ON THE ELASTOPLASTIC STABILITY PROBLEM OF THE CYLINDRICAL PANELS SUBJECTED TO THE COMPLEX LOADING WITH THE SIMPLY SUPPORTED AND CLAMPED BOUNDARY CONSTRAINTS

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Abstract. In this paper, an elastoplastic stability problem of the cylindrical panels under the action of the compression force along the generatrix and external pressure has been investigated. By the Bubnov-Galerkin method, we have established the expression for determining the critical loads. The sufficient condition of extremum for a long cylindrical panels was considered. Some numerical results have been also given and discussed.

1. Formulation of the stability problem and fundamental equations

Let us consider a round cylindrical panel of thickness h and radius of the middle surface equal to R. We choose a orthogonal coordinate system $Oxyz$ so that the plane Oxy coincides with the middle surface and the axis Ox lies along the generatrix of cylindrical panel while $y = R\theta_1$ with θ_1-the angle circular arc and z in the direction of the normal to the middle surface. Denote the sides of cylindrical panel by a and b respectively to the axis Ox and Oy.

Suppose that the cylindrical panel is simultaneously subjected to the compression force of intensity $p(t)$ along the generatrix and external pressure of intensity $q_1(t)$ increasing monotonously and depending arbitrarily on any loading parameter t. We have to find the critical values $t = t_*$, $p_* = p(t_*)$, $q_{1*} = q_1(t_*)$ at which an instability of the structures appears. In order to investigate the proposed we will use the criterion of bifurcation of equilibrium states and dont take into account the unloading in the cylindrical panel. Afterhere we will present the fundamental equations of stability problem.

1.1. Pre-buckling process

Suppose that at any moment t in the pre-buckling stage, there exists a membrane plane stress state

$$\sigma_{xx} = -p(t) \equiv -p, \quad \sigma_{yy} = -q_1(t)\frac{R}{h} \equiv -q(t) \equiv -q,$$
$$\sigma_{12} = \sigma_{13} = \sigma_{23} = \sigma_{33} = 0;$$
$$\sigma = -\frac{1}{3}(p+q), \quad \sigma_u = (p^2 - pq + q^2)^{1/2}. \tag{1.1}$$

The material is assumed to be incompressible

$$\varepsilon_{33} = -(\varepsilon_{11} + \varepsilon_{22}).$$

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The components of the strain velocity tensor determined according to the theory of elastoplastic processes are of the form [1]

$$\begin{aligned} \dot{\varepsilon}_{11} &= \frac{1}{N}(-\dot{p} + \frac{1}{2}\dot{q}) - Q(s,t)(p - \frac{1}{2}q), \\ \dot{\varepsilon}_{22} &= \frac{1}{N}(-\dot{q} + \frac{1}{2}\dot{p}) - Q(s,t)(q - \frac{1}{2}p), \\ \dot{\varepsilon}_{33} &= -(\dot{\varepsilon}_{11} + \dot{\varepsilon}_{22}); \quad \dot{\varepsilon}_{12} = \dot{\varepsilon}_{13} = \dot{\varepsilon}_{23} = 0, \end{aligned} \tag{1.2}$$

where

$$Q(s,t) = \Big(\frac{1}{\phi'} - \frac{1}{N}\Big)\frac{p\dot{p} + q\dot{q} - \frac{1}{2}p\dot{q} - \frac{1}{2}\dot{p}q}{p^2 - pq + q^2}, \quad \phi' = \phi'(s), \quad N = \frac{\sigma_u}{s}.$$

The arc-length of the strain trajectory is respectively calculated by the formula

$$\frac{ds}{dt} = \frac{2}{\sqrt{3}}\big(\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{11}\dot{\varepsilon}_{22} + \dot{\varepsilon}_{22}^2\big)^{1/2} \equiv F(s,t). \tag{1.3}$$

So the equilibrium equations associated with the relations (1.2), (1.3) and boundary conditions entirely define the stress and strain state at any point M in the structure at any moment of pre-buckling process.

