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A BLACK-BOX MODEL OF OPTICAL AMPLIFIER CHAINS IN OPTICAL FIBRE TELECOMMUNICATIONS SYSTEMS

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Abstract. A simple model is proposed to evaluate the number of amplifiers and the required power in optical fibre transmission systems using a chain of EDFA. The model requires only knowledge of the overall gain of the amplifier chain, the noise figure and the amplifier spacing beside the line's parameters. The influence of attenuation and dispersion of the fiber cable are also taken into account.

Index terms. Power budget of optical fibre transmission systems, fibre attenuation and dispersion, technical parameters of in-line EDFA, bit error rate (BER).

I. Introduction

In recent years, the erbium-doped fiber amplifiers (EDFA) have revolutionized the field of long haul transmission. Typical long distance links installed today contain an optical power amplifier at the transmitter, a series of in-line amplifiers (chain of EDFA) along the transmission line and a preamplifier at the receiver (figure 1). The number of 3-R regenerators (normal repeaters) along the transmission line has substantially dropped in comparison with traditionally equipped links. Therefore, the predictability of the amplifier figures in general, the in-line amplifier figures in particular plays an important role for the layout of such transmission systems.

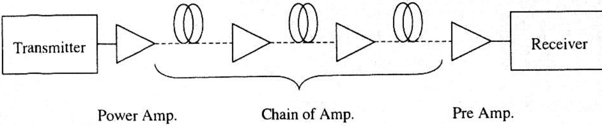

Figure 1. A typical optical transmission system

In this paper, a simple model is proposed to compute the number of chain's amplifiers based on the overall gain, to compute the launched power when amplifier spacing of the chain is given. In short transmission systems using in-line amplifiers, the number of amplifiers can simply be found by dividing the transmission length to the amplifier spacing. But in long haul links, the influence of attenuation and dispersion might take place during the operation. Thus, to predict the number of amplifiers in the chain, one should

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take the attenuation, the dispersion and some technical parameters of in-line EDFA [2,3] into account. And a possible approach is following the power budget [1,6].

There are some solutions to overcome the effects of attenuation and dispersion. One might be adding a penalty power P_{Pen} to the power budget [2]. This solution ensures to remain BER unchanged. In optical transmission systems using EDFA, BER is usually measured in terms of the Q factor [4]. In turn, the Q factor can be calculated through signal to noise ratio SNR, launched power P_{in}, inversion parameter n_{sp}, etc. Briefly, the number of amplifiers and the launched power depend on many parameters. In other words, they are the solutions of a complicated equations system. However, some EDFA's parameters (n_{sp}, SNR,...) can be considered as intermediate parameters, thus can be skipped in the final solutions.

II. Basic equations

The simplest way to analyze a cascade of optical amplifiers is to assume that all amplifiers have the same gain and that the loss between amplifiers exactly matches the amplifier gain. The signal output power at the end of the chain is supposed to be equal to the signal input power at the beginning of the chain. It means that the amplified spontaneous emission factor (ASE) generated is small in comparison with the signal power. And each amplifier generates an equal amount of ASE as the other amplifiers, and this ASE propagates transparently to the output of the chain, much as the signal does. Thus, the ASE at the output of the chain is linear addition of the ASE generated by each amplifier. There are two conceptual configurations for using the amplifier chain in optical transmission systems. We consider the case of the configuration shown in fig. 2, where amplifier placed before the lossy fiber span.

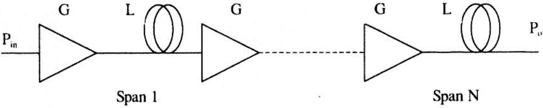

Figure 2. Amplifier before the lossy fiber span

Suppose that the input power, P_{in}, to the chain is independent of the amplifier gain G and of the span loss L. The signal input to the receiver is P_{in}/L. The SNR at the output of the amplifier chain can then be written as

$$SNR = \frac{P_{in}}{4Nn_{sp}h\nu B_e(G-1)L}, \qquad (1)$$

where P_{in} is launched power at the beginning of the chain; N is the number of amplifiers in the chain; n_{sp} is the inversion parameter; h is Planck's constant; ν is optical frequency (Hz); G is gain of the optical amplifiers; L is the span loss; B_e is electrical bandwidth (Hz).

The total gain G_{tot} and the total loss L_{tot} of the system are given by

$$G_{tot} = \frac{1}{L_{tot}} = G^N = \frac{1}{L^N}. \tag{2}$$

Therefore, SNR can be written otherwise

$$SNR = \frac{P_{in}}{4n_{sp}h\mu B_e} \frac{G_{tot}^{1/N}}{(G_{tot}^{1/N} - 1)N}. \tag{3}$$

Eq. 3 yields the result that the SNR of the system is maximized when $N = 1$, i.e. there should be only one amplifier and the amplifier spacing should be as long as possible. But the long amplifier spacing requires the high gain. In fact, practical limitation renders it impossible to construct an amplifier having too high gain or too high output power.

