On equations of motion, boundary conditions and conserved energy-momentum of the rigid string

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Abstract. The correct forms of the equations of motion, of the boundary conditions and of the reconserved energy - momentum for the a classical rigid string are given. Certain consequences of the equations of motion are presented. We also point out that in Hamilton description of the rigid string the usual time evolution equation $\dot{F} = \{F, H\}$ is modified by some boundary terms

1. Introduction

The modified string model, so-called rigid or smooth strings, has been discussed [1 − 11]. The action functional in this model contains in addition to the usual Nambu-Gato the term proportional to the external curvature of the world sheet of the string.These models are expected to have many different applications in string interpretation of QCD, in a statistical theory of random surfaces, in connection with two dimensional, quantized gravity [12].

Our main goal in this paper is to re-derive the classical equations of motion, boundary conditions and conserved energy - momentum of the rigid string, obtained by [4 − 6]. The first reason to discuss in detail such basis is that rigid model is an example of a Lagrangian field theory with higher order derivatives. In such case the seemingly standard derivations contain many interesting points which in our opinion, have not been sufficient emphasized. The second reason is that one can find in the literature many misleading or even erroneous statements concerning in equations of motion, the boundary conditions and the energy-momentum.

The plan of our paper is the following. In Section 2 we present the derivation of the Euler -Lagrange equations of motion, of the boundary conditions and of the conserved energy-momentum in the case of genetic Lagrangian with second order derivatives . In Section 3 we present the corresponding formulae in the case of rigid string, i. e. for the specific Lagrangian given at the beginning of Section 3. There we also derive some simple consequences of the equations of motion. In the Section 4 we point out the peculiar features of the Hamiltonian formalism appearing in the case of the open string.

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2. The Formalism

Let us suppose that the Lagrangian density L depends on the field function $x_\mu(\tau, \sigma)$ and on their first and second derivatives.

$$S = \int_\Omega d^2 u L(x_\mu, x_{\mu,i}, x_{\mu,ij}) = \int_{\tau_1}^{\tau_2} d\tau \int_{\sigma_1}^{\sigma_2} d\sigma L(x_\mu, x_{\mu,i}, x_{\mu,ij}). \tag{1}$$

For the partial derivatives we introduce the following notation:

$$x_{\mu,i} = \frac{\partial x_\mu}{\partial u^i}, \qquad x_{\mu,ij} = \frac{\partial x_\mu}{\partial u^i \partial u^j}; \qquad i,j = 1,2;$$

$$\dot{x}_\mu = \frac{\partial x_\mu}{\partial \tau}; \qquad x'_\mu = \frac{\partial x_\mu}{\partial \sigma}. \tag{2}$$

where $x_\mu = x_\mu(\tau, \sigma)$ are fields in the two -dimensional space-time $u^0 = \tau$; $u^1 = -\infty < \tau < +\infty$; $\mu = 0, 1, 2, ..., D - 1..$ The following formula for the full variation of the action S is given

$$\delta S = \int_\Omega \{\Lambda_\mu + \epsilon^{ij} \partial_i \Pi_j + \partial_0 \partial_1 Z\}, \tag{3}$$

where

$$\varepsilon^{i,j} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\varepsilon^{i,j} \partial_i \Pi_j = \partial_0 \Pi_j - \partial_1 \Pi_0 \tag{4}$$

$$\Lambda_\mu(\tau, \sigma) = \frac{\partial L}{\partial x_\mu} - \frac{\partial}{\partial \tau} \left[\frac{\partial L}{\partial \dot{x}_\mu} - \frac{\partial}{\partial \tau} \left(\frac{\partial L}{\partial \ddot{x}_\mu} \right) + \frac{\partial}{\partial \sigma} \left(\frac{\partial L}{\partial \dot{x}'_\mu} \right) \right] = \tag{5}$$