1.2. Post-buckling process

The system of stability equations of the thin cylindrical panel established in [5] is written in the form

$$\alpha_1\frac{\partial^4\delta w}{\partial x^4} + \alpha_3\frac{\partial^4\delta w}{\partial x^2\partial y^2} + \alpha_5\frac{\partial^4\delta w}{\partial y^4} + \frac{9}{h^2N}\Big(p\frac{\partial^2\delta w}{\partial x^2} + q\frac{\partial^2\delta w}{\partial y^2} - \frac{1}{R}\frac{\partial^2\varphi}{\partial x^2}\Big) = 0, \tag{1.4}$$

$$\beta_1\frac{\partial^4\varphi}{\partial x^4} + \beta_3\frac{\partial^4\varphi}{\partial x^2\partial y^2} + \beta_5\frac{\partial^4\varphi}{\partial y^4} + \frac{N}{R}\frac{\partial^2\delta w}{\partial x^2} = 0, \tag{1.5}$$

where the coefficients α_i, β_i $(i = 1,3,5)$ are calculated as follows

$$\begin{aligned} &\alpha_1 = 1 - \frac{3}{4}\Big(1 - \frac{\phi'}{N}\Big)\frac{p^2}{\sigma_u^2}; \quad \alpha_3 = 2 - \frac{3}{2}\Big(1 - \frac{\phi'}{N}\Big)\frac{pq}{\sigma_u^2} \\ &\alpha_5 = 1 - \frac{3}{4}\Big(1 - \frac{\phi}{N}\Big)\frac{q^2}{\sigma_u^2}; \quad \beta_1 = 1 + \frac{1}{4}\Big(\frac{N}{\phi'} - 1\Big)\frac{(2q-p)^2}{\sigma_u^2}; \\ &\beta_3 = 2 + \frac{1}{2}\Big(\frac{N}{\phi'} - 1\Big)\frac{(2p-q)(2q-p)}{\sigma_u^2}; \quad \beta_5 = 1 + \frac{1}{4}\Big(\frac{N}{\phi'} - 1\Big)\frac{(2p-q)^2}{\sigma_u^2}. \end{aligned} \tag{1.6}$$

For solving the stability problem of cylindrical panel, we consider two types of kinematic constraints following

* The cylindrical panel is simply supported at the four edges $x = 0$, $x = a$, $y = 0$, $y = b$.

* The cylindrical panel is simply supported at the edges $y = 0$, $y = b$ and clamped at the edges $x = 0$, $x = a$.

2. The solving method for the simply supported cylindrical panel at four edges

We find the increment of deflection δw in the form

$$\delta w = \sum_{m=1}^{M}\sum_{n=1}^{M} A_{mn} \sin\frac{n\pi x}{a} \sin\frac{2n\pi y}{b} \tag{2.1}$$

it is easy to see that this solution satisfies the kinematic boundary conditions.

Substituting the expression of δw into (1.5) we receive the particular solution φ as follows

$$\varphi = \sum_{m=1}^{M}\sum_{n=1}^{M} B_{mn} \sin\frac{m\pi x}{a} \sin\frac{2n\pi y}{b}, \tag{2.2}$$

where

$$B_{mn} = \frac{N}{R}\Big(\frac{m\pi}{a}\Big)^2 A_{mn}\Big[\beta_1\Big(\frac{m\pi}{a}\Big)^4 + \beta_3\Big(\frac{m\pi}{a}\Big)^2\Big(\frac{2n\pi}{b}\Big)^2 + \beta_5\Big(\frac{2n\pi}{b}\Big)^4\Big]^{-1}. \tag{2.3}$$

It is seen that the system of functions

$$\delta w_{mn} = \sin\frac{m\pi x}{a} \sin\frac{2n\pi y}{b} \quad (m, n = 1, 2, \ldots, M)$$

is linearly independent. Therefore we can apply the Bubnov-Galerkin method for establishing an expression of critical forces.

