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The Transverse Hall Effect in a Quantum Well with High Infinite Potential in the Influence of Confined Optical Phonons

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Abstract: The Transverse Hall effect (THE) has been theoretical studied in a quantum well (QW) with high infinite potential subjected to a crossed dc electric field and a magnetic field (MF) which is oriented perpendicularly to the confinement direction in the present of an intense electromagnetic wave (EMW). The analytical expression of the transverse hall coefficient (THC) which depends not only on the parameters of the system but especially on the quantum number m characterizing confined phonons, is obtained by using the quantum kinetic equation method for confined electrons - confined optical phonons interaction. The analytic expression of THC is numerically evaluated, plotted and discussed for a specific case of the AlAs/GaAs/AlAs QW. Results show the THC depends strong nonlinearly on the EMW amplitude and the MF. All results are compared with that in case of unconfined phonons to see differences.

Keywords: Transverse Hall effect, Confined phonons, Quantum Well.

1. Introduction

In recent years, the low-dimensional system (LDS) is the great interest in researches because of these unusual behaviors. This due to that the motion of both electrons and phonon are restricted and their energy levels become discrete [1-2]. The strong effect of electron and phonon confinement enhanced the nonlinear kinetic properties of LDS have been investigated. For example, the influence of confined electrons and confined phonons on the nonlinear absorption coefficient of EMW [3], the parametric interactions and transformations of excitations [4] and the Acoustoelectric effect [5] have been studied by using the quantum equation method.

The Hall effect is the production of a voltage difference (the Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current. The Hall effect has been studied extensively both in experiment [6-9] and theoretical in LDS with case of unconfined phonons [10-12]. In this work, we study the THE under the impact of confined optical phonons in the QW. It is considered that an infinite potential QW subjected to a

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crossed dc electric field and a MF $\vec{B}$ which is oriented perpendicularly to the confinement direction in the present of an intense EMW $\vec{E}_1$ a laser radiation $\vec{E} = \vec{E}_o \sin\Omega t$. We achieve analytical expressions for the Transverse Hall Conductivity Tensor (THCT) and the THC in the next section. Numerical results and discussions are given in Sec.3. Finally, Sec.4 shows remarks and conclusions.

2. The transverse hall effect in an infinite potential quantum well under the influence of confined optical phonons

We consider an infinite potential **QW** structure subjected to a crossed dc electric field $\vec{E} = (0,0,E_1)$, a **MF** $\vec{B} = (0,B,0)$ and a laser radiation applied along the z direction with the electric field vector $\vec{E} = (\vec{E}_o \sin\Omega t,0,0)$. Under the impact of these external fields and the material confined potential, the motion of carriers is restricted, thus the electron wave function and its discrete energy are now modified [13]:

$$\Psi(\vec{r}) = \frac{1}{2\pi}\exp\left(i\vec{k}_\perp \vec{r}_\perp\right) H_N\left(z - z_o\right)\exp\left(\frac{-\left(z - z_o\right)^2}{2}\right) \tag{1}$$

$$\varepsilon_N\left(k_x\right) = \hbar\omega_p\left(N + \frac{1}{2}\right) + \frac{1}{2m^*}\left[\hbar^2 k_x^2 - \left(\frac{\hbar k_x \omega_c + e.E_1}{\omega_p}\right)^2\right],$$
$$\omega_p^2 = \omega_z^2 + \omega_c^2; \quad N = 0,1,2...; \quad z_o = \frac{\left(\hbar k_x \omega_c + \mathrm{eE}_1\right)}{m^* \omega_p^2} \tag{2}$$

where N is the Landau level index; is the Planck constant; m^* is the effective mass of an electron; k_X being the wave vector of the electron along the x axis; $\omega_C = eB/m^*$ is the cyclotron frequency; $H_N\left(z\right)$ is Hermite polynomials. When the phonons are confined, the wave vector of phonon and its frequency are quantized [14, 15]:

$$\hbar\omega_{m,\vec{q}_\perp} = \hbar\omega_o - \beta\left[q_\perp^2 + \left(\frac{m\pi}{L}\right)^2\right]; \quad \vec{q} = \vec{q}_\perp + \vec{q}_z; \vec{q}_\perp = \vec{q}_x + \vec{q}_y; q_z = \frac{m\pi}{L} \tag{3}$$

