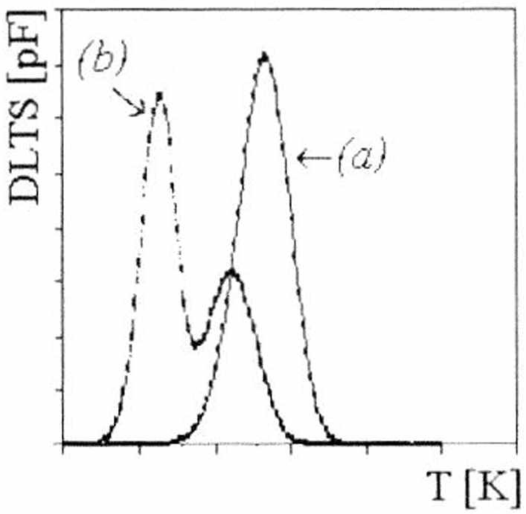

Fig. 5. DLTS finger-prints obtained from (a) $C_n(t)_\varepsilon$ and (b) $C_n(t)_\xi$. The first discovers the real center and the second shows the false ones. Noise=2% of C_n unit

b) Simulation for sample with two close-spaced deep levels

The above procedure has been tested on simulation for a sample with two preset close-spaced deep levels at 0.30 eV and 0.38 eV. Capture-cross sections have been set at $1.0 \times 10^{-15} cm^2$ and $2.0 \times 10^{-15} cm^2$, respectively. Both level concentrations were $0.1 \times 10^{-15} cm^{-3}$. Constant random noise at 2%, 3% and 5% of signal maximum was added to output. Then the output was filtered-off a) using lock-in and b) using $p(\epsilon)$ statistics.

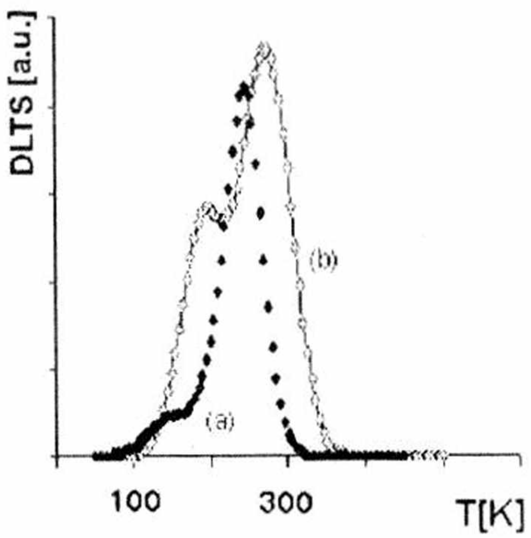

Fig. 6. DLTS spectra obtained using (a) lock-in filtered signal and (b) $p(\varepsilon)$ filtered signal

Level analysis was carried out using the classical Lang's DLTS scheme [3] for two cases: a) filtered by lock-in; b) filtered by $p(\epsilon)$. Fig.6 shows the resulting DLTS spectra for these cases at noise 2%. As seen $p(\epsilon)$-filtered signal reveals the two preset close-spaced levels while the lock-in filtered signal sees only their average at 0.34 eV. As noise increases the spectrum of un-filtered signal becomes unstable and failed to provide meaningful result.

c) Measurement with SiAu sample

The measurement was carried on SiAu sample. This sample has been investigated by Fourier DLTS [4] on BIO-RAD's DLTS equipment at Center for Materials Science, Faculty of Physics, Hanoi University of Science and 3 different levels were shown. The re-examination of the widdening of $p(\epsilon)$ peak has reveal the interference of a constant white random noise at 1.2% of maximal signal. After filtering off noise the reconstructed noise-free data was used for Fourier calculation and the resulting b1 coefficient is plotted in Fig.7. As seen there are at least 2 more levels. All of them are close-spaced to the existing ones and did not appear in the original Fourier calculation using un-filtered signal.

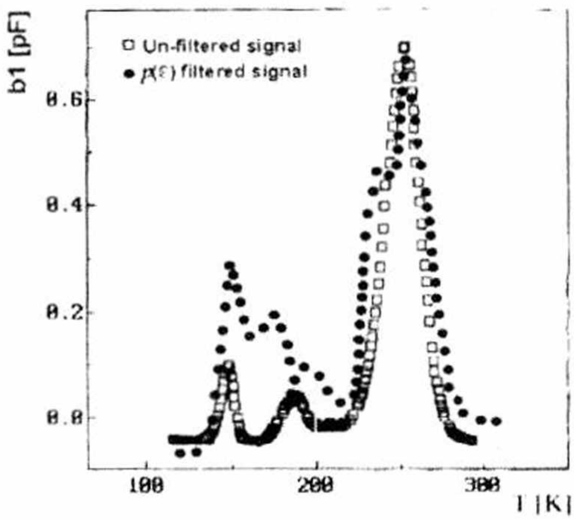

Fig. 7. Temperature dependence of fourier coeficient b1 for (a) unfiltered signal and (b) p(ε) filtered signal

IV. Conclusion

The method is efficient to recover signals from noise in heavy noisy environment when signal-to-noise ratio drops below 10. Unlike averaging techniques, which take averages of signals and noise over certain time period and usually remove the local smooth structure of signals within this period, the present method is able to separate signals directly from noise due to the difference in their statistical behaviours. The method can reveal the real values of signals while reserving the signal original smooth structure, which is significantly important for obtaining the information about the existence of close-spaced deep levels. Some modern method like Laplace DLTS [5, 6] is extremely sensible for noise so the reconstructed noise-free data would be helpful to reduce instability of such methods.

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MỘT PHƯƠNG PHÁP MỚI TÁCH NHIỄU TỪ TÍN HIỆU PHỔ QUÁ ĐỘ TÂM SÂU

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Bài báo này giới thiệu một phương pháp thống kê để tách nhiễu ngẫu nhiên từ tín hiệu điện dung trong phép đo phổ quá độ các tâm sâu (DLTS). Để tách biệt nhiễu ngẫu nhiên ξ với tín hiệu điện dung $c(t)$ dạng hàm mũ $C_0e^{-\epsilon t}$, các tác giả đã chỉ ra nhiễu ξ và hệ số phát xạ ϵ là có thể tách biệt. Phương pháp này đã được thử cho các tỷ số tín hiệu trên tạp khác nhau từ 1000 đến 10. Sự mô phỏng và các ví dụ đã được chỉ ra.