First of all, substitute the expressions of δw and φ from (2.1), (2.2) into (1.4), afthward multiply both sides of the just received equation by $\delta w_{ij} = \sin\frac{i\pi x}{a}\sin\frac{2j\pi y}{b}$ and integrate that equation following x and y. Finally we get

$$\int_0^a\int_0^b \Big[\alpha_1\frac{\partial^4\delta w}{\partial x^4} + \alpha_3\frac{\partial^4\delta w}{\partial x^2\partial y^2} + \alpha_5\frac{\partial^4\delta w}{\partial y^4} + \frac{9}{h^2N}\Big(p\frac{\partial^2\delta w}{\partial x^2} + q\frac{\partial^2\delta w}{\partial y^2} - \frac{1}{R}\frac{\partial^2\varphi}{\partial x^2}\Big)\Big]$$
$$\sin\frac{i\pi x}{a}\sin\frac{2j\pi y}{b}dxdy = 0 \quad (i, j = 1, 2, \ldots, M). \tag{2.4}$$

For taking this integral, it needs to use the result

$$\int_0^a\int_0^b \sin\frac{m\pi x}{a}\sin\frac{i\pi x}{a}\sin\frac{2n\pi y}{b}\sin\frac{2j\pi y}{b} = \begin{cases} 0 & \text{with } m \neq i;\ n \neq j \\ \dfrac{ab}{4} & \text{with } m = i,\ n = j. \end{cases}$$

After series of calculations, the relation (2.4) gives us

$$\frac{ab}{4}\Big\{\alpha_1\Big(\frac{m\pi}{a}\Big)^4 + \alpha_3\Big(\frac{m\pi}{a}\Big)^2\Big(\frac{2n\pi}{b}\Big)^2 + \alpha_5\Big(\frac{2n\pi}{b}\Big)^4 - \frac{9}{h^2N}\Big[p\Big(\frac{m\pi}{a}\Big)^2 + q\Big(\frac{2n\pi}{b}\Big)^2\Big]$$
$$+ \frac{9}{h^2R^2}\Big(\frac{m\pi}{a}\Big)^4\Big[\beta_1\Big(\frac{m\pi}{a}\Big)^4 + \beta_3\Big(\frac{m\pi}{a}\Big)^2\Big(\frac{2n\pi}{b}\Big)^2 + \beta_5\Big(\frac{2n\pi}{b}\Big)^4\Big]^{-1}\Big\}A_{mn} = 0. \tag{2.5}$$

Because of the condition on the existence of non-trivial solution i.e. $A_{mn} \neq 0$ then we receive the expression for determining critical loads

$$\alpha_1\left(\frac{m\pi}{a}\right)^4+\alpha_3\left(\frac{m\pi}{a}\right)^2\left(\frac{2n\pi}{b}\right)^2+\alpha_5\left(\frac{2n\pi}{b}\right)^4-\frac{9}{h^2N}\left[p\left(\frac{m\pi}{a}\right)^2+q\left(\frac{2n\pi}{b}\right)^2\right]$$
$$+\frac{9}{h^2R^2}\left(\frac{m\pi}{a}\right)^4\left[\beta_1\left(\frac{m\pi}{a}\right)^4+\beta_3\left(\frac{m\pi}{a}\right)^2\left(\frac{2n\pi}{b}\right)^2+\beta_5\left(\frac{2n\pi}{b}\right)^4\right]^{-1}=0. \quad (2.6)$$

Putting $X=\left(\dfrac{mb}{2na}\right)^2$, $Y=n^2$, $i=\dfrac{3b}{h}$; the relation (2.6) can be rewritten in the other form

$$i^2=\frac{4N\pi^2\left(\alpha_1X+\alpha_3+\dfrac{\alpha_5}{X}\right)\left(\beta_1X+\beta_3+\dfrac{\beta_5}{X}\right)Y^2}{Y(pX+q)\left(\beta_1X+\beta_3+\dfrac{\beta_5}{X}\right)-\dfrac{b^2N}{4\pi^2R^2}}. \quad (2.7)$$

Minimizing i^2, it means $\dfrac{\partial i^2}{\partial X}=0$, $\dfrac{\partial i^2}{\partial Y}=0$, that yields

$$Y=\frac{b^2N}{2\pi^2R^2\left(p+\dfrac{q}{X}\right)\left(\beta_1X+\beta_3+\dfrac{\beta_5}{X}\right)}; \quad (2.8)$$

$$\left(\alpha_1-\frac{\alpha_5}{X^2}\right)\left(\beta_1X+\beta_3+\frac{\beta_5}{X}\right)-\left(\beta_1-\frac{\beta_5}{X^2}\right)\left(\alpha_1X+\alpha_3+\frac{\alpha_5}{X}\right)$$
$$+\frac{2q}{X^2\left(p+\dfrac{q}{X}\right)}\left(\alpha_1X+\alpha_3+\frac{\alpha_5}{X}\right)\left(\beta_1X+\beta_3+\frac{\beta_5}{X}\right)=0. \quad (2.9)$$

