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Determining thermal expansion coefficients of three-phase fiber composite material reinforced by spherical particles

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Abstract. Thermal expansion property of three-phase fiber composite material reinforced by spherical particles is one of important properties of this material. In this paper, we would like to propose a way in order to determine thermal expansion coefficients of three-phase composite reinforced by fibres and spherical particles.

Keywords: thermal expansion coefficients, three-phase composite material, aligned fibres, spherical particles, effective matrix phase.

1. Introduction

Composite material is commonly used in modern structures by more advanced advantages than other types of composite material [1]. One of investigated materials is three-phase fiber composite material reinforced by spherical particles. In it, the fibre phase is taken to compose of a number of long circular cylinders embedded into a continous matrix phase. The third phase is the particle phase which is assumed by means of isotropic homogeneous elastic spheres of equal radii and embedded into the matrix phase of this composite material.

For three-phase composite material reinforced by fibres and spherical particles, there are many relative problems necessary to solve. Algorithm determining technique modulus of three-phase fiber composite material reinforced by spherical particles is presented by [2]. Authors in [3] have brought out the expression determining Young modulus E_{11}^{*} of three-phase composite material of aligned fibres and spherical particles. In the paper, we only force to investigate the thermal expansion behaviour of composite because it is one of very important specificity necessary to consider when investigating every material. Assumption is that phases of three-phase composite material reinforced by fibres and spherical particles consist of the fibre, matrix, particle phase having elastic specificities $E_i, \upsilon_i, \alpha_i$ as well as volume fractions ξ_i for $i = \overline{1,3}$, respectively.

Problem set up is determining thermal expansion coefficients of three-phase fiber composite material reinforced by spherical particles through technique parameters of constituent materials, or bringing out the expression of $\hat{\alpha}\left(E_i, \upsilon_i, \alpha_i, \xi_i\right)$ as a function of elastic specificities E_i, υ_i, constituent thermal expansion coefficients α_i, constituent volume fractions of the fibre and particle phase ξ_1, ξ_3.

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Main idea for solving the three-phase problem is that we convert it into two two - phase problems, then combine them in order to give final results. Firstly, we combine original matrix phase with particle phase in order to creat a new matrix phase, called effective matrix phase. In fact, this effective matrix phase is assumed as a spherical particle - reinforced composite material. After that, we seek the solution for case where this material is made of the effective matrix phase and fibre phase embedded into that.

2. Determining thermal expansion coefficients of three-phase fiber composite material reinforced by spherical particles

2.1. Thermal expansion coefficient of the effective matrix phase

By composite sphere model, using theory of thermoelasticity [4] and method of volume approximation [5], authors in [6] have brought out the expression determining thermal expansion coefficient of composite material of spherical particles as the following

$$\overline{\alpha_2} = \alpha_2 + (\alpha_3 - \alpha_2)\frac{K_3(3K_2 + 4G_2)\xi_3}{K_2(3K_3 + 4G_2) + 4(K_3 - K_2)G_2\xi_3}, \tag{1}$$

where: α_2, α_3 : elastic thermal expansion coefficients of matrix and particle phase.

K_2, K_3 : bulk moduli of matrix and particle phase.

$G_2 = \mu_2$: shear modulus of matrix phase.

ξ_3 : volume fraction of particle phase.

$\overline{\alpha_2}$: thermal expansion coefficient of the effective matrix phase.

2.2. Thermal expansion coefficients of two – phase composite material reinforced by fibres

Continuing the way in section 2.1 but applying composite cylinder model, authors in [7] have brought out expressions determining thermal expansion coefficients of this type of material. Specifically, they have been brought out as the following

$$\alpha_t^* = \frac{\alpha_2(1-\xi_1)K_2(k_1+\mu_2) + \alpha_1 K_1(k_2+\mu_2)\xi_1}{(1-\xi_1)(k_2-k_1)\mu_2 + (k_2+\mu_2)k_1}\frac{k_t^*}{\left(k_t^* - \frac{\mu_a^*}{3}\right)} \tag{2}$$

$$\alpha_a^* = \frac{\mu_a^*}{(k_t^* - \mu_a^*)E_a^*}\frac{1}{(1-\xi_1)(k_2-k_1)\mu_2 + (k_2+\mu_2)k_1}\{\alpha_2(1-\xi_1)K_2[\lambda_2(k_1+\mu_2) + \xi_1(\lambda_1-\lambda_2)\mu_2] + \alpha_1\xi_1 K_1[\lambda_1 k_2 + \lambda_2\mu_2 + \xi_1(\lambda_1-\lambda_2)\mu_2]\} \tag{3}$$

where:

α_1, α_2 : elastic thermal expansion coefficients of fibre and matrix phase.

k_1, k_2 : plane strain bulk moduli of fibre and matrix phase.