We now turn our attention to the case of the configuration illustrated in figure 3

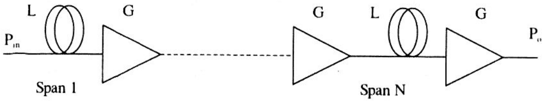

Figure 3. Amplifier after the lossy fiber span

The SNR at the output of the transparent amplifier chain is given by

$$SNR = \frac{P_{in}}{4Nn_{sp}h\nu B_e(G-1)}. \tag{4}$$

The difference between two configurations is that in the first case, P_{in} is measured at the input of the amplifier, whereas in the second case, P_{in} is measured at the beginning of the chain. Similarly to the previous derivation, we can write

$$SNR = \frac{P_{in}}{4n_{sp}h\nu B_e} \frac{1}{(G_{tot}^{1/N} - 1)N} = \frac{P_{in}}{4n_{sp}h\nu B_e} \frac{1}{Ln(G_{tot})} \frac{L(G)}{(G-1)}. \tag{5}$$

From Eq. 5, we can see that SNR is improved by increasing the number of amplifiers N and reducing the gain G, correspondingly. And when the number N goes to infinity, the SNR is maximized and equal to

$$SNR = \frac{P_{in}}{4n_{sp}h\nu B_e} \frac{1}{Ln(G_{tot})}. \tag{6}$$

Generally speaking, quality of optical transmission systems can be evaluated through BER. In term of the Q factor [3], BER can be well approximated by:

$$BER = \frac{1}{\sqrt{2\pi}} \frac{\exp(-Q^2/2)}{Q}. \tag{7}$$

Additionally, the definition of Q is given by:

$$Q = \frac{I_s(1) - I_s(0)}{\sqrt{N_{tot}(1)} + \sqrt{N_{tot}(0)}}, \tag{8}$$

where $I_s(1), I_s(0)$ and $N_{tot}(1), N_{tot}(0)$ are the signal and total noise for a bit "1" and a bit "0", respectively. Assume an infinite extinction ration, such that $I_s(0) = 0$ and that the noise on the raise "1" is the same as that on the raise "0", i.e. $N_{tot}(1) = N_{tot}(0) = N_{tot}$. The Q factor is then presented as

$$Q = \frac{I_s(1)}{2\sqrt{N_{tot}}}, \tag{9}$$

where N_{tot} is the total noise power. Squaring both sides of Eq. 4, we obtain:

$$Q^2 = \frac{I_s^2(1)}{4N_{tot}} = \frac{SNR}{4}. \tag{10}$$

The noise power P_{ASE} at the signal wavelength is calculated from the expressions for spontaneous emission noise n_{sp} [2,3]:

$$P_{ASE} = 2n_{sp}(G-1)h\nu B_0. \tag{11}$$

In this expression, B_o is equal to bandpass of the optical spectrum and G is the gain of amplifiers at the signal frequency ν. The gain of an amplifier is expressed as the ratio between the input signal level and the output signal level, typically expressed in dB.

$$G(dB) = 10\log_{10}\left(\frac{P_{signal-out}}{P_{signal-in}}\right), \tag{12}$$

when taking the effect of P_{ASE} into account, G can be generally written as

$$G = 10\log_{10}\frac{(P_{signal-out} + P_{ASE}) - P_{ASE}}{P_{signal-in}}. \tag{13}$$

A common way to characterize the performance of an amplifier chain is through its noise figure NF_{sys}. The noise figure of an amplifier NF is defined as the ratio of the SNR at the input to that at the output of the amplifier:

$$NF = \frac{(SNR)_{in}}{(SNR)_{out}}. \tag{14}$$

The noise figure can also be simply written in terms of the ASE power:

$$NF = \frac{P_{ASE}}{h\nu B_0 G} + \frac{1}{G}, \tag{15}$$

where B_0 is the optical bandwidth. Consider a system of N amplifiers where SNR_i denotes the SNR after amplifier i. Assume that each amplifier provides a gain G to exactly compensate the loss L between amplifiers. The overall noise figure of the system is given by:

$$NF_{sys} = \frac{SNR_0}{SNR_N} = \frac{SNR_0}{SNR_1}\frac{SNR_1}{SNR_2}\frac{SNR_{N-1}}{SNR_N}, \tag{16}$$

where SNR_0 is the SNR at the input of the chain. Thus, the SNR ratios in Eq. 16 are the noise figures of each amplifier divided to L. In logarithmic units, the system noise figure is presented by:

$$NF_{sys} = GNF_1 + GNF_2 + \ + GNF_N = NGNF. \tag{17}$$

where we assume each amplifier has the same noise figure and $G = L$.