Substituting the expressions (2.8) and (2.9) into (2.7) we obtain

$$i^2=\frac{4N^2b^2}{R^2\left(p+\dfrac{q}{X}\right)^2}\left\{\alpha_1X+\alpha_3+\frac{\alpha_5}{X}\right\}\left\{\beta_1X+\beta_3+\frac{\beta_5}{X}\right\}^{-1} \quad (2.10)$$

where X is found from the equation (2.9).

Applying the loading parameter method [1], and solving simultaneously the equation (1.3) and (2.10) we can find the critical values t_*, $p_*=p(t_*)$, $q_*=q(t_*)$.

For long cylindrical panel, i.e. $Y=1$, $X\ll 1$ we have from (2.7)

$$i^2=\frac{4N\pi^2\alpha_5\beta_5}{(pX+q)\beta_5-C_0X^2}; \quad C_0=\frac{b^2N}{4\pi^2R^2}. \quad (2.11)$$

Calculating $\dfrac{\partial i^2}{\partial X}=0$, leads us $X=\dfrac{p\beta_5}{2C_0}\equiv X_*$. In addition

$$\left.\frac{\partial^2 i^2}{\partial X^2}\right|_{X=X_*}=\frac{8C_0N\pi^2\alpha_5\beta_5}{\left(q\beta_5+\dfrac{p^2\beta_5^2}{4C_0}\right)^2}>0.$$

So the sufficient condition of extremum is verified.

Substituting the values of α_5, β_5 and $X = X_*$ into (2.11) we obtain

$$i^2 = \frac{4N^2b^2}{R^2}\left\{\frac{1-\frac{3}{4}\left(1-\frac{\phi'}{N}\right)\frac{q^2}{p^2-pq+q^2}}{p^2\left[1+\frac{1}{4}\left(\frac{N}{\phi'}-1\right)\frac{(2p-q)^2}{p^2-pq+q^2}\right]+\frac{b^2N}{\pi^2R^2}q}\right\}. \tag{2.12}$$

3. The solving method for the simple supported cylindrical panel at $y = 0$, $y = b$ simultaneously clamped at the sides $x = 0$, $x = a$

The kinematic boundary constraints of stability problem are satisfied completely by choosing

$$\delta w = \sum_{m=1}^{M}\sum_{n=1}^{M} C_{mn}\left(1-\cos\frac{2m\pi x}{a}\right)\sin\frac{2n\pi y}{b}. \tag{3.1}$$

Using the equation (1.5) and the expression of δw we can find the particular solution φ in the form

$$\varphi = \sum_{m=1}^{M}\sum_{n=1}^{M} D_{mn}\cos\frac{2m\pi x}{a}\sin\frac{2n\pi y}{b} \tag{3.2}$$

where

$$D_{mn} = -\frac{N}{R}\left(\frac{2m\pi}{a}\right)^2\left[\beta_1\left(\frac{2m\pi}{a}\right)^4+\beta_3\left(\frac{2m\pi}{a}\right)^2\left(\frac{2n\pi}{b}\right)^2+\beta_5\left(\frac{2n\pi}{b}\right)^4\right]^{-1}. \tag{3.3}$$

It is possible to prove that the system of functions $\delta w_{mn} = \left(1-\cos\frac{2m\pi x}{a}\right)\sin\frac{2n\pi y}{b}$ is linearly independent. Then we can use the Bubnov-Galerkin. By the same method presented in the above part we change the equation (1.4) into a relation as follows

$$\int_0^a\int_0^b\left\{\alpha_1\frac{\partial^4\delta w}{\partial x^4}+\alpha_3\frac{\partial^4\delta w}{\partial x^2\partial y^2}+\alpha_5\frac{\partial^4\delta w}{\partial y^4}+\frac{9}{h^2N}\left(p\frac{\partial^2\delta w}{\partial x^2}+q\frac{\partial^2\delta w}{\partial y^2}\right)\right.$$
$$\left.-\frac{9}{h^2NR}\frac{\partial^2\varphi}{\partial x^2}\right\}\left(1-\cos\frac{2i\pi x}{a}\right)\sin\frac{2j\pi y}{b}dxdy = 0 \quad (i,j=1,2,\ldots,M). \tag{3.4}$$