$$= -\frac{\partial}{\partial \sigma} \left[\frac{\partial L}{\partial x'_\mu} - \frac{\partial}{\partial \sigma} \left(\frac{\partial L}{\partial x''_\mu} \right) \right];$$

$$\Pi_0(\tau, \sigma) = \left[-\frac{\partial L}{\partial x^\mu_{,1}} + \partial_i \left(\frac{\partial L}{\partial x^\mu_{,0i}} \right) \right] \delta x^\mu - \frac{\partial L}{\partial x^\mu_{,11}} \delta x_{\mu,1}; \tag{6}$$

$$\Pi_1(\tau, \sigma) = \left[\frac{\partial L}{\partial x^\mu_{,0}} - \partial_i \left(\frac{\partial L}{\partial x^\mu_{,0i}} \right) \right] \delta x^\mu + \frac{\partial L}{\partial x^\mu_{,00}} \delta x_{\mu,00} \delta x_{\mu,0}; \tag{7}$$

$$Z(\tau, \sigma) = \frac{\partial L}{\partial x^\mu_{,01}} \delta x_\mu. \tag{8}$$

Using Stokes theorem we can write δS in the following form

$$\delta S = \int_\Omega d^2 u \Lambda_\mu \delta u^\mu + \int_{\delta\Omega} U_i du^i + [Z(\tau_2, \pi) - Z(\tau_2, 0) + Z(\tau_1, \pi) - Z(\tau_1, 0)], \tag{9}$$

where $\delta\Omega$ denotes the boundary of the rectangle Ω. The advantage of the form of the variation δS is that it involves the least possible number of derivatives of the variations δx_μ. The remaining derivatives of δx in formula (9) cannot be removed by any partial integrations. The Z-terms in formula (9) for δS can be regarded as a contribution from the corner points of the rectangle R. For the closed string they cancel each other. However, for the open string they give a nonvanishing contribution if the Lagrangian L depends on $\frac{\partial^2 x_\mu}{\partial u_0 \partial u_1}$

The Z-terms have appeared because in this case of open rigid string we encounter a coincidence of the following two mathematical obstacles: the presence of the high derivatives in the Lagrangian , and the fact that the field $x_\mu(\tau, \sigma)$ is defined on the finite strip $0 \preceq \sigma \preceq \pi$, and $-\infty < \tau < \infty$, which has boundaries. The classical equations of the motion and the boundary conditions for the open rigid string follow from the requirement

$$\delta S = 0 \Leftrightarrow \Lambda_\mu(\tau, \sigma) = 0, \tag{10}$$

for the any variation δx_μ obeying following conditions

$$\delta x_\mu(\tau, \sigma) = 0, \tau = \tau_1, \tau_2; \sigma \in [0, \pi]; \tag{11a}$$

$$\delta x_{\mu,0}(\tau, \sigma) = 0, \tau = \tau_1, \tau_2; \sigma \in [0, \pi]. \tag{11b}$$

This conditions $(11b)$ is due to the fact that Lagrangian contains the second order derivatives with respect to the evolution τ. From $(2.11b)$ it follows that

$$\delta x_{\mu,1}(\tau, \sigma) = 0, for \qquad \tau = \tau_1, \tau_2; \sigma \in [0, \pi]. \tag{12}$$

On the other hand, neither δx_μ nor $\delta x_{\mu,1}$, are fixed for $\sigma = 0, \quad \sigma = \pi, \quad \tau \in (\tau_1, \tau_2)$. Now, it is clear that the requirement (10) implies the following equations of motion

$$\Lambda_\mu(\tau, \sigma) = 0, \tag{13}$$

and the following boundary conditions

$$B_\mu(\tau, \sigma = 0) = 0, B_\mu(\tau, \sigma = \pi) = 0, \tag{14}$$

$$C_\mu(\tau, \sigma = 0) = 0, C_\mu(\tau, \sigma = \pi) = 0, \tag{15}$$

where

$$B_\mu(\tau, \sigma) = \frac{\partial L}{\partial x_{\mu,1}} + \partial_i \left(\frac{\partial L}{\partial x_{\mu,1i}} \right), \tag{16}$$

$$B_\mu(\tau, \sigma) = \frac{\partial L}{\partial x_{\mu,1}} + \partial_i \left(\frac{\partial L}{\partial x_{\mu,1i}} \right), \tag{16}$$

and

$$C_\mu(\tau, \sigma) = \left(\frac{\partial l}{\partial x_{\mu,11}} \right). \tag{17}$$