where ν is velocity parameter and *m* being the quantum number characterizing the phonon confinement. The confined electron - confined optical phonon interaction constant $C_{m,\vec{q}_\perp} J_{NN'}\left(u\right) I_{nn'}^m$:

$$\left|C_{m,\vec{q}_\perp}\right|^2 = \frac{2\pi e^2 \hbar\omega_{m,\vec{q}_\perp}}{\varepsilon_o V_o}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_o}\right)\frac{1}{q_\perp^2 + q_z^2};$$
$$\left|J_{N,N'}(u)\right|^2 = \left(N'!/N!\right)e^{-u}u^{N'-N}\left[L_N^{N'-N}(u)\right]^2; u = l_B^2 q_\perp^2 / 2 \tag{4}$$
$$I_{nn'}^m = \sqrt{\frac{2}{L}}\int_0^L\left(\eta\left(m\right)\cos\frac{m\pi z}{L} + \eta\left(m+1\right)\sin\frac{m\pi z}{L}\right)\phi_{n'}^*\left(z\right)\phi_n\left(z\right)dz,$$

where ε_o is the electric constant; V_o is the normalization volume of specimen; χ_o and χ_∞ are the static and the high frequency dielectric constants; $J_{N,N'}(u)$ is the associated Laguerre polynomial, $l_B = \sqrt{\hbar / m^* \omega_c}$ is the radius of the Landau orbit in the x − y plane, and $h(m)$=1 if m is even, $h(m)$=0 if m is odd; $f_n(z)$ and $f^*_{n'}(z)$ are the electron subband wave functions in the initial and final states.

Using Hamiltonian of the confined electrons - confined phonons in such a quantum well, we establish the quantum kinetic equation for electron distribution function. Following that, the equation for the current density $\vec{R}(\varepsilon)$ is obtained:

$$\frac{\vec{R}(\varepsilon)}{\tau(\varepsilon)} + \omega_c \left[\vec{h} \wedge \vec{R}(\varepsilon) \right] = \vec{Q}(\varepsilon) + \vec{S}(\varepsilon) \qquad \text{with } \vec{R}(\varepsilon) = \sum_{N,k_x} \frac{e}{m^*} k_x f_{N,k_x} \delta(\varepsilon_{N,k_x} - \varepsilon_{N',k'_x}) \tag{5}$$

$$\vec{Q}(\varepsilon) = -\frac{e}{m^*} \sum_{N,k_x} k_x (\vec{F} \frac{f_{N,k_x}}{\partial k_x}) \delta(\varepsilon_{N,k_x} - \varepsilon_{N',k'_x}); \vec{F} = e.\vec{E} \tag{6}$$

$$\begin{aligned} \vec{S}(\varepsilon) = \sum_{N,N',k_x} \frac{2\pi e m^*}{\hbar} \left| C_{m,\vec{q}_\perp} \right|^2 \left| I^m_{n,n'} \right|^2 \left| J_{N,N'}(u) \right|^2 N_{m,\vec{q}_\perp} k_x q \left\{ -\vec{\chi}(\varepsilon_{N,k_x}) f^{o'}_{N,k_x} \left[\left(1 - \frac{\Lambda^2}{2\Omega^2} \right) \right] \right\} \times \Delta_2 + \\ + \left(\frac{\Delta^2}{4\Omega^2} \times \Delta_1 \right) \Bigg] + \vec{\chi}(\varepsilon_{N,k_x}) f^{o'}_{N,k_x} \left[1 - \frac{\Delta^2}{2\Omega^2} \right] \times \Delta_1 + \left[\frac{\Delta^2}{4\Omega^2} \times \Delta_1 \right] \times \delta(\varepsilon_{N,k_x} - \varepsilon_{N',k'_x}) \end{aligned} \tag{7}$$

