Race and Hazard algebra in asynchronous system

Nguyen Quy Thuong[*]

Vietnam National University, Hanoi, 144 Xuan Thuy, Cau Giay, Hanoi, Vietnam

Received 5 September 2007; received in revised form 12 December 2008

Abstract. Under the name of ideal circuit network, it is easy to make us understand that digital circuits are independent of the inner status and their outputs just only depend on the inputs. They are time constants and therefore Boolean algebra is a special suitable means of describing. In fact propagation delay time of circuit system should be in thought firstly in designing circuit network. In that case Boolean algebra does not seems to be fully suitable for describing actual circuit systems. Furthermore, Boolean algebra cannot help to locate and repair hazard errors in a circuit thoroughly when the circuit function is minimized with Distributive Law. On the other hand, Boolean algebra can create new hazard errors in the circuit. This problem can be solved by using hazard algebra.

1. General view of Race problem and Hazard

The running time difference of input signals or intermediate signals causes a race. In fact, many micro circuits which depend on time have been used and we should recognize the time in signal processing solution. Time processing is different for every micro circuit type and even one micro circuit itself under various environment conditions (temperature, humidity...) may lead to different drifted time relations. Besides, signal pulse frames of digital circuits require a τ period of time for progressing to the necessary signal level rather than following a square pulse frame. Such times also can not be defined precisely. The accumulation of propagation delay time, delay time of components themselves, line delay... and somewhat caused race phenomenon.

Hence, there is a signal transition upon the time during the signal transmission from input to output. If change of input signal caused a change of the output then we called it a dynamic transition, and if it's a un-change output then we called a static transition. This phenomenon is a dynamic characteristic of circuit systems and being called as Hazard.

If the signal change is made an *instant undesirable change of another variable according to laws* $(0 \rightarrow 1 \rightarrow 0)$ *or* $(1 \rightarrow 0 \rightarrow 1)$, there's a *static hazard*. We may say also that there's a static hazard ξ, if the desired signal has φ value ($\varphi \in \{0,1\}$) and the attach error signal has ξ value in a short term.

After switching on an input, if *output signal has multiple transitions* for a short time, then there would be a *dynamic hazard.*

If a hazard occurs due to the time delay of different individual input signals, then there would be a *functional hazard*. Vise versa, if it occurs due to circuit structure, then there would be a *structural hazard.*

---

[*] E-mail: nguyenquythuong@yahoo.com

2. Hazard algebra

As mentioned above, Boolean algebra in fact is not completely suitable for designing circuit network if Distributive Law is used. Study this example:

Given a circuit function correlative with the logic schema in figure 1a:

$$y = xc + \bar{x} \qquad (1)$$

from the logic schema and the circuit function (1) we can see that there exists only static hazard because:

- with c = 1, $y = x1 + \bar{x} = x + \bar{x}$ ⇒ static – 1 hazard.

- with c = 0, $y = x0 + \bar{x} = \bar{x}$ ⇒ not Hazard

After Distributive Law is applied we have circuit function (2) and the logic schema in figure 1b:

$$y = xc + \bar{x} = (c + \bar{x})(x + \bar{x}) \qquad (2)$$

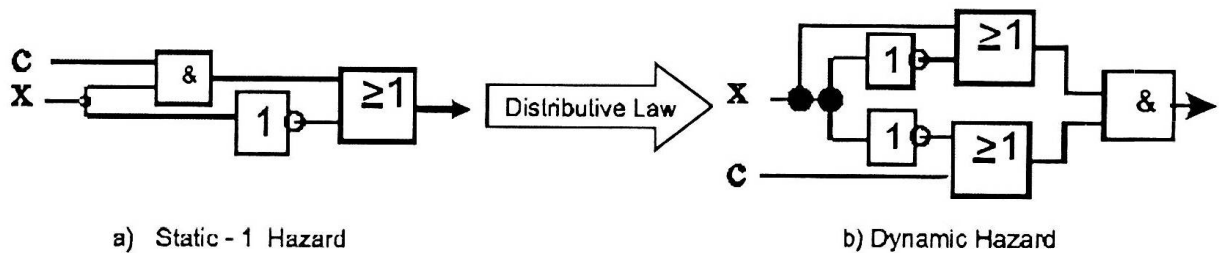a) Static - 1 Hazard   b) Dynamic Hazard

Fig. 1. The distributive Law changing static hazards to dynamic hazards.