In the case of the closed string $\delta_\mu(\tau, \sigma)$ obey the conditions $\delta_{\mu\mu}(\tau, \sigma = 0) = \delta_\mu(\tau, \sigma = \sigma\pi)$. Then, the variation principle implies only the equations motion $[13]$.

Now, let us pass to the derivation of the energy-momentum four-vector corresponding to the action . We again use the formula

$$\delta x_\mu = \epsilon_\mu = const. \tag{18}$$

Assume the Lagrangian is invariant, $\frac{\partial L}{\partial x_\mu} = 0$ and $\delta S = 0$ with the conditions $x_\mu(\tau, \sigma)$ obeys the equations of motion (13), and conditions (14) and (15). From (9) we have

$$P_\mu = \int_0^\pi d\sigma \left[-\frac{\partial L}{\partial x_{\mu,0}} + \partial_i \left(\frac{\partial L}{\partial x_{\mu,0i}} \right) \right] - \frac{\partial L}{\partial x_{\mu,01}} \bigg|_{\sigma=\pi} + \frac{\partial L}{\partial x_{\mu,1}} \bigg|_{\sigma=0}, \tag{19}$$

is constant during the τ-evolution .We notice that the two last terms on the right hand side of formula (19) cancel with the term $\int_0^\pi d\sigma \partial_1 (\frac{\partial L}{\partial_{\mu,01}})$. Therefore the final formula for the energy- momentum four -vector has form

$$P_\mu = \int_0^\pi d\sigma p_\mu; p_\mu = \left[-\frac{\partial L}{\partial x_{\mu,0}} + \partial_i \left(\frac{\partial L}{\partial x_{\mu,0i}} \right) \right]. \tag{20}$$

where

$$\partial_1 B + \partial_0 p_\mu = 0, \tag{21}$$

Integrating formula (21) over σ, and taking into account boundary conditions (14) we again obtain that

$$\partial_0 p_\mu = 0. \tag{22}$$

This is a check that our formulae (21) and (22) are correct. By a similar reasoning we obtain a conserved angular-momentum tensor $M_{\mu\nu}$ for rigid string. The only difference is that now

$$\delta x_\mu = \omega_{\mu\nu} x_\nu. \tag{23}$$

instead of formula (18). Here $\omega_{\mu\nu} = -\omega_{\nu\mu}$ are the six infinitesimal parameters of Lorentz transformations. After a partial integration, contribution of the Z-terms is canceled by each other.The final formula for $M_{\mu\nu}$ has the following form.

$$M_{\mu\nu} = \int_0^\pi d\sigma (x_\mu p_\nu - x_\nu p_\mu) + \int_0^\pi d\sigma \left(\frac{\partial L}{\partial x_{\mu,0i}} x_{\nu,i} - \frac{\partial L}{\partial x_{\nu,0i}} x_{\mu,i} \right), \tag{24}$$

where p_μ is the momentum density given by formula (20)