where $\vec{h} = \vec{B}/B$ is the unit vector along the magnetic field; the notation '$\wedge$' represents the vector product; τ is the electron momentum relaxation time, which is assumed to be a constant; $\Delta_1 = \delta(\varepsilon_{N',k_x+q_x} - \varepsilon_{N,k_x} - \hbar\omega_{m,\vec{q}_\perp} - s\hbar\Omega)$ and $\Delta_2 = \delta(\varepsilon_{N',k_x+q_x} - \varepsilon_{N,k_x} - \hbar\omega_{m,q_x} - s\hbar\Omega)$; $\delta(...)$ being the Dirac's delta function and $f_{N,k_x} = f^o_{N,k_x} - k_x c(e_{N,k_x}) f^{o'}_{N,k_x}$ is the time-independent component of the distribution function of electrons, $\vec{\chi}(\varepsilon) = \frac{\tau(\varepsilon)}{m^*} \left\{ 1 + \omega_c^2 \tau^2(\varepsilon) \right\}^{-1} \left\{ \vec{F}(\varepsilon) - \omega_c \tau(\varepsilon) \left[\vec{h} \wedge \vec{F}(\varepsilon) \right] + \omega_c^2 \tau^2(\varepsilon) \vec{h}(\vec{h}, \vec{F})(\varepsilon) \right\}$ and f^o_{n,k_x} is the equilibrium electron distribution function.

For simplicity, we limit the problem to the cases of s = −1, 0, 1. This means that the processes with more than one photon are ignored. Let us consider the electron gas is non-degenerate $f^o_{N,k_x} = e^{b(e_F - e_{N,k_x})}, b = \frac{1}{k_B T}$, here ε_F is the Fermi level, and k_B is the Boltzmann constant. After some manipulation, the expression for the THCT is obtained:

$$s_{lm} = \frac{t}{\left\{ 1 + w_c^2 t^2 \right\}} \left[d_{ij} - w_c t e_{ijk} h_k + w_c^2 t^2 h_i h_j \right] \times \left\{ a d_{jm} + \frac{b e t}{m^*} \left[1 + w_c^2 t^2 \right]^{-1} d_{jl} \left[d_{lm} - w_c t e_{lm} h_p + w_c^2 t^2 h_l h_m \right] \right\} \tag{8}$$

here δ_{ij} is the Kronecker delta; ε_{ijk} being the antisymmetric Levi - Civita tensor; symbols i, j, k, l, p corresponding the components x, y, z of the Cartesian coordinates,

$$a = \frac{e^2 L_x}{2\pi m^* \hbar}\sqrt{\frac{\pi}{\alpha\beta}}\sum_N \exp\left\{\beta\left[\varepsilon_F - \left(N + 1/2\right)\hbar\omega_p + \frac{e^2 E_1^2}{2m^*\omega_p^2} + \frac{\gamma^2}{4\alpha}\right]\right\}; \quad with \quad \alpha = \frac{\hbar^2}{m*}\left(1 - \frac{\omega_c^2}{\omega_p^2}\right); \tag{9}$$

$$b = \frac{\beta A L_x e N_o}{16\pi^2\alpha^2 m^*\hbar}\sum_{n,n'} I_{nn'}^m J_{N,N'}(u)(B_1 + B_2 + B_3 + B_4 + B_5 + B_6 + B_7 + B_8) \tag{10a}$$

$$B_1 = e^{\beta\left[\varepsilon_F - \left(N+\frac{1}{2}\right)\hbar\omega_p + \frac{e^2E_1^2}{2m\omega_p^2} - \beta\left(\frac{C_1}{2} - \frac{\gamma^2}{4\alpha}\right)\right]} \times \left[\alpha\left(\frac{C_1^2}{\alpha^2}\right)^{\frac{1}{4}} K_{\frac{1}{2}}\left(\frac{\beta|C_1|}{2}\right) - \gamma K_0\left(\frac{\beta|C_1|}{2}\right) + C_1\left(\frac{C_1^2}{\alpha^2}\right)^{\frac{1}{4}} K_{\frac{1}{2}}\left(\frac{\beta|C_1|}{2}\right)\right] \tag{10b}$$