From logic schema 1b and the circuit function (2) we have not only static -1 hazard but also a dynamic hazard.

- with c = 1, $y = (1 + \bar{x})(x + \bar{x}) = x + \bar{x}$ ⇒ hazard – 1 static.

- with c = 0, $y = (0 + x)(x + \bar{x}) = \bar{x}(x + \bar{x})$ ⇒ dynamic hazard.

It is clear that a new dynamic hazard has arises from the given static hazard when Distributive Law is used.

Hence, it is necessary to establish a new algebra to solve hazard problem.

Some other authors such as Fantuazze [4] who had used nine-valued algebra to analyze logic system, John Knight with seven-valued algebra, or Levis [5], and Thomson [6] who also have mentioned this issue. Here we bring out a method to establish hazard algebra in order to detect as well as repair hazard errors (structure). Before giving definition of Hazard algebra, we should start with Boolean algebra.

According to Boolean algebra's definition [4] we have:

$$A := (X, \blacktriangle, \blacktriangledown) \qquad (3)$$

Whereas ▲ and ▼ are dyadic operation and X is a finite set. In other words

$$A := (X, \blacktriangle, \blacktriangledown) \to A := (X, \blacktriangle, \blacktriangledown, n_{\blacktriangle}, n_{\blacktriangledown}) \qquad (4)$$

Whereas "-" is a single operation, that performed monovalent by X set and having following relation:

$$x \blacktriangle \bar{x} = n_{\blacktriangle}, \quad x \blacktriangledown \bar{x} = n_{\blacktriangledown}, \quad \bar{\bar{x}} = x \qquad (5)$$

According to the above definition, the *negative operation* has been occurred and the negative operation itself shall be a basic factor to review the definition of hazard algebra as well as to

distinguish between Boolean and Hazard algebras. Note that under this definition *x and $\overline{x}$ are only one variable and* $\overline{x}$ *is a negative of x.*

On the other hand, we understand that hazard algebra is constructed based on the cause of hazard and cause of hazard is the time delay. Reasons of time delay are many, and main reason is caused by negative circuits. It is illustrated in Figure 2.

Fig. 2. Delay caused by the negator.

Based on the electric circuit we have the circuit equation as :

$$y = x \wedge N = x \wedge \overline{x} \qquad (6)$$

For negator 2n+1 , we may write $\overline{x}$ based on identical power law and $n \in N$ is a natural number. Note that $N = \overline{x}$ is generated by adding delay time of participated negative gate 2n+1 and AND - gate propagation delay time included.

In summary, when the input is attached to a negator, then x will be placed after the negator. Time delay presented via negator is $\overline{x}$ and that is one of the reasons causing hazard. Hence, *in hazard algebra x and $\overline{x}$ are two individual variables.* In Boolean algebra, under the above definition x and $\overline{x}$ are only one variable and $\overline{x}$ is a negative of x. This is the main difference between Boolean and hazard algebra: all laws related to the negation of one variable in Boolean algebra could not be applied in hazard algebra, Distributive law is one example.

If we construct a formal system and take time into consideration, it will be very complicated for actual use. Time, therefore, should also be formalized for convenient computation.

Herein, B = {0,1} value in Boolean algebra is extended with third values of τ^H and τ^L represents the running of temporary signal of logic gates. Temporary value of τ^H and τ^L is defined by the nodded up and down changes of input signals to gate and now outputs are still remained stable or either changed, short for appear error hazard ξ.