3. The Rigid String

For the rigid string the Lagrangian has the form

$$L = \sqrt{-g}(-\gamma + \alpha \Box x^\mu \Box x_\mu), \tag{25}$$

$$\Box x_\mu = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial u^i} \left(\sqrt{-g} g^{ij} \frac{\partial x_\mu}{\partial u^j} \right) = g^{ij} \frac{\partial^2 x^\mu}{\partial u^i \partial u^j} + \frac{1}{\sqrt{-g}} \frac{\partial}{\partial u^i} \left(\sqrt{-g} g^{ij} \right) \frac{\partial x}{\partial u^j} = \tag{26}$$

$$= \frac{1}{\sqrt{-g}} \left\{ \frac{\partial}{\partial \tau} \left[\frac{(\dot{x}x')x'^\mu - x'^2 x''}{\sqrt{-g}} \right] + \frac{\partial}{\partial \tau} \left[\frac{(\dot{x}x')x'' - \dot{x}'^\mu - x'^2 x'^\mu}{\sqrt{-g}} \right] \right\},$$

where $\gamma > 0$ is the constant with dimension of the squared mass, $\alpha \neq 0$ is the dimensionless constant which specifies the rigidity of the string world sheet. $\Box$ is the Laplace-Beltrami operator for the metric tensor $g_{ij}.g = det \parallel g_{ij} \parallel$.In the Minkowski space-time the metric with signature $\eta_{\mu\nu} = diag(+1, -1, -1, -1, ...)$ For $\alpha = 0$ we would obtain the usual Nambu-Gato string.In the case of Lagrangian (25) equations of motion have the form

$$(\gamma - \alpha \Box x^\mu \Box x_\mu) \Box x_\mu + 2\alpha \left[\Box(\Box x_\mu) - g^{ij} x_{,i}^\nu x_{\mu,j} \Box(\Box x_\nu) \right] - 4\alpha g^{ij} g^{kz} (\Box x_\nu)_{,j} x_{,k}^\nu \nabla_i x_{\mu,z} = 0 \tag{27}$$

where $\nabla_a x$

$$\nabla_i x_{\mu,z} = x_{\nu,ij} \left(\eta_{\mu\nu} - g^{kz} x_{\nu,z} x_{\mu,k} \right), \tag{28}$$

Equation (27) are very complicated . They contain fourth -order partial derivatives and nonlinearities . For $\alpha = 0$ they reduce to equations of motion for the Nambu-Gato string.

$$\Box x_\mu = 0. \tag{29}$$

Equations (29) are also nonlinear . However, it is a well-known fact that they can be locally linearized by choosing so called orthonormal coordinates on the world sheet with following conditions

$$x\dot{x}' = 0, \dot{x}^2 > 0, x'^2 < 0, \dot{x}^2 = -x'^2 \tag{30}$$

$$\Box x_\mu = 0 \Leftrightarrow \left(\partial_0^2 - \partial_1^2\right) x_\mu(\tau, \sigma) = 0. \tag{31}$$

The functions $B_\mu(\tau, \sigma), C_\mu(\tau, \sigma)$ which appear in boundary conditions in the case of Lagrangian (25) have following form

$$B_\mu(\tau, \sigma) = \sqrt{-g}(\gamma - \alpha\Box x^\mu\Box x_\mu)g^{1i}x_{\mu,i} + 2\alpha\sqrt{-g}g^{jk}x_{,i}^\lambda x_{\lambda,jk}\Box x_\mu + eqno(3.8)$$

$$+4\alpha\sqrt{-g}\Box x_\sigma x_{,ij}^\sigma g^{1j}G^{ik}x_{\mu,k} + 2\alpha\partial_0\left(\sqrt{-g}g^{01}\Box x_\mu\right) + 2\alpha\partial_j\left(\sqrt{-g}g^{1j}\Box x_\mu\right);$$

$$C_\mu(\tau, \sigma) = 2\alpha\sqrt{-g}g^{11}\Box x_\mu. \tag{32}$$

The energy-momentum density p_μ has the following form

$$P_\mu = \sqrt{-g}g^{0j}(\gamma - \alpha\Box x^\sigma\Box x_\sigma)x_{\mu,j} + 2\alpha\partial_0\left(\sqrt{-g}g^{00}\Box x_\mu\right) + \tag{33}$$

$$+2\alpha\sqrt{-g}g^{0i}g^{jk}\left(2\Box x_\sigma x_{ij}^\sigma x_{\mu,k} + x_{,jk}^\lambda x_{\lambda,i}\Box x_\mu\right)$$

