# ON THE AMPLIFICATION OF SOUND (ACOUSTIC PHONONS) BY ABSORPTION OF LASER RADIATION IN CYLINDRICAL QUANTUM WIRES WITH PARABOLIC POTENTIAL IN THE PRESENCE OF MAGNETIC FIELD

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**Abstract** Based on the quantum kinetic equation for phonon, the amplification of sound (acoustic phonons) due to the absorption of laser radiation in cylindrical quantum wires with parabolic potential in the presence of magnetic field is studied. Analytical expressions for the rate of acoustic phonons excitations, the conditions for the amplification of sound and the momentum conditions for electron to participate in the amplification of sound are obtained for the two cases: singlephoton absorption process and multiphoton absorption process. The differences between the two cases of singlephoton absorption and multiphoton absorption are discussed; the numerical computations and plots are carried out for a GaAs/GaAsAl quantum wire. The results are compared with bulk semiconductors and quantum wells.

## 1. Introduction

It is well known that the interaction of a laser radiation with materials can lead to the excitations of higher harmonics and the amplification of phonons. The problem has been widely investigated in the past in a number of papers [1-6]. In [1-3] the problem has been studied in bulk semiconductors with singlephoton absorption process and multiphoton absorption process for degenerate and non-degenerate electron system. In [4,5] the problem has been considered for quantum wells with singlephoton absorption process and multiphoton absorption process and in the presence of magnetic field. All these authors shown that the phonon population grows with time under some conditions.

In this paper, we study the influence of magnetic field on the amplification of sound (acoustic phonons) due to the absorption of laser radiation in cylindrical quantum wires with asymmetric parabolic potential. Based on the quantum kinetic equation for phonon, we obtain analytical expressions for the rate of change of the population of the phonon states, the conditions for the amplification of sound, and momentum conditions for electron to participate in the amplification of sound with the presence of magnetic field in the two cases: singlephoton absorption process and multiphoton absorption process. All results are numerically computed and plotted for a GaAs/GaAsAl quantum wire.

## 2. Rates of acoustic phonons excitation

Consider a quantum wire with elliptical cross section and asymmetric parabolic confining potential:

$$V(x,y) = \frac{m^*}{2}(\Omega_x^2 x^2 + \Omega_y^2 y^2).$$

Here $\Omega_x, \Omega_y$ are the effective frequencies of the potential and $m^*$ is the effective electron mass.

Assume the vector potential $\wp$ of the titled magnetic field $B$ in the form $\wp = (0, B_z x, B_x y - B_y x)$, electron wave function and energy can by written as [7]:

$$\Psi_{n,l,\vec{k}}(x,y,z) = \frac{e^{ikz}}{\sqrt{L}} \frac{1}{\sqrt{2^n n! l_x \sqrt{\pi}}} e^{-\frac{x^2}{2l_x^2}} H_n\left(\frac{x}{l_x}\right) \frac{1}{\sqrt{2^l l! l_y \sqrt{\pi}}} e^{-\frac{y^2}{2l_y^2}} H_l\left(\frac{y}{l_y}\right), \quad (1)$$

$$\epsilon_{n,l}(k_z) = \frac{\hbar^2 k^2}{2M} + \hbar\omega_1 (n + 1/2) + \hbar\omega_2 (l + 1/2), \quad (2)$$

where $M = m^*\left[1 + \left(\frac{\omega_x}{\Omega_y}\right)^2 + \left(\frac{\omega_y}{\Omega_x}\right)^2\right]$, $\omega_i = \frac{eB_i}{m^* c}$, $l_i = \sqrt{\frac{\hbar}{4m^*\Omega_i}}$, $i = (x,y)$, $\omega_c = \frac{eB}{m^* c}$ is the cyclotron frequency, $H_n(x)$ is Hermitte polynomial of order $n$, $L$ is the length of the wire,

$$\omega_{1,2} = \frac{1}{2}\left\{\sqrt{\Omega_x^2 + \Omega_y^2 + \omega_c^2 + 2\Omega_x\Omega_y\sqrt{1 + \left(\frac{\omega_x}{\Omega_y}\right)^2 + \left(\frac{\omega_y}{\Omega_x}\right)^2}} \pm \right.$$

$$\left. \pm \sqrt{\Omega_x^2 + \Omega_y^2 + \omega_c^2 - 2\Omega_x\Omega_y\sqrt{1 + \left(\frac{\omega_x}{\Omega_y}\right)^2 + \left(\frac{\omega_y}{\Omega_x}\right)^2}}\right\}^{1/2}$$