$$B_2 = \frac{\theta}{2}\exp\left\{\beta\left[\varepsilon_F - (N + \frac{1}{2})\hbar\omega_p + \frac{e^2E_1^2}{2m^*\omega_p^2} - \frac{C_1}{2} + \frac{\gamma^2}{4\alpha}\right]\right\}\times$$
$$\times\left\{\alpha\left(\frac{C_1^2}{\alpha^2}\right)^{\frac{3}{4}} K_{\frac{3}{2}}\left(\frac{\beta|C_1|}{2}\right) - \gamma\left(\frac{\beta|C_1|}{2}\right)^{\frac{1}{2}} K_1\left(\frac{\beta|C_1|}{2}\right) + C_1\left(\frac{C_1^2}{\alpha^2}\right)^{\frac{1}{4}} K_{\frac{1}{2}}\left(\frac{\beta|C_1|}{2}\right)\right\}; \quad with \quad \theta = \frac{e^2E_0^2}{m^{*2}\Omega^4}\left(1 - \frac{\omega_c^2}{\omega_p^2}\right) \tag{10c}$$

$$B_3 = \frac{-\theta}{4}\exp\left\{\beta\left[\varepsilon_F - (N + \frac{1}{2})\hbar\omega_p + \frac{e^2E_1^2}{2m^*\omega_p^2} - \frac{C_2}{2} + \frac{\gamma^2}{4\alpha}\right]\right\}\times$$
$$\times\left\{\alpha\left(\frac{C_2^2}{\alpha^2}\right)^{\frac{3}{4}} K_{\frac{3}{2}}\left(\frac{\beta|C_2|}{2}\right) - \gamma\left(\frac{\beta|C_2|}{2}\right)^{\frac{1}{2}} K_1\left(\frac{\beta|C_2|}{2}\right) + C_2\left(\frac{C_2^2}{\alpha^2}\right)^{\frac{1}{4}} K_{\frac{1}{2}}\left(\frac{\beta|C_2|}{2}\right)\right\} \tag{10d}$$

$$B_4 = \frac{-\theta}{4}\exp\left\{\beta\left[\varepsilon_F - (N + \frac{1}{2})\hbar\omega_p + \frac{e^2E_1^2}{2m^*\omega_p^2} - \frac{C_3}{2} + \frac{\gamma^2}{4\alpha}\right]\right\}\times$$
$$\times\left\{\alpha\left(\frac{C_3^2}{\alpha^2}\right)^{\frac{3}{4}} K_{\frac{3}{2}}\left(\frac{\beta|C_3|}{2}\right) - \gamma\left(\frac{\beta|C_3|}{2}\right) + C_3\left(\frac{C_3^2}{\alpha^2}\right)^{\frac{1}{4}} K_{\frac{1}{2}}\left(\frac{\beta|C_3|}{2}\right)^{\frac{1}{4}} K_{\frac{1}{2}}\left(\frac{\beta|C_3|}{2}\right)\right\} \tag{10e}$$

$B_5 = B_1$ ($C_1 \rightarrow D_1$), $B_6 = B_2$ ($C_1 \rightarrow D_1$), $B_7 = B_3$ ($C_2 \rightarrow D_2$), $B_8 = B_4$ ($C_3 \rightarrow D_3$) (10f)

And the **THC** is given by the formula [16]:

$$R_H = \frac{\rho_{zx}}{B} = -\frac{1}{B}\frac{\sigma_{zx}}{\sigma_{zx}^2 + \sigma_{zz}^2}; \sigma_{zx} = \frac{\tau}{\left\{1 + \omega_c^2\tau^2\right\}}\left\{a + \frac{be\tau}{\mathrm{m}^*}\left[1 + \omega_c^2\tau^2\right]^{-1}\left[1 - \omega_c^2\tau^2\right]\right\};$$
$$\sigma_{zz}\frac{\tau}{\left\{1 + \omega_c^2\tau^2\right\}}\left\{a + \frac{be\tau}{\mathrm{m}^*}\left[1 + \omega_c^2\tau^2\right]^{-1}\left[1 - \omega_c^2\tau^2\right]\right\} \tag{11}$$

Formulae (8) and (11) show the dependence of the THCT and the THC on the external fields, the temperature T of the system, the quantum well width L, and especially the quantum numbers n, m characterizing the electron and phonon confinement, respectively. When m goes to zero, we obtain results as the case of bulk phonon.

