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On a Five-dimensional Scenario of Massive Gravity

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Abstract: A study on a five-dimensional scenario of a ghost-free nonlinear massive gravity proposed by de Rham, Gabadadze, and Tolley (dRGT) will be presented in this article. In particular, we will show how to construct a five-dimensional massive graviton term using the Cayley-Hamilton theorem. Then some cosmological solutions such as the Friedmann-Lemaitre-Robertson-Walker, Bianchi type I, and Schwarzschild-Tangherlini-(A)dS spacetimes will be solved for the five-dimensional dRGT theory thanks to the constant-like behavior of massive graviton terms under an assumption that the reference metric is compatible with the physical one.

Keywords: Massive gravity, higher dimensions, Friedmann-Lemaitre-Robertson-Walker, Bianchi type I, and Schwarzschild-Tangherlini-(A)dS spacetimes.

1. Introduction

Recently, an important nonlinear extension of the Fierz-Pauli massive gravity [1] has been proposed by de Rham, Gabadadze, and Tolley (dRGT) [2], which has been confirmed to be free of the so-called Boulware-Deser (BD) ghost, a negative energy mode arising from nonlinear terms [3], by several approaches [4]. It turns out that a number of cosmological implications of dRGT theory have been investigated extensively. For example, the dRGT theory has been expected to provide an alternative solution to the cosmological constant problem. Besides the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric, some anisotropic metrics such as the Bianchi type I metric along with some black holes such as the Schwarzschild, Kerr, and charged black holes have also been shown to exist in the context of dRGT theory [5, 6]. Since the dRGT theory has been proved to be free of the BD ghost for arbitrary reference metrics, a very interesting extension of the dRGT theory called a massive bigravity, in which the reference metric is introduced to be dynamical, has been proposed by Hassan and Rosen in Ref. [7]. For up-to-date reviews on massive gravity, see Ref. [5].

It is worth noting that it is possible to extend the dRGT theory to higher dimensional spacetimes [8]. As far as we know, however, most of previous papers on the dRGT massive gravity have worked only in four-dimensional spacetimes [5]. Hence, we would like to study higher dimensional scenarios of dRGT theory. In particular, we have systematically investigated some cosmological implications of a five-dimensional dRGT theory in Ref. [9]. As a result, we have used a simple method based on the

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Cayley-Hamilton theorem for square matrix [10] to construct higher dimensional graviton terms (or interaction terms), for example, L_5 existing in five- (or higher) dimensional spacetimes. It is worth noting that we have been able to show that higher dimensional massive graviton terms $L_{n>4}$ all vanish in four-dimensional spacetimes but do survive in spacetimes, whose dimension number is larger than or equal to n [2, 7, 9]. Hence, we should not ignore their existence when studying higher dimensional dRGT theories. For example, we have introduced the five-dimensional graviton term, L_5, to a five-dimensional dRGT theory. Then, the corresponding field and constraint equations have been derived in order to see whether the FLRW, Bianchi type I, and Schwarzschild-Tangherlini metrics act as physical solutions to the five-dimensional dRGT theory [9].

In the present article, we will summarize basic results of our recent study [9]. The article is organized as follows: A very brief introduction of our research has been written in section 1. The Cayley-Hamilton theorem, which is used to construct the graviton terms, will be mentioned in section 2. Then, we will present a basic setup and simple physical solutions of a five-dimensional massive gravity in sections 3 and 4, respectively. Finally, concluding remarks will be given in section 5.

2. Cayley-Hamilton theorem and ghost-free graviton terms

As mentioned above, we would like to show a connection between the Cayley-Hamilton theorem and the graviton terms $L_{n\geq 2}$ of the dRGT massive gravity. In linear algebra, there exists the Cayley-Hamilton theorem [10] stating that any square matrix must obey its characteristic equation. Particularly, for an arbitrary $n\times n$ matrix K, we have the following characteristic equation [10]

$$P(K)\equiv K^n - D_{n-1}K^{n-1} + D_{n-2}K^{n-2} - \cdots + (-1)^{n-1} D_1 K + (-1)^n \det(K) I_n = 0, \quad (1)$$

where $D_{n-1} = trK \equiv [K]$, $D_{n-j}\,(2\leq j\leq n-1)$ are coefficients of the characteristic polynomial, and I_n is a $n\times n$ identity matrix. Now, we apply this theorem to the following matrix K of dRGT theory, whose definition is given by

$$K^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\alpha} f_{ab}\partial_\alpha \phi^a \partial_\nu \phi^b}\,, \quad (2)$$