Definition 1: Function A(x) has a hazard error in variable x when x receives value x = τ^H or x = τ^L, if the result of function A(x) is $\xi(\tau^H, \tau^L)$.

$$\text{Hazard } (x,A) \equiv (\exists a \in \{\tau^H, \tau^L\} \quad \text{if } A[\, x =: a\,] = \xi) \qquad (7)$$

Definition 2: During the level change of variable x $\in \{\tau^H, \tau^L \}$, with any possible value of the independent variables $x_i \in \{0, 1 \} \neq x$, if the result of function A (x) $\neq \xi$, we say that function A(x) has no hazard.

$$\text{Safe}(x,A) \equiv (\, \forall a \in \{\tau H, \tau L\} \quad \text{then } A[x = a] \neq \xi) \qquad (8)$$

Based on these comments and rely on Boolean algebra_ the combine between ternary algebra and John Knight's recommendations [3] we will solve the formal definition issue of static and dynamic hazards by using octal algebra, called as the hazard algebra. Variable range and value are 8:

$$H = B \vee \{\tau H, \tau L, \xi, +, ., -, 0, 1\} \qquad (9)$$

whereas:

B ∈ B{0,1}, with 1(0) represent constant high(low) signal in the inactive time as in Boolean algebra

- 0,1 are intermediate elements (not integers) that related to "+" and "." operations of circuit algebra.
- $\tau^L \equiv$ represents falling signal
- $\tau^H \equiv$ represents rising signal
- ξ represents undefined signal which may cause hazard.
- Negative operation "-" isn't a single operation as in Boolean algebra_ but a two-status computation which helps make clear that x and $\overline{x}$ are independent variables
- The temporary values τ^H and τ^L are defined by the rising and falling of the signals of gate input.
- "▼" and "▲" are extended Boolean computations, so they are suitable in Boolean algebra computing laws for τ^H, τ^L, ξ, - , 0, 1.

Based on the above conceptions as in Boolean algebra we construct the basic laws of the hazard algebra:

- Commutative Laws:

$$\tau^H \blacktriangle \tau^L = \tau^L \blacktriangle \tau^H \qquad \tau^H \blacktriangledown \tau^L = \tau^L \blacktriangledown \tau^H \qquad (10)$$

- Operation with intermediate elements

$$0 \blacktriangle [\tau^H, \tau^L, \xi] = 0 \qquad 0 \blacktriangledown [\tau^H, \tau^L, \xi] = [\tau^H, \tau^L, \xi]$$
$$1 \blacktriangle [\tau^H, \tau^L, \xi] = [\tau^H, \tau^L, \xi] \qquad 1 \blacktriangledown [\tau^H, \tau^L, \xi] = 1 \qquad (11)$$

- Absorption laws

$$\tau^H \blacktriangle \tau^H = \tau^H \qquad \tau^L \blacktriangle \tau^L = \tau^L$$
$$\tau^H \blacktriangledown \tau^H = \tau^H \qquad \tau^L \blacktriangledown \tau^L = \tau^L \qquad (12)$$

- Hazard laws

$$\xi \blacktriangle x = \xi \ \text{ if } \ x \neq 0 \qquad \xi \blacktriangledown x = \xi \ \text{ if } \ x \neq 1$$
$$\tau^H \blacktriangle \tau^L = \xi \qquad \tau^H \blacktriangledown \tau^L = \xi \qquad (13)$$

- Negative property

$$\overline{\tau^H} = \tau^L \qquad \overline{\tau^L} = \tau^H \qquad (14)$$

In this case τ^H and τ^L are completely independent variables, not like x and $\overline{x}$ are only one variable.

Based on the above conceptions we construct the truth table for common characteristics of hazard algebra:

Table 1. The truth table of hazard algebra.