In the orthonormal coordinates this formula is simplified to

$$p_\mu = \dot{x}_\mu\left[\gamma(NG) - \alpha\frac{\bar\partial^2 x^\mu\bar\partial^2 x_\mu}{\left(\dot{x}^2\right)^2} + 4\alpha\frac{\bar\partial^2 x_\sigma\ddot{x}^\sigma}{\left(\dot{x}^2\right)^2}\right] + 2\alpha\partial_0\left(\frac{1}{\dot{x}^2}\bar\partial^2 x_\mu\right) - \frac{4\alpha}{\left(\dot{x}^2\right)}\bar\partial^2 x_\sigma\dot{x}'x_\mu'. \tag{34}$$

In the Nambu-Gato $\alpha = 0$

$$p_\mu = \gamma\dot{x}_\mu. \tag{35}$$

Investigations of the rigid string model are not easy to carry out because equations of motion of the classical string and the corresponding canonical structure are rather complicated.

4. Hamilton description of the open rigid string

Discussion of Hamilton formulation of dynamics of systems with reparametrization invariance, which is a special case of local gauge invariance, is complicated by a problem of constraints. In order to avoid this obstacle we shall discuss the Hamilton description of the rigid string in the physical gauge, which is defined by the requirement that the evolution parameter τ is equal to the physical time x_0

$$x_0(\tau, \sigma) = \tau. \tag{36}$$

In this gauge, the independent dynamical variables are $x_i(t, \sigma), i = 1, 2, 3 \, t = x_0$. Variations are now replaced by

$$\vec{x}(\tau, \sigma) \to \vec{x}(\tau, \sigma) + \delta\vec{x}(\tau, \sigma), \tag{37}$$

where $\vec{x} = x_i$. The considerations of section 2 can be repeated with the only difference that the index $\mu = 0, 1, 2, 3$ is now replaced by the index $i = 1, 2, 3$. In particular,the equations of motion (13) and the boundary conditions have the form given by formula $(14 - 15)$ with the replacement $\mu \to i$. From the invariance under the spatial translations

$$\delta\vec{x} = \vec{\epsilon} = const, \tag{38}$$

$$S = \int_{t_1}^{t_2} dt \int_0^\pi d\sigma L \left(\vec{x}, \dot{\vec{x}}, \vec{x}', \ddot{\vec{x}}, \dot{\vec{x}}', \vec{x}'' \right). \tag{39}$$

The result is

$$P_0 = \int_0^\pi d\sigma \left\{ \ddot{\vec{x}} \frac{\partial L}{\partial \ddot{\vec{x}}} + \dot{\vec{x}} \left[\frac{\partial L}{\partial \dot{\vec{x}}} - \partial_0 \left(\frac{\partial L}{\partial \ddot{\vec{x}}} \right) \right] + \dot{\vec{x}} \frac{\partial L}{\partial \dot{\vec{x}}} - L \right\}, \tag{40}$$

In order to obtain this formula , the equations of motion and and the boundary conditions have been used. Also some partial integrations over σ have been performed

In the case of Lagrangian L with second order derivatives there are two independent "configuration space-type" variables

$$q_{1a} = x_a, q_2 = \dot{x}_i, \tag{41}$$

and the corresponding canonical momenta

$$p_{1a} = -\frac{\partial L}{\partial q_{2a}} + \frac{\partial}{\partial \tau} \left(\frac{\partial L}{\partial \dot{q}_2} \right) + \frac{\partial}{\partial \sigma} \left(\frac{\partial L}{\partial q'_{2a}} \right). \tag{42}$$