where $g_{\mu\nu}$ is the physical metric, while f_{ab} is the (non-dynamical) reference (or fiducial) metric. In addition, ϕ^a's are the Stuckelberg scalar fields, which will be chosen to be in a unitary gauge, i.e., $\phi^a = x^a$ in the rest of this paper. As a result, it is straightforward to recover the first three massive graviton terms, $L_2 = 2\det K_{2\times 2}$, $L_3 = 2\det K_{3\times 3}$, and $L_4 = 2\det K_{4\times 4}$ corresponding to $n = 2$, 3, and 4, respectively. Similarly, we are able to define a five-dimensional ($n = 5$) graviton term L_5 to be [9]

$$L_5 = 2\det K_{5\times 5} = \frac{1}{60}[K]^5 - \frac{1}{6}[K]^3[K^2] + \frac{1}{3}[K]^2[K^3] - \frac{1}{3}[K^2][K^3] + \frac{1}{4}[K][K^2]^2 - \frac{1}{2}[K][K^4] + \frac{2}{5}[K^5]. \quad (3)$$

Generally, we have the following relation: $L_{n\geq 2} = 2\det K_{n\times n}$, which is a key to construct arbitrary dimensional dRGT theory. For instance, the definition of L_6 and L_7 can be seen in Ref. [9].

3. Basic setup of five-dimensional nonlinear massive gravity

In this section, we would like to present basic details of five-dimensional nonlinear massive gravity, whose action is given by [9]

$$S = \frac{M_p^2}{2}\int d^5x\sqrt{-g}\left\{R + m_g^2\left(L_2 + \alpha_3 L_3 + \alpha_4 L_4 + \alpha_5 L_5\right)\right\}, \tag{4}$$

where M_p the Planck mass, $m_g \neq 0$ the mass of graviton, $\alpha_{3,4,5}$ the field parameters, and $L_{2,3,4,5}$ the graviton terms (or interaction terms) whose definitions are given by

$$L_2 = [K]^2 - [K^2], \tag{5}$$

$$L_3 = \frac{1}{3}[K]^3 - [K][K^2] + \frac{2}{3}[K^3], \tag{6}$$

$$L_4 = \frac{1}{12}[K]^4 - \frac{1}{2}[K]^2[K^2] + \frac{1}{4}[K^2]^2 + \frac{2}{3}[K][K^3] - \frac{1}{2}[K^4], \tag{7}$$

$$L_5 = \frac{1}{60}[K]^5 - \frac{1}{6}[K]^3[K^2] + \frac{1}{3}[K]^2[K^3] - \frac{1}{3}[K^2][K^3] + \frac{1}{4}[K][K^2]^2 - \frac{1}{2}[K][K^4] + \frac{2}{5}[K^5]. \tag{8}$$

As a result, the corresponding Einstein field equations of physical metric will be defined by varying the action (4) with respect to the inverse metric $g^{\mu\nu}$:

$$\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + m_g^2\left(X_{\mu\nu} + \sigma Y_{\mu\nu} + \alpha_5 W_{\mu\nu}\right) = 0, \tag{9}$$

with the following tensors:

$$X_{\mu\nu} = -\frac{1}{2}\left(\alpha L_2 + \beta L_3\right) g_{\mu\nu} + \tilde{X}_{\mu\nu}, \tag{10}$$

$$\tilde{X}_{\mu\nu} = K_{\mu\nu} - [K]g_{\mu\nu} - \alpha\left\{K_{\mu\nu}^2 - [K]K_{\mu\nu}\right\} + \beta\left\{K_{\mu\nu}^3 - [K]K_{\mu\nu}^2 + \frac{L_2}{2}K_{\mu\nu}\right\}, \tag{11}$$

$$Y_{\mu\nu} = -\frac{L_4}{2} g_{\mu\nu} + \tilde{Y}_{\mu\nu},\ \tilde{Y}_{\mu\nu} = \frac{L_3}{2}K_{\mu\nu} - \frac{L_2}{2}K_{\mu\nu}^2 + [K]K_{\mu\nu}^3 - K_{\mu\nu}^4, \tag{12}$$

$$W_{\mu\nu} = -\frac{L_5}{2} g_{\mu\nu} + \tilde{W}_{\mu\nu},\ \tilde{W}_{\mu\nu} = \frac{L_4}{2}K_{\mu\nu} - \frac{L_3}{2}K_{\mu\nu}^2 + \frac{L_2}{2}K_{\mu\nu}^3 - [K]K_{\mu\nu}^4 + K_{\mu\nu}^5. \tag{13}$$