3. Numerical results and discussions

In this section, we present the numerical evaluation of THC for the AlAs/GaAs/AlAs QW. Parameters used in this calculation are as follows: $m^* = 0.067m_o$, (m_o is the free mass of an electron), $\chi_\infty = 10.9$, $\chi_o = 12.9$, $\varepsilon_F = 50$meV, $\tau = 10^{-12}$s, $\nu = 8.73 \times 10^4 \text{ms}^{-1}$, $n_o = 10^{20}\text{m}^{-3}$, $\omega_o = 36.6$meV , $\Omega = 6.5 \times 10^{12}\text{s}^{-1}$, T = 290K, $L_x = L_y = 100$nm, $E_1 = 2.10^2$V/m.

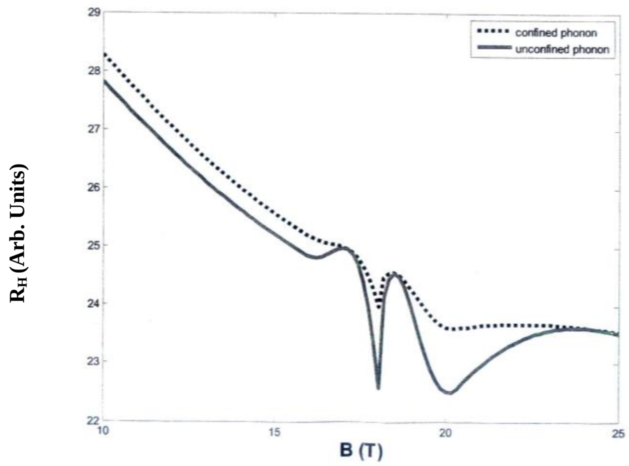

Figure 1. The dependence of THC on the B of MF in both case of confined phonon (dashed curve) and unconfined phonon (solid curve).

Figure 1 shows that the strong and nonlinear dependence of the THC on the B of MF for both cases of confined and unconfined phonons. Especially, there are appearing clearly two resonant peaks of the THC at B = 18,5T and B = 20,5T. The resonants pick in case of confined phonon is higher than that in case of unconfined phonons. This results shows that the confined phonons have increased the value of THC about 5 percentage.

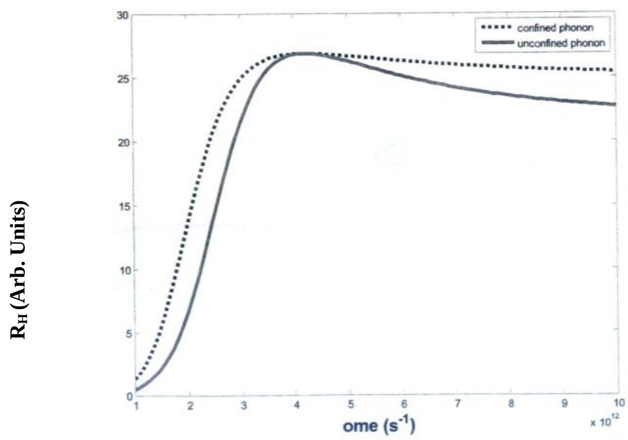

Figure 2. The dependence of THC on the Ω of a laser radiation in both case of confined phonon (dashed curve) and unconfined phonon (solid curve).

Figure 2 shows that the THC is nonlinear fuction of the frequency of the laser radiation for both cases of confined and unconfined phonons. THC' value increases fast as the value of the frequency of the laser radiation (from $1.10^{12}s^{-1}$ to $4.10^{12}s^{-1}$) and decreases as the value of the frequency of the laser radiation (from $4.10^{12}s^{-1}$ to $10.10^{12}s^{-1}$). The results also show that THC increases about 20 percentage in case of confined phonons.

4. Conclusion

In this work, the influence of confined optical phonons on the THE in a QW with high infinite potential under the presence of an intense EMW is studied by using quantum kinetic equation method. The analytical expressions for the THCT and the THC are obtained. The THCT and the THC dependence complex on the external fields, the temperature T of the system, the QW width L, and especially the quantum numbers n, m characterizing the electron and phonon confinement. When m goes to zero we have results as the case of bulk phonon in the QW. Numerical calculation is also applied for AlGaAs/GaAs/AlGaAs QW. Results show that the strong and nonlinear dependence of the THC on the B of MF and the Ω of laser radiation. All the results show that the THC has been enhanced much strongly in value under influences of confined phonons and a EMW but not in the posture.

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