With bulk phonon assumption, the Hamiltonian for the electron-phonon system of a quantum wire in the external field can be written as:

$$H(t) = \sum_{n,l,\vec{k}} \varepsilon_{n,l}\left(\vec{k} - \frac{e}{\hbar c}\vec{A}(t)\right) + \sum_{\vec{q}} \hbar\omega_{\vec{q}} b_{\vec{q}}^+ b_{\vec{q}} +$$

$$+ \sum_{n,l,n',l',\vec{k},\vec{q}} C_{n,l,n',l'}(\vec{q}) a^+_{n,l,\vec{k}+\vec{q}} a_{n',l',\vec{k}} (b^+_{-\vec{q}} + b_{\vec{q}}), \quad (3)$$

where $a^+_{n,l,\vec{k}}$ and $a_{n,l,\vec{k}}$ ($b^+_{\vec{q}}$ and $b_{\vec{q}}$ ) are the creation and annihilation operators of electron (phonon); $\vec{k} = (0, 0, k_z)$ is the electron wave vector (along the wire's axis: z axis); $\vec{q}$ is the phonon wave vector; $\vec{A}(t) = \frac{c}{\Omega} E_0 cos(\Omega t)$ is the potential vector, depends on the laser radiation; $C_{n,l,n',l'}(\vec{q}) = C_{\vec{q}} I_{n,l,n',l'}$ is the electron-acoustic phonon interaction coefficient, where $|C_{\vec{q}}|^2 = \frac{\hbar q \xi^2}{\rho v_s V}$, $V$ is the principal volume of the crystal, $\xi$ is the deformation acoustic potential, $\rho$ is the density of the material, $v_s$ is the sound velocity in the material, $R$ is the wire's radius, $I_{n,l,n',l'}$ is the form factor.

From Hamiltonian (3), we obtain the quantum kinetic equation for acoustic phonons in quantum wires:

$$\frac{\partial}{\partial t}\langle b_{\vec{q}}\rangle_t + i\omega_{\vec{q}}\langle b_{\vec{q}}\rangle_t =$$
$$-\frac{1}{\hbar^2}\sum_{n,l,n',l'}|C_{n,l,n',l'}(\vec{q})|^2\sum_{\vec{k}}\left(n_{n,l}(\vec{k}-\vec{q})-n_{n',l'}(\vec{k})\right)\int_{-\infty}^{t}\langle b_{\vec{q}}\rangle_{t_1}\times$$
$$\times\sum_{\nu,s=-\infty}^{\infty}J_\nu\left(\frac{\lambda}{\hbar\Omega}\right)J_s\left(\frac{\lambda}{\hbar\Omega}\right)exp\left(\frac{i}{\hbar}[\varepsilon_{n',l'}(\vec{k})-\varepsilon_{n,l}(\vec{k}-\vec{q})](t_1-t)-i\nu\Omega t_1+is\Omega t\right)dt_1, \quad (4)$$

Where the symbol $\langle x\rangle_t$ means the usual thermodynamic average of operator $x$; $n_{n,l}(\vec{k}) = \langle a^+_{n,l,\vec{k}}a_{n,l,\vec{k}}\rangle_t$; $\lambda = \frac{e\hbar\vec{q}\vec{E_0}}{m^*\Omega}$; $J_\nu(x)$ is the Bessel function of the first kind.

Perform Fourier transformation, from the dispersion equation for phonon, we obtain the rate of acoustic phonons excitations:

$$\alpha(\vec{q}) = -\frac{\pi}{\hbar}\sum_{n,l,n',l'}|C_{n,l,n',l'}(\vec{q})|^2\sum_{\vec{k}}[n_{n',l'}(\vec{k})-n_{n,l}(\vec{k}-\vec{q})]\times$$
$$\times\sum_{\nu=-\infty}^{\infty}J_\nu^2\left(\frac{\lambda}{\hbar\Omega}\right)\delta\{\varepsilon_{n',l'}(\vec{k})-\varepsilon_{n,l}(\vec{k}-\vec{q})-\hbar\omega_{\vec{q}}-\nu\hbar\Omega\}, \quad (5)$$

with $\delta(x)$ is the Dirac function.

From (5), we process with non-degenerate electron system assumption to obtain the rate of acoustic phonons excitations in the case of singlephoton absorption process and the case of multiphoton absorption process.

