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ORIENTATION ALGORITHMS OF A STRAP-DOWN INERTIAL NAVIGATION SYSTEM USING RODRIG - HAMILTON PARAMETERS

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Abstract. The paper presents the positioning algorithms of flying objects (FOs) compared with ground coordinate system based on data from inertial sensors on board of the FOs. The solutions of the positioning algorithms are based on dynamic functions with Rodrig - Hamilton parameters. With them the positioning algorithms do not depend on the rotation angles of the FOs, and the above algorithms allow the application of low-cost, less accurate 2DOF gyros as angular rate MEMS gyros.

Introduction

Positioning algorithms have been used in navigation and guiding of the FOs. The solution of these algorithms is done through continuous Euler integration of the dynamic functions, or Poisson's function of direction cosines of object based coordinate system with an another coordinate system. The specifications of the above algorithms fit only in some limits of rotation angles of the FOs due to the use of trigonometric functions or fast decreasing functions when rotation angles of the FOs approach some values.

The use of Rodrig - Hamilton parameters (q_0, q_1, q_2, q_3) allows to overcome the disadvantages of the above solutions [1].

Problem statement

The structure of a strap-down inertial navigation system has been chosen in Figure 1

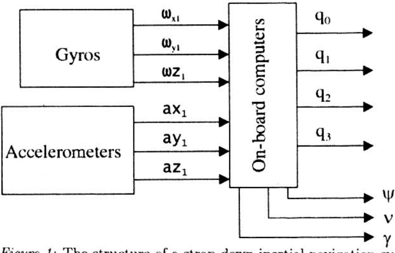

Figure 1: The structure of a strap-down inertial navigation system

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Parts of the inertial navigation system are angular rate measurement systems of the FOs compared with the $(\omega x_1, \omega y_1, \omega z_1)$ coordinate system, three axis acceleration measurement systems and the on-board computer. The task of the system is to estimate the Rodrig - Hamilton parameters and Euler angles of the FOs compared to local earth surface coordinate system, including the curve form of the earth (Figure 2).

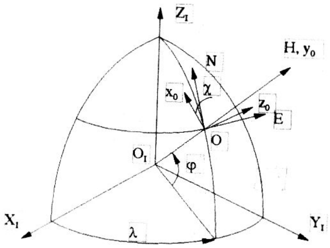


Figure 2: Coordinate systems

In Figure 2: $0_I X_I Y_I Z_I$ - Inertial coordinate system.

$ONEH$ - Geographic coordinate system where base O coincides with object centre point, ON shows the North Pole.

OH - Vertical axis. OEN lays on horizontal plane at O.

$0x_0y_0z_0$ - Horizontal coordinate system where $0y_0$ - vertical axis.

$0xyz$ coordinate system (Figure 3) is fixed to the FO, defined by Euler angles ψ, ν and γ with $0x_0y_0z_0$.

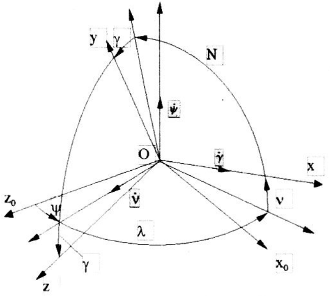
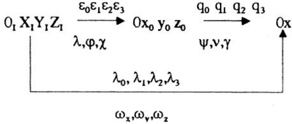


Figure 3: Schema of $0X_0Y_0Z_0$ coordinate system, in line with $0xyz$.



(1)

where ϵ_1 defines $0x_0y_0z_0$ related to $0_I X_I Y_I Z_I$ and q_i, λ_i define the position of the FO related to $0x_0y_0z_0$ and $0_I X_I Y_I Z_I$,. $\omega = (\omega_x, \omega_y, \omega_z)$ are angle rates of the FO related to the components of the fixed coordinate system.

In many real cases, it is necessary to estimate only q_1.

Position parameters of the coordinate system connect $Oxyx$ with $0x_0y_0z_0$.

Positioning Algorithms:

Rodrig - Hamilton parameter q_1 got from the following equation (2)

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = 0.5 \begin{bmatrix} 0 & -\omega_{rx} & -\omega_{ry} & \omega_{rz} \\ \omega_{rx} & 0 & \omega_{rz} & -\omega_{ry} \\ \omega_{ry} & \omega_{rx} & 0 & -\omega_{rz} \\ \omega_{rz} & \omega_{ry} & -\omega_{rx} & -0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_2 \\ q_2 \\ q_3 \end{bmatrix} \tag{2}$$

where $\omega_\Gamma = \omega - \omega_e$ - relative rotation speed of $0x_0y_0z_0$ related to the inertial coordinate system because of the earth's rotation $(\omega_e = 0), \omega_r = \overline{\omega}$, for low flying objects some corrections to the change of the earth acceleration vector, caused by the curve form of the earth, are needed. The most simple earth model in navigation technics is a globe with radius $R = 6.371.10^6 m$ and free fall acceration at the surface $g = 9,81 m/s^2$. In this case we get the following positioning algorithms [2]