Apply the method as that in previous paper [6], i.e. apply the link loss budget method and take the attenuation and the dispersion into consideration, we obtain:

$$P_T = P_S - P_R = (\alpha_f + \alpha_j)D + \alpha_{cr} + D_L + System\ margin, \tag{18}$$

where P_T is total optical power loss (no repeater); P_S is optical power emerging from the end of a fiber flylead attached to the light source; P_R is receiver sensitivity; α_f is fibre attenuation; α_j is splice loss; α_{cr} is connector loss; D is transmission length; D_L is the loss penalty; $System margin$ concerns components aging due to temperature fluctuation.

D_L can be calculated through the root-mean-square spectral width σ and the bit rate B_T:

$$D_L(dB) = (2\sqrt{2}\sigma B_T)^4. \tag{19}$$

Combine with the above amplifier chain equations, we can easily predict the number of amplifiers in the chain as well as the minimal input power launched to the chain.

III. Simulation and serults

The black box model proposed in this paper bases on main amplifier chain parameters (G, NF, P_{in}, P_{ASE}, etc.), on line parameters (optical power, receiver sensitivity, wavelength, attenuation, dispersion, transmission length, bit rate, BER, system margin, etc.). The simulation packages are written in Visual Basic language and executed on Window platform [5].

From the above equations, we can find out the relation between the number of amplifiers in the chain and the gain of the amplifiers (figure 4).

- Spectrum width: 0.1 nm
- Wavelength: 1550 nm
- Attenuation: 0.20 dB/km
- Dispersion: 15 ps/nm/km
- Bit rate: 2.5 Gb/s
- BER: 10-12
- NF: 6 dB
- Launched power: 2.5 mW

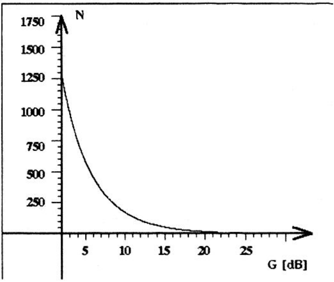

Figure 4. Maximum number of amplifiers cascaded in a chain in function of gain

From Fig. 4, one can see that when increasing the gain, the number of amplifiers reduces. It is due to the effect of the noise power. By increasing G, noise power generated at the output of each amplifier grows. Therefore, the total noise power at the output of the chain, which is the sum of all individual noise powers, increases. It makes the SNR (i.e. the BER) reduced. In other words, the SNR degradation in a cascaded amplifier chain is seen to be linear with the number of amplifiers. In order to remain the BER unchanged, one should reduce the number of amplifiers.

The relation between required power at the beginning of the chain and the amplifier spacing is illustrated in figure 5.

- Transmission distance: 2000 km
- Spectrum width: 0.1 nm
- Wavelength: 1550 nm
- Attenuation: 0.20 dB/km
- Dispersion: 15 ps/nm/km
- Bit rate: 2.5 Gb/s
- BER: 10^{-12}
- NF: 6 dB

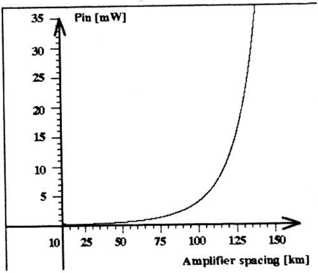

Figure 5 Required signal power in function of amplifier spacing

One can be seen from Fig 5, the required signal power needed to maintain BER get higher with increasing the amplifier spacing. When increasing the amplifier spacing, the loss between adjacent amplifiers grows. Thus, one should increase the gain of each amplifier to compensate the loss. Unfortunately, increasing gain makes noise power increased. To maintain BER, input signal power should be increased too.

For the given overall gain, noise figure, launched power and transmission's length, in combining Fig. 4 and Fig. 5, we can easily predict the necessary number of EDFAs as well as the amplifier spacing (i.e. we will know how many EDFAs should be used and where to place them).

Practically, the output power of the light source is about some mW. Thus, the launched power is not very high. The typical value of EDFA's gain is around 10 dB. When the launched power or the EDFA's gain is rather high, the effect of EDFA's saturation may take placed.

The relations between the number of amplifiers in the chain and the amplifier gain , between the required power at the beginning of the chain and the amplifier spacing are considered [2], where some other EDFA's parameters (SNR, n_{sp},...) are mentioned while the effect of the distortions are rejected. The difference in this model is that the parameters are taken from the technical point of view.

IV. Conclution

A black box model of optical amplifier chains has been derived. The relations between the number of amplifiers and the amplifier gain, between input required power and the amplifier spacing have been shown. With these simulations, we can predict the number of amplifiers as well as the required power at the beginning of the chain. Accompany with simulations presented in previous papers [6, 7], we have proposed some packaged simulations in the hope that these may facilitate the choice of suitable EDFA, of EDFA's number and thus minimize the number of needed components, reduce the system cost.

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