Here we have introduced some additional parameters such as $\alpha = \alpha_3 + 1$, $\beta = \alpha_3 + \alpha_4$, and $\sigma = \alpha_4 + \alpha_5$ for convenience. Besides the field equations of physical metric, we have also derived the following constraint equations due to the existence of reference metric [9]:

$$t_{\mu\nu} \equiv \tilde{X}_{\mu\nu} + \sigma\tilde{Y}_{\mu\nu} + \alpha_5\tilde{W}_{\mu\nu} - \frac{1}{2}\left(\alpha_3 L_2 + \alpha_4 L_3 + \alpha_5 L_4\right) g_{\mu\nu} = 0. \tag{14}$$

As a result, due to the constraint equations (14), the Einstein field equations (9) can be reduced to the simpler form [9]:

$$\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) - \frac{m_g^2}{2}L_M g_{\mu\nu} = 0, \tag{15}$$

where $L_M \equiv L_2 + \alpha_3 L_3 + \alpha_4 L_4 + \alpha_5 L_5$ is the total massive graviton term. We observe that L_M will act as an effective cosmological constant, $\Lambda_M \equiv -m_g^2 L_M / 2$, due to the Bianchi constraint that $\partial^\nu L_M = 0$. Indeed, this claim will be the case for a number of metrics, which will be discussed in the next section.

4. Simple cosmological solutions

In this section, we would like to examine the validity of our claim in the section 3 that the total graviton term L_M turns out to be an effective cosmological constant for a number of physical metrics and compatible reference ones. It is worth noting that some metrics such as FLRW and Bianchi type I have been found in the four-dimensional dRGT theory in Ref. [6], in which the physical metrics have also been assumed to be compatible with the reference ones.

4.1. Friedmann-Lemaitre-Robertson-Walker metrics

As a result, the following FLRW physical and reference metrics are given by [9]

$$ds^2\left(g_{\mu\nu}\right) = -N_1^2\left(t\right)dt^2 + a_1^2\left(t\right)\left(d\vec{x}^2 + du^2\right), \tag{16}$$

$$ds^2\left(f_{ab}\right) = -N_2^2\left(t\right)dt^2 + a_2^2\left(t\right)\left(d\vec{x}^2 + du^2\right). \tag{17}$$

Given these FLRW metrics, the total graviton term L_M becomes as

$$\begin{aligned} L_M = {} & 2\left[\left(\gamma\alpha_4 - \gamma_3\right)\Sigma^3 + 3\left(\gamma_2 - \gamma\gamma_3\right)\Sigma^2 + 3\left(\gamma\gamma_2 - \gamma_1\right)\Sigma - \gamma\gamma_1 + \left(3\gamma_1 - 3\gamma_2 + \gamma_3\right)\right] + 2\left\{\alpha_4\Sigma^3 - \alpha_5\left(\gamma-1\right)\left(\Sigma-1\right)^3\right. \\ & \left. -3\left[\gamma_3 - \left(\gamma-1\right)\alpha_4\right]\Sigma^2 + 3\left[\gamma_2 - \left(\gamma-1\right)\left(\gamma_3+\alpha_4\right)\right]\Sigma + \left(\gamma-1\right)\left(3\gamma_3+1\right) - \gamma_1\right\}\left(\Sigma-1\right), \end{aligned} \tag{18}$$

with $\gamma = N_2/N_1$, $\Sigma = a_2/a_1$, $\gamma_1 = 3 + 3\alpha_3 + \alpha_4$, $\gamma_2 = 1 + 2\alpha_3 + \alpha_4$, and $\gamma_3 = \alpha_3 + \alpha_4$. Armed with these results, we will solve the following constraint equations (14), which turn out to be equivalent with the Euler-Lagrange equations of scale factors of reference metric [9]:

$$\frac{\partial L_M}{\partial N_2} = \frac{\partial L_M}{\partial a_2} = 0 \Leftrightarrow \frac{\partial L_M}{\partial \gamma} = \frac{\partial L_M}{\partial \Sigma} = 0. \tag{19}$$

As a result, once these constraint equations are solved, the corresponding values of L_M and then that of effective cosmological constant, $\Lambda_M = -m_g^2 L_M/2$, will be determined. For detailed calculations, one can see Ref. [9]. Once the value of Λ_M is figured out, we will solve the following Einstein field equations of physical metric (15) to obtain the following FLRW solution [9]:

$$a_1 = \exp\left[\sqrt{\frac{\Lambda_M}{6}}t\right]. \tag{20}$$

It turns out that for a case of positive Λ_M we will have the de Sitter solution, which describes the expanding universe in five dimensions.