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} (1-v^2), & -\omega_x - k\mu_1 - v_1, & -\omega_y - k\mu_2, & -\omega_z - k\mu_3 - v_3 \\ \omega_x + k\mu_1 + v_1, & \rho(1-v^2), & \omega_z - k\mu_3 - v_3, & -\omega_y + k\mu_2 \\ \omega_y + k\mu_2, & \omega x - k\mu_1 - v_1, & \rho(1-v^2), & -\omega_z + k\mu_3 + v_3 \\ \omega_z + k\mu_3 + v_3, & \omega_y - k\mu_2, & -\omega_x + k\mu_1 + v_1, & \rho(1-v^2) \end{bmatrix} * \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{3}$$

$$\begin{aligned} \dot{v}_1 &= l\mu_1 \\ \dot{\nu}_3 &= l\mu_3 \\ \mu_1 &= -\frac{W_{z0}}{g}; \mu_3 = +\frac{W_{x0}}{g}; \mu_0 = \mu_2 = 0 \end{aligned} \tag{4}$$

$$\begin{bmatrix} W_{x0} \\ W_{y0} \\ W_{z0} \end{bmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} * \begin{bmatrix} W_x \\ W_y \\ W_z \end{bmatrix} \tag{5}$$

$$\begin{array}{lll} a_{11} = q_0^2 + q_1^2 - q_2^2 - q_3^2; & a_{12} = 2(q_1 q_2 - q_0 q_3); & a_{13} = 2(q_0 q_2 + q_1 q_3) \\ a_{21} = 2(q_1 q_2 - q_0 q_3); & a_{22} = q_0^2 + q_2^2 - q_1^2 - q_3^2; & a_{23} = 2(q_0 q_1 + q_2 q_3) \\ a_{31} = 2(q_1 q_3 - q_0 q_2); & a_{32} = 2(q_0 q_1 + q_2 q_3); & a_{33} = q_0^2 + q_3^2 - q_1^2 - q_2^2; \end{array} \quad (6)$$

$$\nu^2 = q_0^2 + q_1^2 + q_2^2 + q_3^2$$

k, l, ρ- corrective coefficients for position, differentiation and standardization, chosen during design process.

After receiving the estimated Rodrig - Hamilton parameters from (3) - (6), the specific Euler angles for positioning of $Ox_0y_0z_0$ can be computed:

$$\begin{aligned} \psi &= \mathrm{arctg}\left(-2\frac{q_1 q_3 - q_0 q_2}{q_0^2 + q_1^2 - q_2^2 - q_3^2}\right) \\ \theta &= \mathrm{arctg}\left(\frac{\sin\theta}{\sqrt{1 - \sin^2\theta}}\right); \sin\theta = 2(q_0 q_3 + q_1 q_2) \\ \gamma &= \mathrm{arctg}\left(-2\frac{q_2 q_3 - q_0 q_1}{q_0^2 + q_2^2 - q_1^2 - q_3^2}\right) \end{aligned} \quad (7)$$

Results when modelling the algorithms for a strap-down inertial navigation system according to (3) - (7) in Matlab - Simulink (Figure 4) will be compared with solutions of the dynamic equations of the FO [3]

$$\begin{aligned} \dot{\psi} &= \frac{1}{\cos\nu}(\omega_y \cos\gamma - \omega_z \sin\gamma), \\ \dot{\nu} &= \omega_y \sin\gamma + \omega_z \cos\gamma \\ \dot{\gamma} &= \omega_x - \mathrm{tg}\gamma(\omega_y \cos\gamma - \omega_x \sin\gamma) \end{aligned} \quad (8)$$

with same input parameters $\omega_x, \omega_y, \omega_z$ and a_x, a_y, a_z received from 3D simulation of a specific FO [3].

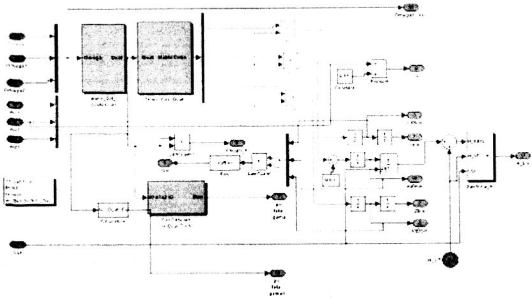


Figure 4: Positioning Algorithms in Simulink

Results have been plotted in Figure 5.

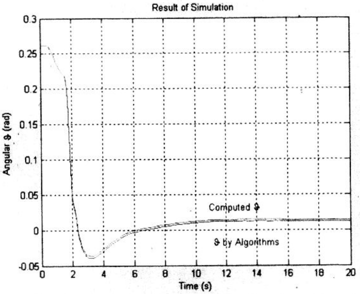


Figure 5: Simulated results with conventional and position algorithms

It can be seen on the figures that computations results are highly accurate according to positioning algorithms.

$$\Delta\psi = \frac{\psi - \psi_{tt}}{\psi} \le 0.1\%$$

$$\Delta\nu = \frac{\nu - \nu_{tt}}{\nu} \le 0.2\%$$

$$\Delta\dot{\gamma} = \frac{\gamma - \gamma_{tt}}{\gamma} \le 0.1\%.$$

Remained errors are caused by computation errors and round-up errors of the computer.

Conclusion

Positioning algorithms (3) - (6) of a strap-down inertial navigation system Is working stable and do not depend on rotation angle of the object as well as the global take-up position.

Algorithms (3) - (6) can be realized in on-board computer, satisfy the required accuracy and quick response of modern FOs.

References

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