For taking this integral above all we substitute δw and φ represented by (3.1) and (3.2) into (3.4), afterwards integrate that received expression. We will obtain a system of linear algebraic equations with the unknowns C_{ij} which is written in the matrix form

$$[a_{ij}][C_{ij}] = 0; \quad i,j = 1,2,\ldots,M. \tag{3.5}$$

Because of the condition on the existence of non-trivial solution i.e. $C_{ij} \neq 0$ then the determinant of the coefficients of C_{ij} must be equal to zero

$$\det[a_{ij}] = 0; \quad i,j = 1,2,\ldots,M. \tag{3.6}$$

Associating this expression with (1.3) we can determine the critical values t_*, $p_* = p(t_*)$, $q_* = q(t_*)$.

Note that a development of the determinant (3.6) in general case is complicated mathematically therefore we will take the solution in the first approximation.

In this case we choose δw and φ in the form

$$\delta w = C_{mn}\left(1 - \cos\frac{2m\pi x}{a}\right)\sin\frac{2n\pi y}{b},$$
$$\varphi = \frac{-\dfrac{N}{R}\left(\dfrac{2m\pi}{a}\right)^2 C_{mn}}{\beta_1\left(\dfrac{2m\pi}{a}\right)^4 + \beta_3\left(\dfrac{2m\pi}{a}\right)^2\left(\dfrac{2n\pi}{b}\right)^2 + \beta_5\left(\dfrac{2n\pi}{b}\right)^4}\cos\frac{2m\pi x}{a}\sin\frac{2n\pi y}{b}. \tag{3.7}$$

Substituting δw and φ into (3.4), integrating that relation and taking into account the condition $C_{mn} \neq 0$, leads us

$$\frac{ab}{4}\left\{\alpha_1\left(\frac{2m\pi}{a}\right)^4 + \alpha_3\left(\frac{2m\pi}{a}\right)^2\left(\frac{2n\pi}{b}\right)^2 + 3\alpha_5\left(\frac{2n\pi}{a}\right)^4 - \frac{9}{h^2N}\left[p\left(\frac{2m\pi}{a}\right)^2 + 3q\left(\frac{2n\pi}{b}\right)^2\right]\right.$$
$$\left. + \frac{9}{h^2R^2}\left(\frac{2m\pi}{a}\right)^4\left[\beta_1\left(\frac{2m\pi}{a}\right)^4 + \beta_3\left(\frac{2m\pi}{a}\right)^2\left(\frac{2n\pi}{b}\right)^2 + \beta_5\left(\frac{2n\pi}{b}\right)^4\right]^{-1}\right\} = 0. \tag{3.8}$$

Using notations $\xi = n^2$; $\eta = \left(\dfrac{mb}{na}\right)^2$; $i = \dfrac{3b}{h}$, the equation (3.8) is rewritten as follows

$$i^2 = \frac{4N\pi^2\left(\alpha_1\eta + \alpha_3 + \dfrac{3\alpha_5}{\eta}\right)\left(\beta_1\eta + \beta_3 + \dfrac{\beta_5}{\eta}\right)\xi^2}{\xi\left(p + \dfrac{3q}{\eta}\right)\left(\beta_1\eta + \beta_3 + \dfrac{\beta_5}{\eta}\right) - \dfrac{b^2N}{4\pi^2R^2}}. \tag{3.9}$$