K_1, K_2 : bulk moduli of fibre and matrix phase.

λ_1, λ_2: Lame's ratio of fibre and matrix phase.
μ_1, μ_2: shear moduli of fibre and matrix phase.
υ_1, υ_2: Poisson's ratio of fibre and matrix phase.
k_t^*: plane strain bulk modulus of composite material of aligned fibres.
μ_a^*: shear modulus of composite material of aligned fibres.
E_a^*: Young's modulus of composite material of aligned fibres.
ξ_1: volume fraction of fibre phase.
α_t^*: transverse linear thermal expansion coefficient of two - phase composite material reinforced fibres.
α_a^*: axial linear thermal expansion coefficient of two - phase composite material reinforced fibres.

Moreover, according to [8], we have

$$k_i = K_i + \mu_i/3 = \lambda_i + \mu_i \left(i = \overline{1,2}\right) \tag{4}$$

$$k_t^* = \lambda + \mu = K_{23} = K_2 + \frac{\mu_2}{3} + \frac{\xi_1}{\dfrac{1}{\left[K_1 - K_2 + \frac{1}{3}(\mu_1 - \mu_2)\right]} + \dfrac{1-\xi_1}{(\lambda_2 + \frac{4}{3}\mu_2)}} \tag{5}$$

$$\mu_a^* = \mu = \mu_2 \frac{\mu_1(1+\xi_1) + \mu_2(1-\xi_1)}{\mu_1(1-\xi_1) + \mu_2(1+\xi_1)} \tag{6}$$

$$E_a^* = E_{11} = \xi_1 E_1 + (1-\xi_1)E_2 + \frac{4\xi_1(1-\xi_1)(\upsilon_1 - \upsilon_2)^2 \mu_2}{\dfrac{(1-\xi_1)\mu_2}{K_1 + \frac{\mu_1}{3}} + \dfrac{\xi_1 \mu_2}{K_2 + \frac{\mu_2}{3}} + 1} \tag{7}$$

2.3. Thermal expansion coefficients of three-phase fiber composite material reinforced by spherical particles

By combining two problems above, we'd like to propose a way in order to bring out expressions of transverse and axial thermal expansion coefficients of three-phase fiber composite material reinforced by spherical particles. In it, we note the variance of elastic specificities of the effective matrix phase replacing the old matrix phase in expressions (2) and (3). Expressions of tranverse $\hat{\alpha}_t$ and axial $\hat{\alpha}_a$ thermal expansion coefficients of this type of material are determined as the following

$$\hat{\alpha}_t = \frac{\overline{\alpha_2}(1-\xi_1)\overline{K_2}\left(k_1 + \overline{\mu_2}\right) + \alpha_1 K_1\left(\overline{k_2} + \overline{\mu_2}\right)\xi_1}{(1-\xi_1)\left(\overline{k_2} - k_1\right)\overline{\mu_2} + \left(\overline{k_2} + \overline{\mu_2}\right)k_1} \frac{\overline{k_t^*}}{\left(\overline{k_t^*} - \dfrac{\overline{\mu_a^*}}{3}\right)} \tag{8}$$

$$\hat{\alpha}_a = \frac{\overline{\mu_a^*}}{\left(\overline{k_t^*} - \overline{\mu_a^*}\right)\overline{E_a^*}} \frac{1}{(1-\xi_1)\left(\overline{k_2} - k_1\right)\overline{\mu_2} + \left(\overline{k_2} + \overline{\mu_2}\right)k_1} \left\{\overline{\alpha_2}(1-\xi_1)\overline{K_2}\left[\overline{\lambda_2}\left(k_1 + \overline{\mu_2}\right) + \right.\right.$$

$$\xi_1\left(\lambda_1-\overline{\lambda_2}\right)\overline{\mu_2}\Big]+\alpha_1\xi_1 K_1\Big[\lambda_1\overline{k_2}+\overline{\lambda_2}\overline{\mu_2}+\xi_1\left(\lambda_1-\overline{\lambda_2}\right)\overline{\mu_2}\Big]\Big\} \quad (9)$$