▼	0	1	τ^H	τ^L	ξ
0	0	1	τ^H	τ^L	ξ
1	1	1	1	1	1
τ^H	τ^H	1	τ^H	ξ	ξ
τ^L	τ^L	1	ξ	τ^L	ξ
ξ	ξ	1	ξ	ξ	ξ

▲	0	1	τ^H	τ^L	ξ
0	0	0	0	0	0
1	0	1	τ^H	τ^L	ξ
τ^H	0	τ^H	τ^H	ξ	ξ
τ^L	0	τ^L	ξ	τ^L	ξ
ξ	0	ξ	ξ	ξ	ξ

-	
0	1
1	0
τ^H	τ^L
τ^L	τ^H
ξ	ξ

With the base of definition 1, definition 2 and the truth table we can define static and dynamic hazard in hazard algebra.

Definition 3: Static - 0 hazard $\xi(0)$ is a status represented by an impulse signal with form ⎍ and defined by the product of τ^H and τ^L

$$\xi = \tau^H \blacktriangle \tau^L = \xi(0) \tag{15}$$

This definition is constructed on the basic of the typical electrical circuit of static – 0 hazard as shown in figure 3
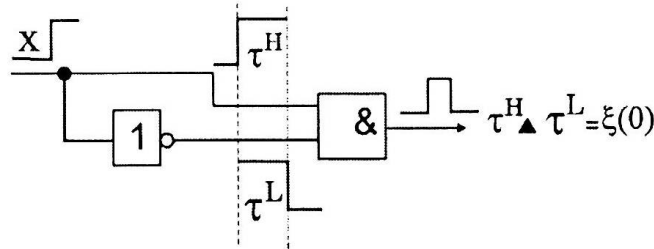

Fig. 3. Circuit showing static – 0 hazard.

Definition 4: Static - 1 hazard $\xi(1)$ is a status represented by an impulse signal ⎎ and defined by the sum of τ^H and τ^L

$$\xi = \tau^H \blacktriangledown \tau^L = \xi(1) \tag{16}$$

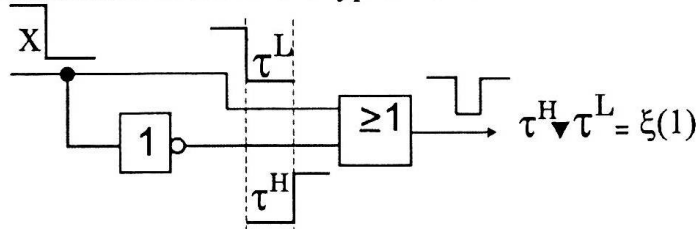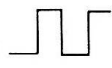This definition is constructed based on the typical electrical circuit of static hazard -1 (figure 4)

Fig. 4. Circuit showing static – 1 hazard.

Definition 5: Dynamic hazard is a status showing signals of logic circuit:

- If dynamic hazard is dependent on static – 0 hazard, the signal is impulsive ⎍⎍ and defined by :

$$\xi = \tau^H \blacktriangle \tau^L \blacktriangledown \tau^H = \xi(0) \blacktriangledown \tau^H = \xi_\theta(0) \tag{17}$$

This definition is constructed based on the typical electrical circuit of the dynamic hazard dependent on static – 0 hazard as shown in figure 5
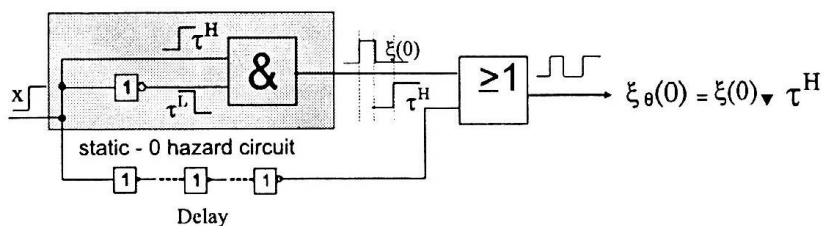

Fig. 5. Circuit showing errors of dynamic hazard dependent on static – 0 hazard.