The Lagrangian L is regarded as a function of variables $q_1, \dot{q}_1, q_2, \dot{q}_2, \acute{q}_2$.The Hamilton is defined by the formula

$$P_{2a} = -\frac{\partial L}{\partial \dot{q}_{2a}}, \tag{43}$$

where $\dot{q}_2$ is unique function of p_2 and of the other variables obtained by solving for $\dot{q}_2$. The function $\dot{q}_2$ is unique because we have fixed the gauge. The equations of motion (13) are equivalent to the following set of Hamilton equations of motion:

$$\bar{H} = -p_{1a}q_{2a} - p_{2a}\dot{q}_{2a} - L \left(q_1, q'_1, q''_1, q_2, \dot{q}_2, q'_2 \right), \tag{44}$$

$$\dot{q}_1 = -\frac{\delta H}{\delta p_1}; \dot{q}_2 = -\frac{\delta H}{\delta p_2}, \tag{45}$$

$$\dot{p}_1 = \frac{\delta H}{\delta q_1}; \dot{p}_2 = \frac{\delta H}{\delta q_2}, \tag{46}$$

where

$$\bar{H} = \bar{H} \left(q_1, q'_1, q''_1, q_2, \dot{q}_2, q'_2 \right)$$

is Hamilton functional

$$H = \int_0^\pi d\sigma \bar{H} = \int_0^\pi d\sigma \left\{ \ddot{\vec{x}} \frac{\partial L}{\partial \ddot{\vec{x}}} + \dot{\vec{x}} \left[\frac{\partial L}{\partial \dot{\vec{x}}} - \partial_0 \left(\frac{\partial L}{\partial \ddot{\vec{x}}} \right) - \partial_1 \left(\frac{\partial L}{\partial \dot{\vec{x}}} \right) \right] - L \right\}, \tag{47}$$

and

$$\frac{\delta H}{\delta q_1} = \frac{\partial \bar{H}}{\partial q_1} - \frac{\partial}{\partial \sigma} \left(\frac{\partial \bar{H}}{\partial q'_1} \right) + \frac{\partial^2}{\partial \sigma^2} \left(\frac{\partial \bar{H}}{\partial q''_1} \right), \tag{48}$$

$$\frac{\delta H}{\delta q_2} = \frac{\partial \bar{H}}{\partial q_2} - \frac{\partial}{\partial \sigma} \left(\frac{\partial \bar{H}}{\partial q'_2} \right); \frac{\delta H}{\delta p_1} = \frac{\partial \bar{H}}{\partial p_1}; \frac{\delta H}{\delta p_2} = \frac{\partial \bar{H}}{\partial p_2},$$

are variational derivatives of the functional H. Comparing H with the energy P_0 we see that

$$H = P_0 - \int_0^\pi d\sigma \partial_1 \left(\dot{\vec{x}} \frac{\partial L}{\partial \dot{\vec{x}}} \right) = P_0 - \dot{\vec{x}} \frac{\partial L}{\partial \dot{\vec{x}}} \bigg|_{\sigma=0}^{\sigma=\pi}. \tag{49}$$

Thus, in the case of the open string H differs from P_0.

$$F = \int_0^\pi d\sigma \bar{F} \left(q_1, q'_1, q''_1, q_2, q'_2, p_1, p_2 \right). \tag{50}$$