## 3. The amplification of sound in the case of singlephoton absorption process

In the case of singlephoton absorption process, assume that $\lambda \ll \Omega$, from (5) we obtain the rate of acoustic phonons excitations in quantum wires:

$$\alpha(\vec{q}) = \frac{m^* L\lambda^2}{4\hbar^5 q\Omega^2}\sum_{n,l,n',l'}|C_{n,l,n',l'}(\vec{q})|^2\times$$
$$\times\, exp\left\{-\beta\hbar\omega_1(n+1/2)+\beta\hbar\omega_2(l+1/2)-\frac{m^*\beta}{2\hbar^2q^2}(a^2+\hbar^2\Omega^2)+\frac{\hbar\beta\omega_{\vec{q}}}{2}\right\}\times$$
$$\times\left[exp\left(-\frac{\beta m^* a\Omega}{\hbar q^2}+\frac{\beta\hbar\Omega}{2}\right)sh\left(\hbar\beta\frac{\omega_{\vec{q}}+\Omega}{2}\right)+exp\left(\frac{\beta m^* a\Omega}{\hbar q^2}-\frac{\beta\hbar\Omega}{2}\right)sh\left(\hbar\beta\frac{\omega_{\vec{q}}-\Omega}{2}\right)\right], \quad (6)$$

with $a = \hbar\omega_1(n-n') + \hbar\omega_2(l-l') + \hbar\omega_{\vec{q}} + \frac{\hbar^2q^2}{2m^*}$, $\beta = k_BT$, $k_B$ is Boltzmann constant.

Due to $\delta$ function in (5), only $k$ satisfies condition:

$$k \geq \left|\frac{q}{2}+\frac{m^*}{\hbar^2 q}\left(\hbar\omega_1(n-n')+\hbar\omega_2(l-l')+\hbar\omega_{\vec{q}}\right)\pm\frac{m^*\Omega}{\hbar q}\right|, \quad (7)$$

contributing in the integral, or only electrons with momentum satisfying condition (7) can damp or amplify phonons.

From (6), it is obvious when $\omega_{\overrightarrow{q}} \ll \Omega$, $\alpha$ is negative, we have the amplification of sound:

$$\alpha(\overrightarrow{q}) = -\frac{m^* L\lambda^2}{2\hbar^5 q\Omega^2} \sum_{n,l,n',l'} |C_{n,l,n',l'}(\overrightarrow{q})|^2 exp\left(\frac{\hbar\beta\omega_{\overrightarrow{q}}}{2} - \frac{m^*\beta\Omega^2}{2q^2}\right) \times$$
$$\times\, exp\left(-\beta\hbar\omega_1(n+1/2) + \beta\hbar\omega_2(l+1/2) - \frac{m^*\beta a^2}{2\hbar^2 q^2}\right) \times$$
$$\times\, sh\left(\frac{\beta\hbar\Omega}{2}\right) sh\left(\frac{\beta m^*\Omega}{\hbar q^2}\left(\hbar\omega_{\overrightarrow{q}} + \hbar\omega_1(n-n') + \hbar\omega_2(l-l')\right)\right). \quad (8)$$

## 4. The amplification of sound in the case of multiphoton absorption process

Use the approximate formula as in [8]:

$$\sum_{\nu} J_\nu^2\left(\frac{\lambda}{\Omega}\right)\delta(E-\nu\Omega) = \frac{\theta(\lambda^2 - E^2)}{\pi\sqrt{\lambda^2 - E^2}},$$

with:

$$\theta = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x < 0 \end{cases}.$$

From (4), after several calculations, we obtain the rate of acoustic phonons excitations in the case of multiphoton absorption process:

$$\alpha(\overrightarrow{q}) = \frac{\pi^{\frac{3}{2}} L m^*}{2\hbar^3 q} exp\left(-\frac{\beta m^*\lambda^2}{2\hbar^2 q^2}\right) \sum_{n,l,n',l'} |C_{n,l,n',l'}(\overrightarrow{q})|^2 \times$$
$$\times \sum_{\nu=0}^{\infty} \frac{\Gamma\left(\nu+\frac{1}{2}\right)}{\nu!}\left(\chi_{n,l}^{\nu}\left(-\frac{\hbar^2 q^2}{2m^*}\right) - \chi_{n',l'}^{\nu}\left(\frac{\hbar^2 q^2}{2m^*}\right)\right), \quad (9)$$

with:

$$\chi_{n,l}^{\nu}(x) = exp\left(\hbar\beta\omega_1(n+1/2) + \hbar\beta\omega_2(l+1/2)\right) \times$$
$$\times\, exp\left[-\frac{\beta m^*}{2\hbar^2 q^2}\left(\hbar\omega_1(n-n') + \hbar\omega_2(l-l') + x + \hbar\omega_{\overrightarrow{q}}\right)^2\right] \times$$
$$\times \left(\frac{\lambda}{\hbar\omega_1(n-n') + \hbar\omega_2(l-l') + x + \hbar\omega_{\overrightarrow{q}}}\right)^{\nu} \times$$
$$\times\, I_\nu\left(\frac{\beta m^*\lambda}{\hbar^2 q^2}\left(\hbar\omega_1(n-n') + \hbar\omega_2(l-l') + x + \hbar\omega_{\overrightarrow{q}}\right)\right),$$

$I_\nu(x)$ is the complex Bessel function of the order $\nu$.

In a similar way to (7), we have the momentum condition for electron to participate in the damping or amplification of sound:

$$k < -\frac{q}{2} + \frac{m^*}{\hbar^2 q}\left(\hbar\omega_1(n-n') + \hbar\omega_2(l-l') + \hbar\omega_{\vec{q}} - |\lambda|\right). \tag{10}$$

Note that if:

$$\chi^{\nu}_{n,l}\left(-\frac{\hbar^2 q^2}{2m^*}\right) < \chi^{\nu}_{n',l'}\left(\frac{\hbar q^2}{2m^*}\right), \tag{11}$$

then $\alpha(\vec{q}) < 0$, or the number of phonon grows with time.

## 5. Numerical results and discussion

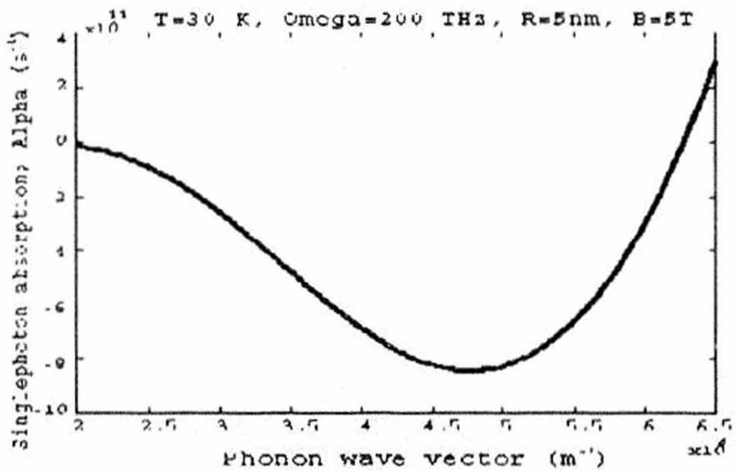


*Figure 1.* Dependence of the rate of phonons excitations on phonon wave vector in the case of singlephoton absorption process; $R = 5nm$, $B = 5T$, $\Omega = 200THz$

From the obtained results, we plot the dependence of the rate of acoustic phonons excitations $\alpha$ on phonon wave vector $\vec{q}$ (Fig.1), on laser frequency $\Omega$ (Fig.2) and on temperature $T$ (Fig.3) for the case of the singlephoton absorption process. Parameters for numerical computation are $m^* = 0.067m_0$; $B = 5Tesla$; $v_s = 4087ms^{-1}$. All figures show that the curves have peaks, which illustrate the maximum of the amplification of sound, at $q = 4.710^6 m^{-1}$, $\Omega = 130THz$, $T = 100K$. Compare these results with quantum wires in the absence of magnetic field, we realize that magnetic field has certain influence on the amplification of sound. However, the numerical results for bulk semiconductor and quantum wells are different at the range of values of $q$ and $\Omega$ for the amplification to happen.

Note that (7) and (10) are the conditions for electron to participate in the damping or the ampification of sound in the wire, but not the conditions for the amplification to happen. In the limiting case when $\omega_{\vec{q}} \ll \Omega$ (for singlephoton absorption process) and (11) (for multiphoton absorption process), $\alpha(\vec{q}) < 0$ and we have the amplification of sound, or the number of phonon increases with time as a direct result of the presence of the laser radiation. These formulae are more complicated than the corresponding formulae in quantum wells [4-6] due to the intricate dependence of the electron wave function and energy spectrum on Hermitte polynomial. With bulk semiconductors, because of the continuance of the energy spectrum, the dependence has a completely different form [1-3].