- If dynamic hazard is dependent on static – 1 hazard, the signal is impulsive ⎏ and defined by:

$$\xi = (\tau^H \blacktriangledown \tau^L) \blacktriangle \tau^H = \xi(1) \blacktriangledown \tau^H = \xi_\theta(1) \qquad (18)$$

This definition is constructed based on the typical electrical circuit of the dynamic hazard dependent on static – 1 hazard as shown in figure 6.

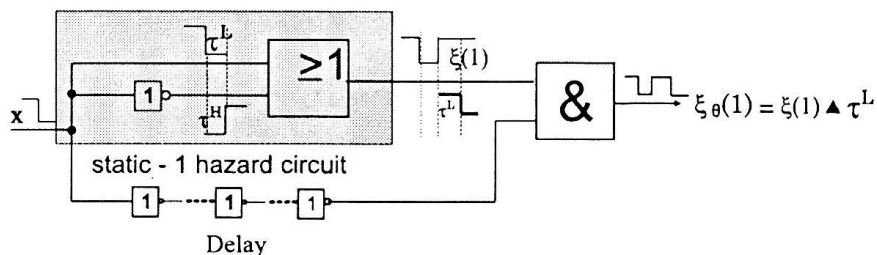If dynamic hazard is dependent on static – 1hazard, the expression will be:

Fig. 6. Circuit showing errors of dynamic hazard dependent on static – 1 hazard.

3. Locating and removing hazard

We can base on hazard algebra to locate hazard as well as remove it from the circuit system. With hazard algebra we can find all hazards in the circuit whose functions are not necessarily changed into minimal terms or maximum terms, that is product of sum forms or sum of product forms as in Boolean algebra.

Locating hazards using algebra follows these steps:

1. Assess the circuit equation. If the circuit equation is complicated, apply De Morgan Law to get the simplest circuit equation.

2. Assess the variables

- Find the variables that can cause hazards. They are those variables having both x and $\bar{x}$ form, in this case $x := \tau^H$ and $\bar{x} := \tau^L$ are independent.

- Give values $x_i | x_1, x_2,..., x_i | \in \{0,1\} \neq [x := \tau^H, \bar{x} := \tau^L]$:

$$x_1.x_2....x_i | x_1 := 0, \; x_2 := 0,...,x_i := 0 | +$$

$$x_1.x_2....x_i | x_1 := 0, \; x_2 := 0,...,x_i := 1 | +$$

$$\cdot$$

$$\cdot$$

$$x_1.x_2....x_i | x_1 := 1, \; x_2 := 1,...,x_i := 1 |$$

After the variables x_i received value 0 or 1 we will get circuit equations either $\tau^H \blacktriangle \tau^L = \xi(0)$, that is, the circuit coefficient contain static – 0 hazard, or $\tau^H \blacktriangledown \tau^L = \xi(1)$, that is the circuit coefficient contain static – 1 hazard, or dynamic hazard $\tau^L \blacktriangle \tau^H \blacktriangledown \tau^H = \xi_\theta(0)$ and $(\tau^L \blacktriangledown \tau^H) \blacktriangle \tau^H = \xi_\theta(1)$, or no equations at all, that is the hazard free circuit.

It is necessary to locate hazard in a circuit system but it is also more difficult than to remove it. To remove hazard we can use Karnaugh board as in Boolean algebra.

4. Example

From the truth table and definitions of hazard now we can find and bring out methods to repair hazard errors (structure). Study this simple example:

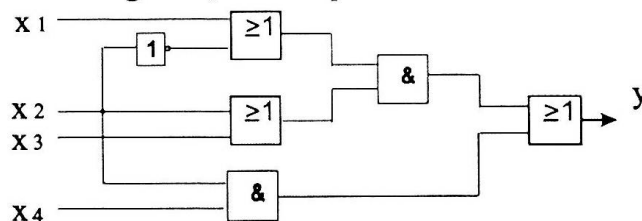Given an electrical circuit as in Figure 7, find all possible hazards in the circuit.