Minimizing this relation i.e. $\dfrac{\partial i^2}{\partial \xi} = 0$, $\dfrac{\partial i^2}{\partial \eta} = 0$, gives us

$$\xi = \frac{b^2N}{2\pi^2R^2\left(p + \dfrac{3q}{\eta}\right)\left(\beta_1\eta + \beta_3 + \dfrac{\beta_5}{\eta}\right)}, \tag{3.10}$$

$$\left(\alpha_1 - \frac{3\alpha_5}{\eta^2}\right)\left(\beta_1\eta + \beta_3 + \frac{\beta_5}{\eta}\right) - \left(\beta_1 - \frac{\beta_5}{\eta^2}\right)\left(\alpha_1\eta + \alpha_3 + \frac{3\alpha_5}{\eta}\right)$$
$$+ \frac{6q}{\left(p + \dfrac{3q}{\eta}\right)\eta^2}\left(\alpha_1\eta + \alpha_3 + \frac{3\alpha_5}{\eta}\right)\left(\beta_1\eta + \beta_3 + \frac{\beta_5}{\eta}\right) = 0. \tag{3.11}$$

Putting the just found values of ξ and η into (3.9) we have

$$i^2 = \frac{4N^2b^2}{R^2\left(p + \dfrac{3q}{\eta}\right)^2}\left\{\alpha_1\eta + \alpha_3 + \frac{3\alpha_5}{\eta}\right\}\left\{\beta_1\eta + \beta_3 + \frac{\beta_5}{\eta}\right\}^{-1}, \tag{3.12}$$

where η is determined by the equation (3.11).

For finding the critical value t_* of loading parameter t, we need to solve simultaneously the equation (1.3) and (3.12).

After determining t_* we can obtain the critical forces as follows

$$p_* = p(t_*), \quad q_* = q(t_*).$$

Now consider the case of a long cylindrical panel. Based on [2] leads us

$$\xi = 1, \quad \eta \ll 1, \quad i^2 = \frac{12N\pi^2\alpha_5\beta_5}{(p\eta + 3q)\beta_5 - C_0\eta^2}; \quad C_0 = \frac{b^2N}{4\pi^2R^2}. \tag{3.13}$$

The minimization of the expression i^2 in (3.13), i.e. $\dfrac{\partial i^2}{\partial \eta} = 0$, yields $\eta = \dfrac{p\beta_5}{2C_0} = \eta_*$.

Moreover

$$\frac{\partial^2 i^2}{\partial \eta^2}\bigg|_{\eta=\eta_*} = \frac{24N\pi^2\alpha_5\beta_5C_0}{\left(3q\beta_5 + \dfrac{p^2\beta_5^2}{4C_0}\right)^2} > 0. \tag{3.14}$$

So the sufficient condition of extremum is satisfied. Taking into account α_5, β_5, η_*, the relation (3.13) becomes

$$i^2 = \frac{12b^2N^2}{R^2}\left\{\frac{1 - \dfrac{3}{4}\left(1 - \dfrac{\phi'}{N}\right)\dfrac{q^2}{p^2 - pq + q^2}}{p^2\left[1 + \dfrac{1}{4}\left(\dfrac{N}{\Phi'} - 1\right)\dfrac{(2p-q)^2}{p^2 - pq + q^2}\right] + \dfrac{3b^2N}{\pi^2R^2}q}\right\} \tag{3.15}$$

Remarks

1) If the cylindrical panel has a very small curvature i.e. $R \to +\infty$; $q = 0$ and $m = 1$, $n = 1$ then the expression (2.7) coincides with the result of [1, 5, 7].

2) If $b = 2\pi R$ that means the cylindrical panel becomes a closed round cylindrical shell, then the expressions (2.10), (2.12), (3.12), (3.15) return respectively to the previous well-known results.

4. Numerical calculations and discussion

A numerical analysis is considered on the long cylindrical panel made of the steel 30ХГСА with an elastic modulus $3G = 2.6 \cdot 10^5$ MPa, an yield point $\sigma_s = 400$ MPa and the material function $\phi(s)$ presented in [1].

The relations for determining the critical loads are given in the form:

* Formulae (2.12) and (1.3) for the part a) of the examples.

* Formulae (3.15) and (1.3) for the part b) of the examples.

The numerical results are realized by the program of MATLAB.