Using Hamilton equations of motion (47) we may write

$$
\frac{dF}{dt} = \int_0^\pi d\sigma \left(\frac{\partial \bar{F}}{\partial q_1} \dot{q}_1 + \frac{\partial \bar{F}}{\partial q_1'} \dot{q}''_1 + \frac{\partial \bar{F}}{\partial q_2} \dot{q}'_2 + \frac{\partial \bar{F}}{\partial p_1} \dot{p}_1 + \frac{\partial \bar{F}}{\partial p_2} \dot{p}_2 \right) = \tag{51}
$$

$$
= \int_0^\pi d\sigma \left(\frac{\delta F}{\delta q_1} \dot{q}_1 + \frac{\delta F}{\delta q_2} \dot{q}_2 + \frac{\delta F}{\delta p_1} \dot{p}_1 + \frac{\delta F}{\delta p_2} \dot{p}_2 \right) +
$$

$$
+ \left[\frac{\partial \bar{F}}{\partial q_1'} - \frac{\partial}{\partial \sigma} \left(\frac{\partial \bar{F}}{\partial q''_1} \right) \right] \dot{q}_1 \Big|_{\sigma=0}^{\sigma=\pi} + \frac{\partial F}{\partial q''_1} \dot{q}_1 \Big|_{\sigma=0}^{\sigma=\pi} + \frac{\partial \bar{F}}{\partial q_2'} \dot{q}_2 \Big|_{\sigma=0}^{\sigma=\pi} \tag{52}
$$

Equation (50) has a rather usual implication that Hamilton H might not be a constant at the motion. From Eq. (50) it follows that

$$
\frac{dF}{dt} = \{F, H\} + \text{"the boundary terms"} \tag{52}
$$

where Poisson bracket $\{F.H\}$ is by definition

$$
\{F, H\} = \int_0^\pi d\sigma \left\{ \frac{\delta F}{\delta p_1} \frac{\delta H}{\delta q_1} - \frac{\delta H}{\delta p_1} \frac{\delta F}{\delta q_1} + \frac{\delta F}{\delta p_2} \frac{\delta H}{\delta q_2} - \frac{\delta H}{\delta p_2} \frac{\delta F}{\delta q_2} \right\} \tag{53}
$$

The boundary terms (the last three terms on the right hand side of formula (50)) vanish in the case of closed string. In the case of open string they give a non-vanishing contribution even in the case of Nambu-Gato string.

$$
\frac{dF}{dt} = \text{"the boundary terms"}, \tag{54}
$$

because of boundary condition (14) which in this case reduces to $\frac{\partial L}{\partial \vec{a}} = 0$ for $\sigma = 0, \pi$. In the case of Lagrangian L with second order derivatives , boundary condition (53) to the form

$$
\frac{\partial \bar{H}}{\partial \vec{x}'} \Big|_{\sigma=0}^{\sigma=\pi} = - \frac{\partial L}{\partial \vec{x}'} \Big|_{\sigma=0}^{\sigma=\pi} = 0. \tag{55}
$$

In the case Nambu-Gato string the boundary terms in Eq. (53) reduce

$$
\frac{dH}{dt} = -\partial_0 \left[\int_0^\pi d\sigma \partial_1 \left(\dot{\vec{x}} \frac{\partial L}{\partial \dot{\vec{x}}} \right) \right]. \tag{56}
$$

The right side of equation (55) does not vanish, in general . Therefore, $\frac{dH}{dt} \neq 0$. From equation (55) it follows that

$$
H + \int_0^\pi d\sigma \partial_1 \left(\dot{\vec{x}} \frac{\partial L}{\partial \dot{\vec{x}}} \right), \tag{57}
$$

is constant during the motion, but this just the energy P_0 is given by formula (39). In general, the boundary terms will also be present in other gauges, because their appearance is due to the facts that the Lagrangian contains second order derivatives and range of the parameter σ is finite.However, in some particular cases the boundary terms can vanish. For example , in papers a gauge is used which is physical, i. e $x_0(\tau, \sigma) = \tau$, and orthogonal, i. e. $\vec{\dot{x}} \vec{x}' = 0$,

5. Conclusion

The equations of motion, of the boundary conditions and of the energy - momentum for the classical rigid string are reconserved. Certain consequences of the equations of motion are presented. We also point out that in Hamilton description of the rigid string the usual time evolution equation $\dot{F} = \{F, H\}$ is modified by some boundary terms.

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