Fig. 7. Example circuit.

From the diagram, we have the following function:

$$y = (x_1 + \overline{x_2})(x_3 + x_2) + x_4 x_2 \tag{19}$$

To find Hazard in the circuit, we do the following steps:

Assess the variables: See which variables have neither negation nor non-negation form. In this case x_1, x_3 and x_4. So, only x_2 needs testing. In accordance with negation property in hazard algebra, x_2 and $\overline{x_2}$ are independent variables, with $x_2 \equiv \tau^H$ and $\overline{x_2} \equiv \tau^L$. From this remark function (19) is expressed like this:

$$y = (x_1 + \tau^L)(x_3 + \tau^H) + x_4.\tau^H \tag{20}$$

According to the defenition 2, to see if the circuit function has hazards or not we have to study the variable $x_i \in (0,1) \neq x_2$, that is, give x_i values 0 and 1 to investigate hazard corresponding x_2. In this case $x_i := (x_1, x_3, x_4)$, so there will be $2^n = 2^3 = 8$ cases to examine, as shown in table 2:

Now we examine individual cases shown in table 2:

Table 2. Cases to be tested to find hazard errors in a circuit function (19)

$$x_1.x_2 x_3 x_4 \begin{Vmatrix} x_1 := 0, \; x_3 := 0, \; x_4 := 0 \\ x_1 := 0, \; x_3 := 0, \; x_4 := 1 \\ x_1 := 0, \; x_3 := 1, \; x_4 := 0 \\ x_1 := 0, \; x_3 := 1, \; x_4 := 1 \\ x_1 := 1, \; x_3 := 0, \; x_4 := 0 \\ x_1 := 1, \; x_3 := 0, \; x_4 := 1 \\ x_1 := 1, \; x_3 := 0, \; x_4 := 1 \\ x_1 := 1, \; x_3 := 1, \; x_4 := 0 \\ x_1 := 1, \; x_3 := 1, \; x_4 := 1 \end{Vmatrix}$$

$1. x_1, x_2, x_3, x_4 := \left|0, x_2, 0, 0\right| \rightarrow$

$$y = (x_1 + \overline{x_2})(x_3 + x_2) + x_4 x_2 \text{ with} \left|(x_1, x_3, x_4) := \{0, 0, 0\}, x_2 \in \{\tau^H, \tau^L\}\right|$$

$$y = (0 \; \blacktriangledown \; \tau^L)(0 \; \blacktriangledown \; \tau^H) + 0 \blacktriangle \tau^H = \tau^L \blacktriangle \tau^H + 0 = \tau^L \blacktriangle \tau^H = \xi(0)$$

$\Rightarrow$ so according to the definition 3 with $x_1, x_2, x_3, x_4 := (0, x_2, 0, 0)$ circuit function y has static - 0 hazard.

Similarly:

$2. x_1, x_2, x_3, x_4 := (0, x_2, 0, 1)$

$$y = (0 \; \blacktriangledown \; \tau^L)(0 \; \blacktriangledown \; \tau^H) \; \blacktriangledown \; 1 \blacktriangle \tau^H = \tau^L \blacktriangle \tau^H \; \blacktriangledown \; \tau^H = \xi_\theta(0)$$

$\Rightarrow$ so according to the definition 5 with $x_1, x_2, x_3, x_4 := (0, x_2, 0, 1)$ circuit function y has dynamic hazard dependent on static – 0 hazard

$3. x_1, x_2, x_3, x_4 := (0, x_2, 1, 0)$

$$y = (0 \; \blacktriangledown \; \tau^L)(1 \; \blacktriangledown \; \tau^H) \; \blacktriangledown \; 0 \blacktriangle \tau^H = \tau^L \blacktriangle 1 = \tau^L$$

$\Rightarrow$ With $x_1, x_2, x_3, x_4 := (0, x_2, 1, 0)$ circuit function is hazard free