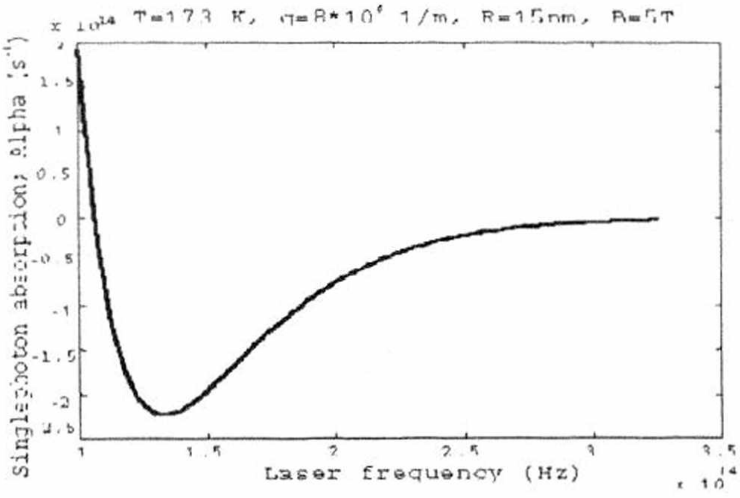


*Figure 2.* The amplification of sound as a function of laser frequency in the case of singlephoton absorption process; $T = -100^oC$, $R = 15nm$, $B = 5T$

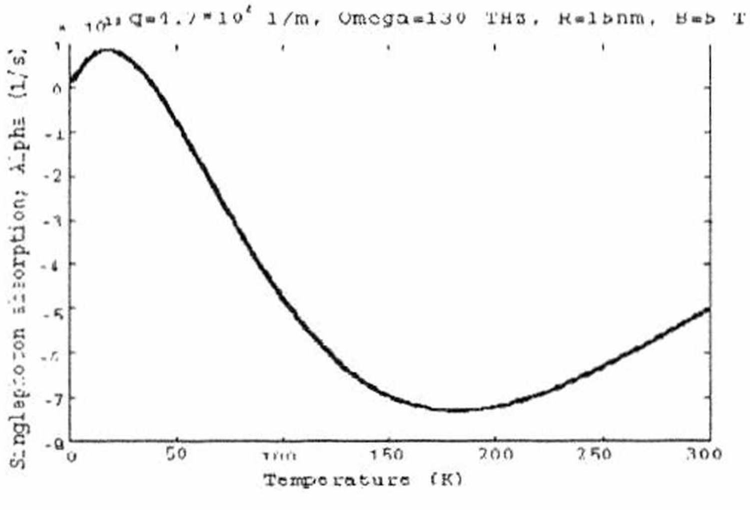


*Figure 3.* Dependence of the rate of phonons excitations on temperature in the case of singlephoton absorption process; $\Omega = 130Thz$, $R = 15nm$, $q = 4.710^6 m^{-1}$, $B = 5T$

In the case of multiphoton absorption process, the rate of acoustic phonons excitations (9) and the momentum condition (10) intricately depend on the variable (the exponent and the complex-Bessel function). The dependence on the complex-Bessel function $I_\nu(z)$, $z$ varies with the intensity of the laser radiation field ($\lambda = \frac{e\hbar q E_0}{m^* \Omega}$), shows that the amplification of sound depends on the intensity of the radiation field with order greater than two, while the dependence in the case of singlephoton absorption process is of the order two (6). When condition (11) is satisfied, we also have the amplification of sound. Compare these formulae with the expression in [1-3] we realize the difference for both the dependence and the momentum conditions.

## 6. Conclusion

In the conclusion, we want to emphasize that:

1. The quantum kinetic equation for phonon in quantum wires in the presence of magnetic field was established, which has a similar form with those in quantum wells and bulk semiconductors.
2. Analytical expressions and conditions for the amplification coefficient of sound were obtained in the case of singlephoton absorption and multiphoton absorption. In proper conditions, the rate of acoustic-phonon excitations is negative and the phonon population grows with time.
3. The numerical computations and plots are carried out for a GaAs/GaAsAl quantum wire. From the results, it is easy to see that there are the ranges of value of phonon wave vector, laser frequency and temperature at which the amplification of sound happens.
4. Magnetic field has certain influences on the amplification of sound.

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