Example 1. The complex loading law is given in the form

$$p \equiv p(t) = p_0 + p_1t^4; \quad q \equiv q(t) = q_0 + q_1t;$$

where $p_0 = 2$ MPa, $p_1 = 0.1$ MPa; $q_0 = 2$ MPa, $q_1 = 0.1$ MPa.

a) Numerical results for the simply supported cylindrical panel

Table 1 : $\frac{b}{R} = \frac{1}{5}$

$\frac{R}{h}$	t_*	$s \cdot 10^3$	p_* MPa	q_* MPa	σ_u^* MPa
100	8.27	2.639	470.7	2.827	469.3
200	8.11	1.780	434.7	2.811	433.3
300	8.03	1.636	418.7	2.803	417.3
400	7.94	1.533	399.8	2.794	398.4
500	7.64	1.308	342.4	2.764	341.1

b) Numerical results for the clamped cylindrical panel

Table 2 : $\frac{b}{R} = \frac{1}{5}$

$\frac{R}{h}$	t_*	$s \cdot 10^3$	p_* MPa	q_* MPa	σ_u^* MPa
100	8.44	4.392	508.3	2.843	506.9
200	8.25	2.440	464.7	2.825	463.3
300	8.13	1.880	439.6	2.813	438.2
400	8.09	1.734	429.7	2.809	428.3
500	8.04	1.647	420.3	2.804	418.9

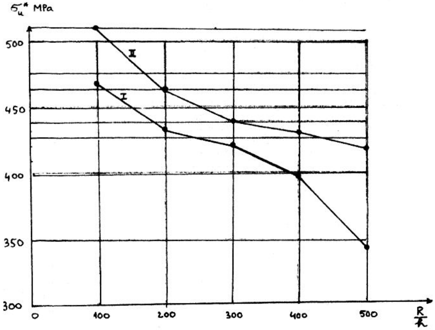

Fig. 1. I - Simply supported, II - Clamped

Example 2. Suppose that the complex loading law is of the form

$p \equiv p(t) = p_0 + p_1 t^3;\ \ p_0 = 2\,\text{MPa},\ \ p_1 = 0.1\,\text{MPa}$

$q \equiv q(t) = q_0 + q_1 t^2;\ \ q_0 = 1\,\text{MPa},\ \ q_1 = 0.1\,\text{MPa}$

a) Results of numerical calculation for the simply supported cylindrical panel

Table 3 : $\dfrac{b}{R} = \dfrac{1}{5}$

$\frac{R}{h}$	t_*	$s \cdot 10^3$	p_* MPa	q_* MPa	σ_u^* MPa
100	16.90	2.581	484.7	30.563	470.2
200	16.47	1.775	448.5	29.117	434.7
300	16.25	1.609	431.1	28.408	417.6
400	15.83	1.479	399.1	27.075	386.2
500	14.54	1.145	309.5	23.146	298.6

b) Results of numerical calculation for the clamped cylindrical panel

Table 4 : $\dfrac{b}{R} = \dfrac{1}{5}$

$\frac{R}{h}$	t_*	$s \cdot 10^3$	p_* MPa	q_* MPa	σ_u^* MPa
100	17.34	4.278	523.1	32.059	507.9
200	16.82	2.373	478.1	30.301	463.7
300	16.52	1.831	452.6	29.282	438.7
400	16.37	1.689	440.8	28.804	427.2
500	16.21	1.588	428.0	28.279	414.6

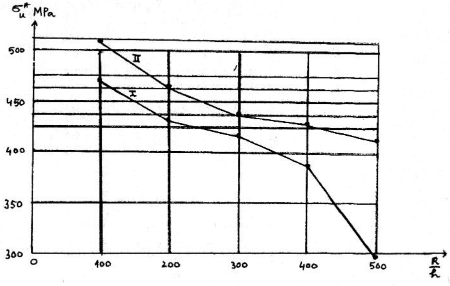

Fig. 2. I - Simply supported, II - Clamped

The above received results leads us to some conclusions

1. By using the Bubnov-Galerkin we have solved the elastoplastic stability problem of the cylindrical panels with two types of kinematic boundary constraints.

2. We have shown, for long cylindrical panel, the sufficient conditions of extremum

3. The critical loads of the simply supported cylindrical panels are always smaller than critical loads of the clamped cylindrical panels (see tables 1, 2, 3, 4 and figures 1, 2).

4. The more the cylindrical panel is thin the more the value of critical stress intensity σ_u^* is small (see table 1, 2, 3, 4).

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