$4. x_1, x_2, x_3, x_4 := (0, x_2, 1, 1)$

$$y = (0 \; \blacktriangledown \; \tau^L)(1 \; \blacktriangledown \; \tau^H) \; \blacktriangledown \; 1 \blacktriangle \tau^H = \tau^L \blacktriangle 1 \; \blacktriangledown \; 1 \blacktriangle \tau^H = \tau^L \; \blacktriangledown \; \tau^H = \xi(1)$$

$\Rightarrow$ so according to the definition 3 *with $x_1, x_2, x_3, x_4 := (0, x_2, 1, 1)$* circuit function y has static - 1 hazard.

$5. x_1, x_2, x_3, x_4 := (1, x_2, 0, 0)$

$$y = (1 \; \blacktriangledown \; \tau^L)(0 \; \blacktriangledown \; \tau^H) \; \blacktriangledown \; 0 \blacktriangle \tau^H = \tau^H \blacktriangle 1 = \tau^H$$

$\Rightarrow$ With $x_1, x_2, x_3, x_4 := (1, x_2, 0, 0)$ circuit function is hazard free.

$6. x_1, x_2, x_3, x_4 := (1, x_2, 0, 1)$

$$y = (1 \; \blacktriangledown \; \tau^L)(0 \; \blacktriangledown \; \tau^H) \; \blacktriangledown \; 1 \blacktriangle \tau^H = 1 \blacktriangle \tau^H \; \blacktriangledown \; \tau^H \blacktriangle 1 = \tau^H$$

$\Rightarrow$ With $x_1, x_2, x_3, x_4 := (1, x_2, 0, 1)$ circuit function is hazard free.

$7. x_1, x_2, x_3, x_4 = (1, x_2, 1, 1)$

$$y = (1 \; \blacktriangledown \; \tau^L)(1 \; \blacktriangledown \; \tau^H) \; \blacktriangledown \; 1 \blacktriangle \tau^H = 1.1 \; \blacktriangledown \; \tau^H \blacktriangle 1 = 1 \; \blacktriangledown \; \tau^H = 1$$

$\Rightarrow$ With $x_1, x_2, x_3, x_4 := (1, x_2, 1, 1)$ circuit function is hazard free.

$8. x_1, x_2, x_3, x_4 := (1, x_2, 1, 0)$

$$y = (1 \; \blacktriangledown \; \tau^L)(1 \; \blacktriangledown \; \tau^H) \; \blacktriangledown \; 0 \blacktriangle \tau^H = 1 \blacktriangle 1 \; \blacktriangledown \; \tau^H \blacktriangle 0 = 1 \blacktriangle 1 = 1$$

$\Rightarrow$ With $x_1, x_2, x_3, x_4 := (1, x_2, 1, 0)$ circuit circuit function is hazard free.

Therefore, we see that in the circuit function exist 3 kinds of hazard corresponding values of x_1, x_3, x_4, which are:

Static - 0 hazard $\xi(0)$ appears when	$x_1, x_3, x_4 = 0, 0, 0$
Dynamic hazard $\xi_\theta(0)$ appears when	$x_1, x_3, x_4 = 0, 0, 1$
Static - 1 hazard $\xi(1)$ appears when	$x_1, x_3, x_4 = 0, 1, 1$

Repairing hazard errors as above meets no difficulty if we use Karnaugh board to supply redundant terms corresponding each kind of hazard.

References

[1] Erik Meijer, Hazard Algebra and the disign of Asynchronous automata, *workshops in computing*, Springer, 1992.
[2] M. Paul, Chirlian, *Analysis and Design of Digital Circuits and Coputer Systems*, 1976.
[3[John Knight, Gliches and Hazard in Digital Circuits, *Eletronics Department, Carleton University*, Carleton, 2003.
[4[G. Scarbata, Synthesis und Analysis Digital Circuits (Synthese und Analyse Digitaler Schaltungen", Oldenbourg, 2000.