In expressions (8) and (9), elastic specificities of the effective matrix phase (we consider it as a composite material of spherical particles) were given by Hasin and Christensen in [8] as the following

$$\overline{G_2}=G_2\left[1-\frac{15(1-\upsilon_2)\left(1-\frac{G_3}{G_2}\right)\xi_3}{7-5\upsilon_2+(8-10\upsilon_2)\frac{G_3}{G_2}}\right] \quad (10)$$

$$\overline{K_2}=K_2+\frac{(K_3-K_2)\xi_3}{1+(K_3-K_2)\left(K_2+\frac{4G_2}{3}\right)^{-1}} \quad (11)$$

According to [6], we have

$$\overline{\alpha_2}=\alpha_2+(\alpha_3-\alpha_2)\frac{K_3(3K_2+4G_2)\xi_3}{K_2(3K_3+4G_2)+4(K_3-K_2)G_2\xi_3} \quad (12)$$

In the other hand

$$\overline{k_t^*}=\overline{K_2}+\frac{\overline{\mu_2}}{3}+\frac{\xi_1}{\frac{1}{\left[K_1-\overline{K_2}+\frac{1}{3}(\mu_1-\overline{\mu_2})\right]}+\frac{1-\xi_1}{(\overline{\lambda_2}+\frac{4}{3}\overline{\mu_2})}} \quad (13)$$

$$\overline{\mu_a^*}=\overline{\mu_2}\frac{\mu_1(1+\xi_1)+\overline{\mu_2}(1-\xi_1)}{\mu_1(1-\xi_1)+\overline{\mu_2}(1+\xi_1)} \quad (14)$$

$$\overline{E_a^*}=\xi_1E_1+(1-\xi_1)\overline{E_2}+\frac{4\xi_1(1-\xi_1)(\upsilon_1-\overline{\upsilon_2})^2\overline{\mu_2}}{\frac{(1-\xi_1)\overline{\mu_2}}{K_1+\frac{\mu_1}{3}}+\frac{\xi_1\overline{\mu_2}}{\overline{K_2}+\frac{\overline{\mu_2}}{3}}+1} \quad (15)$$

where

$\hat{\alpha}_a$: axial thermal expansion coefficient of three-phase fiber composite material reinforced by spherical particles.

$\hat{\alpha}_t$: transverse thermal expansion coefficient of three-phase fiber composite material reinforced by spherical particles.

Like this, (8) and (9) are expressions which determine thermal expansion coefficients of three-phase fiber composite material reinforced by spherical particles necessary to seek, in which thermal expansion coefficients of this material are functions of elastic specificities of constituents, thermal expansion coefficients of constituents, volume fractions of fibre and particle constituent.

3. Numerical example

For illustration, we give an example to calculate. Let composite material have elastic specificities deriving from [1,7] as the following

The glass fibre: $E_1 = 72.38 GPa$; $\upsilon_1 = 0.2$; $\alpha_1 = 5\times10^{-6}\ /°C$

The epoxy resin matrix: $E_2 = 2.75 GPa$; $\upsilon_2 = 0.35$; $\alpha_2 = 54\times10^{-6}\ /°C$

The glass particle: $E_3 = 740 GPa$; $\upsilon_3 = 0.21$; $\alpha_3 = 5.6\times10^{-6}\ /°C$

Case 1: Let sum of volume fractions of the fibre and particle phase be constant and equal to 0.6, or $\xi_1 + \xi_3 = 0.6$. Then, transverse $\hat{\alpha}_t$ and axial $\hat{\alpha}_a$ thermal expansion coefficients of three-phase fiber composite material reinforced by spherical particles are calculated according to expressions (8) and (9). So, we have data presented in table 1 as the following

Table 1. The variance of thermal expansion coefficients of three-phase composite material belonging to volume fractions of constituents

ξ_1	0.05	0.1	0.2	0.3	0.4	0.5	0.55
ξ_3	0.55	0.5	0.4	0.3	0.2	0.1	0.05
$\hat{\alpha}_t(10^{-5})$	2.206	2.303	2.447	2.520	2.518	2.438	2.366
$\hat{\alpha}_a(10^{-6})$	5.183	4.260	3.280	2.718	2.314	1.979	1.825