4.2. Bianchi type I metrics

As a result, the following Bianchi type I metrics, which are homogenous but anisotropic spacetimes, are given by [9]

$$\begin{aligned} ds^2\left(g_{\mu\nu}\right) = {} & -N_1^2\left(t\right)dt^2 + \exp\left[2\alpha_1\left(t\right) - 4\sigma_1\left(t\right)\right]dx^2 \\ & + \exp\left[2\alpha_1\left(t\right) + 2\sigma_1\left(t\right)\right]\left(dy^2 + dz^2\right) + \exp\left[2\beta_1\left(t\right)\right]du^2, \end{aligned} \tag{21}$$

$$\begin{aligned} ds^2\left(f_{ab}\right) = {} & -N_2^2\left(t\right)dt^2 + \exp\left[2\alpha_2\left(t\right) - 4\sigma_2\left(t\right)\right]dx^2 \\ & + \exp\left[2\alpha_2\left(t\right) + 2\sigma_2\left(t\right)\right]\left(dy^2 + dz^2\right) + \exp\left[2\beta_2\left(t\right)\right]du^2, \end{aligned} \tag{22}$$

where $\beta_{1,2}$ are additional scale factors associated with the fifth dimension *u*. Similar to the FLRW case, we define the following total graviton term L_M to be [9]

$$\begin{aligned}L_M &= 2\left[\left(\gamma\alpha_4-\gamma_3\right)AB^2+\left(\gamma_2-\gamma\gamma_3\right)B\left(2A+B\right)+\left(\gamma\gamma_2-\gamma_1\right)\left(A+2B\right)-\gamma\gamma_1+\left(3\gamma_1-3\gamma_2+\gamma_3\right)\right]\\ &+2\left\{\alpha_4AB^2-\alpha_5\left(\gamma-1\right)\left(A-1\right)\left(B-1\right)^2-\left[\gamma_3-\left(\gamma-1\right)\alpha_4\right]B\left(2A+B\right)+\left[\gamma_2-\left(\gamma-1\right)\left(\gamma_3+\alpha_4\right)\right]\left(A+2B\right)\right.\\ &\left.+\left(\gamma-1\right)\left(3\gamma_3+1\right)-\gamma_1\right\}\left(C-1\right),\end{aligned} \tag{23}$$

where $A=\varepsilon\eta^{-2}$, $B=\varepsilon\eta$, $C=\exp\left[\beta_2-\beta_1\right]$, $\varepsilon=\exp\left[\alpha_2-\alpha_1\right]$, and $\eta=\exp\left[\sigma_2-\sigma_1\right]$. Analogous to the FLRW case, the corresponding Euler-Lagrange equations:

$$\frac{\partial L_M}{\partial N_2}=\frac{\partial L_M}{\partial\alpha_2}=\frac{\partial L_M}{\partial\sigma_2}=\frac{\partial L_M}{\partial\beta_2}=0\Leftrightarrow\frac{\partial L_M}{\partial\gamma}=\frac{\partial L_M}{\partial A}=\frac{\partial L_M}{\partial B}=\frac{\partial L_M}{\partial C}=0, \tag{24}$$

need to be solved first in order to determine the following values of Λ_M [9]. Once this task is done, the corresponding Einstein field equations (15) can be solved to give non-trivial solutions [9]:

$$\exp\left[3\alpha_1\right]=\exp\left[3\alpha_0\right]\left[\cosh\left(3\tilde{H}_1t\right)+\frac{\dot{\alpha}_0}{\tilde{H}_1}\sinh\left(3\tilde{H}_1t\right)\right], \tag{25}$$

$$\exp\left[\beta_1\right]=\exp\left[\beta_0\right]\left[\cosh\left(3\bar{H}_1t\right)+\frac{\dot{\beta}_0}{3\bar{H}_1}\sinh\left(3\bar{H}_1t\right)\right], \tag{26}$$

$$\sigma_1=\sigma_0+\sqrt{\dot{\alpha}_0^2+\dot{\alpha}_0\dot{\beta}_0-H_1^2}\int\left\{\left[\cosh\left(3\tilde{H}_1t\right)+\frac{\dot{\alpha}_0}{\tilde{H}_1}\sinh\left(3\tilde{H}_1t\right)\right]\left[\cosh\left(3\bar{H}_1t\right)+\frac{\dot{\beta}_0}{3\bar{H}_1}\sinh\left(3\bar{H}_1t\right)\right]\right\}^{-1}dt, \tag{27}$$

where $\tilde{H}_1^2=4H_1^2/9\left(1-V_0\right)$, $\bar{H}_1^2=V_0\tilde{H}_1^2$, $H_1^2=\Lambda_M/3$, and V_0 is a constant. In addition, parameters with subscript "0" appearing in the above expressions are initial ($t=0$) values of scale factors.