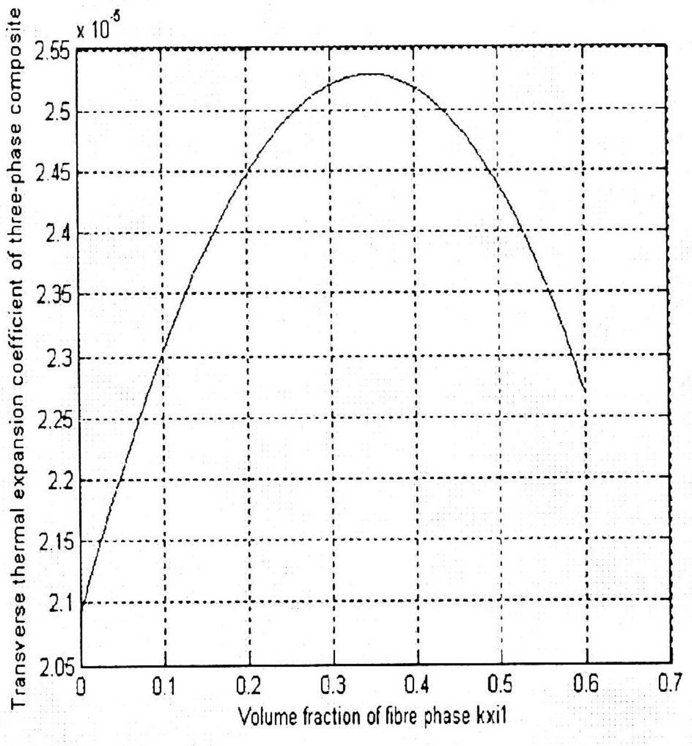

Fig. 1. Graph presenting the dependence of transverse thermal expansion coefficient $\hat{\alpha}_t$ on volume fractions of constituents.

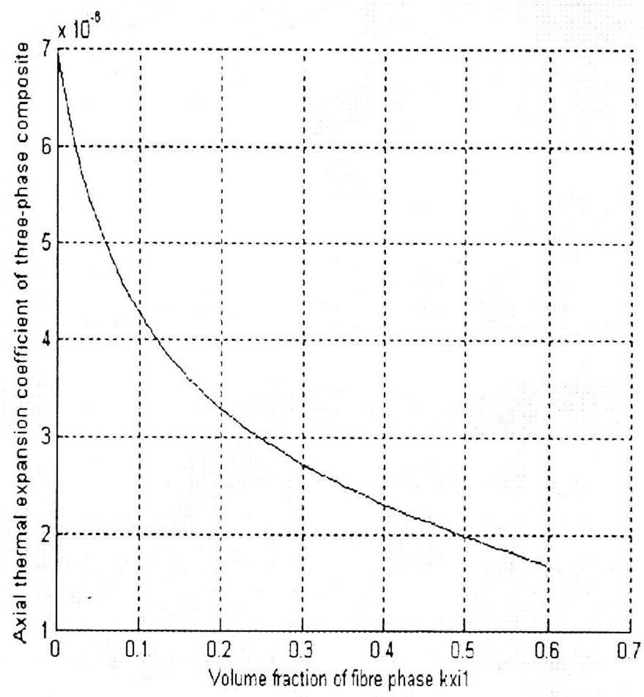

Fig. 2. Graph presenting the dependence of axial thermal expansion coefficient $\hat{\alpha}_a$ on volume fractions of constituents.

Through the detailed calculation and graphs in case 1, three-phase composite material reinforced by fibres and spherical particles is more preeminent than two - phase fiber composite material by means of reducing thermal expansion coefficients of three-phase composite material more than that of two - phase composite material in [7]. So, embedding spherical inclusions into continous matrix phase of two - phase fiber composite material is necessary and meaningful in fact. Besides, we can realize that for every given elastic specificity of constituents, we need to calculate volume fractions ξ_1 and ξ_3 in order to be suitable for requirement and purpose in fact of this type of composite material.

Case 2: Let volume fraction of the fibre phase ξ_1 increase from 0 to 0.6, volume fraction of the particle phase ξ_3 be constant and equal to 0.1. Similarly, we have data presented in table 2 as the following

Table 2. The variance of thermal expansion coefficients of three-phase composite material belonging to volume fraction of the fibre phase

ξ_1	0.05	0.1	0.2	0.3	0.4	0.5	0.55
ξ_3	0.1	0.1	0.1	0.1	0.1	0.1	0.1
$\hat{\alpha}_t(10^{-5})$	4.483	4.238	3.763	3.306	2.864	2.438	2.230
	0.803	0.553	0.356	0.272	0.226	0.198	0.188

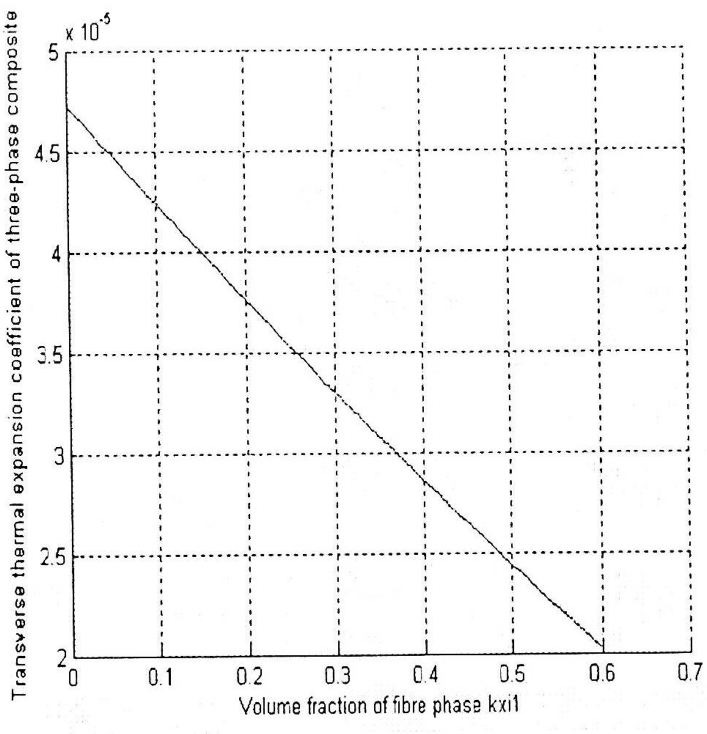

Fig. 3. Graph presenting the dependence of transverse thermal expansion coefficient $\hat{\alpha}_t$ on volume fraction of the fibre phase ξ_1 when volume fraction of the particle phase ξ_3 is constant.

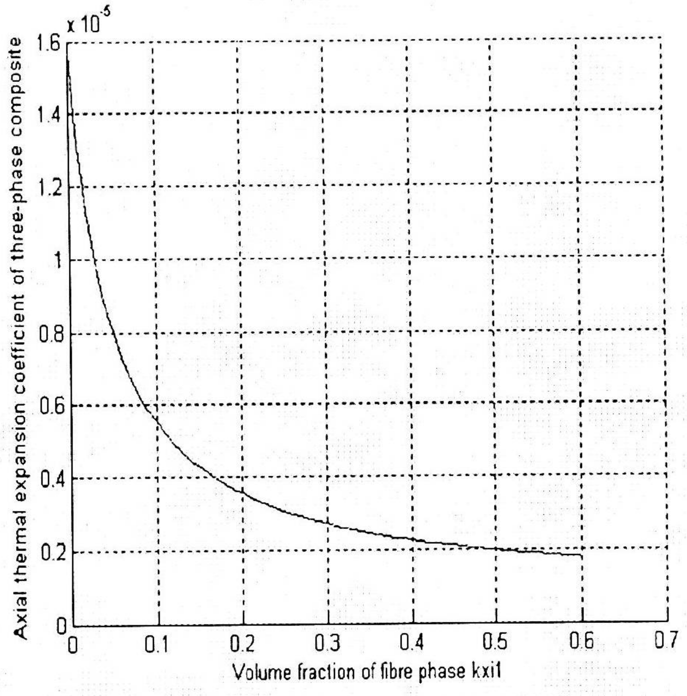

Fig. 4. Graph presenting the dependence of axial thermal expansion coefficient $\hat{\alpha}_a$ on volume fraction of the fibre phase ξ_1 when volume fraction of the particle phase ξ_3 is constant.