4.3. Schwarzschild-Tangherlini metrics

In this subsection, we would like to consider the Schwarzschild-Tangherlini metrics of the following forms [9]:

$$ds^2\left(g_{\mu\nu}\right)=-N_1^2\left(t,r\right)dt^2+\frac{dr^2}{F_1^2\left(t,r\right)}+\frac{r^2d\Omega_3^2}{H_1^2\left(t,r\right)}, \tag{28}$$

$$ds^2\left(f_{ab}\right)=-N_2^2\left(t,r\right)dt^2+\frac{dr^2}{F_2^2\left(t,r\right)}+\frac{r^2d\Omega_3^2}{H_2^2\left(t,r\right)}, \tag{29}$$

where $d\Omega_3^2=d\theta^2+\sin^2\theta d\varphi^2+\sin^2\theta\sin^2\varphi d\psi^2$, $0\le\theta\le\pi$, $0\le\varphi\le\pi$, and $0\le\psi\le2\pi$. As a result, the corresponding total graviton term turns out to be [9]

$$\begin{aligned}L_M &= 2\left\{\left[\alpha_5\left(K_2^2\right)^3+3\alpha_4\left(K_2^2\right)^2+3\alpha_3K_2^2+1\right]K_0^0K_1^1+K_2^2\left[\alpha_4\left(K_2^2\right)^2+3\alpha_3K_2^2+3\right]\left(K_0^0+K_1^1\right)\right.\\ &\left.+\left(K_2^2\right)^2\left(\alpha_3K_2^2+3\right)\right\},\end{aligned} \tag{30}$$

with $K_0^0=1-N_2/N_1$, $K_1^1=1-F_1/F_2$, and $K_2^2=K_3^3=K_4^4=1-H_1/H_2$. Hence, the corresponding Euler-Lagrange equations read

$$\frac{\partial L_M}{\partial K_0^0} = \frac{\partial L_M}{\partial K_1^1} = \frac{\partial L_M}{\partial K_2^2} = 0 . \tag{31}$$

Solving these constraint equations will yield the following values of Λ_M . Furthermore, solving the Einstein field equations (15) will give us the following metric [9]:

$$ds^2\left(g_{\mu\nu}\right) = -f\left(r\right)dt^2 + \frac{dr^2}{f\left(r\right)} + r^2 d\Omega_3^2, \tag{32}$$

here

$$N_1^2\left(t,r\right) = F_1^2\left(t,r\right) = f\left(r\right) = 1 - \frac{\mu}{r^2} - \frac{\Lambda_M}{6}r^2 \text{ and } H_1^2\left(t,r\right) = 1 . \tag{33}$$

It is noted that $\mu \equiv 8G_5 M/3\pi$ is a mass parameter with M and G_5 stand for the mass of source and the five-dimensional Newton constant, respectively. It is also noted that we will have the Schwarzschild-Tangherlini-de Sitter (dS) and Schwarzschild-Tangherlini-anti-de Sitter (AdS) black holes for positive and negative Λ_M , respectively. On the other hand, we will have the (pure) Schwarzschild-Tangherlini black hole for vanishing Λ_M .

5. Conclusions

We have presented basic results of our recent study on the five-dimensional dRGT massive gravity [9]. In particular, we have shown the effective method based on the Cayley-Hamilton theorem to construct the five- (or higher) dimensional graviton term. Then, we have examined, after deriving the corresponding Einstein field and constraint equations, whether the five-dimensional dRGT theory admits some well-known metrics such as FLRW, Bianchi type I, and Schwarzschild-Tangherlini metrics as its cosmological solutions. Our research has indicated that the five-dimensional dRGT theory might play an important role in describing our universe. Of course, many other cosmological aspects, e.g., gravitational waves, should be discussed in the context of the five-dimensional massive gravity in order to improve its cosmological viability. To end this article, we would like to note that a bi-gravity extension of the five-dimensional dRGT theory, in which the reference metric is introduced to be fully dynamical as the physical one [7], has been proposed in our recent paper [11].

Acknowledgments

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