Case 3: Let volume fraction of the particle phase ξ_3 increase from 0 to 0.6, volume fraction of the fibre phase ξ_1 be constant and equal to 0.1. Similarly, we have data presented in table 3 as the following

Table 3. The variance of thermal expansion coefficients of three-phase composite material belonging to volume fraction of the particle phase

ξ_1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
ξ_3	0.05	0.1	0.2	0.3	0.4	0.5	0.55
$\hat{\alpha}_t(10^{-5})$	4.528	4.238	3.695	3.195	2.732	2.303	2.100
$\hat{\alpha}_a(10^{-6})$	5.542	5.535	5.390	5.108	4.724	4.266	4.016

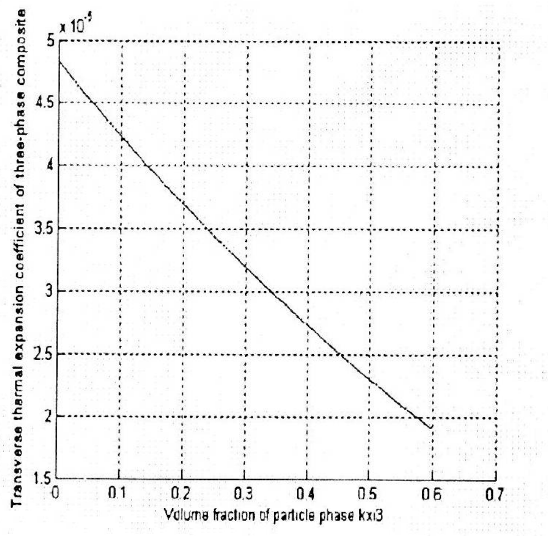

Fig. 5. Graph presenting the dependence of transverse thermal expansion coefficient $\hat{\alpha}_t$ on volume fraction of the particle phase ξ_3 when volume fraction of the fibre phase ξ_1 is constant.

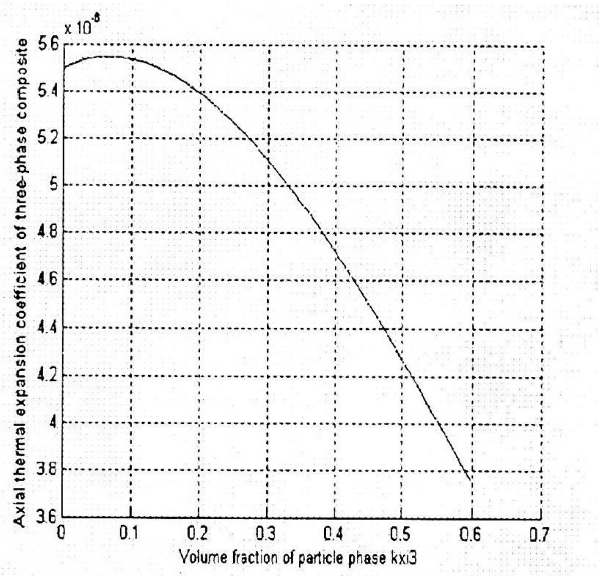

Fig. 6. Graph presenting the dependence of axial thermal expansion coefficient $\hat{\alpha}_a$ on volume fraction of the particle phase ξ_3 when volume fraction of the fibre phase ξ_1 is constant.

In case 2, letting volume fraction of the particle phase be constant and increasing step by step volume fraction of the fibre phase will reduce thermal expansion coefficients of three-phase composite material. This resembles case 3 when letting volume fraction of the fibre phase be constant and increasing step by step volume fraction of the particle phase. When comparing these two cases, we realize that the result of case 3 is better. It means that the more volume fraction of the particle phase we increase, the more thermal expansion coefficients of three-phase composite material reduce.

4. Conclusions

Based on the idea solving the problem of three-phase composite material through problems of known two - phase composite material, this paper has brought out a way in order to determine expressions of thermal expansion coefficients of three-phase fiber composite material reinforced by spherical particles as functions of elastic specificities of constituents, thermal expansion coefficients of constituents, volume fractions of fibre and particle constituent.

For composite material of epoxy resin matrix and glass fibre, three-phase composite is more heatproof than two - phase composite. Calculated results of this material also indicate that when increasing volume fraction of glass particle phase, three-phase composite is more heatproof than itself when increasing volume fraction of glass fibre phase. This is meaningful in manufacturing materials impervious to heat and reducing the prices of products (because the cost of particles is cheaper than that of fibres...).

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