APPLICATING OF SECONDARY CHARGE METHOD FOR SOLVING FAVOURABLE PROBLEM IN PARTIALLY INHOMOGENEOUS MODELS

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Abstract. In this paper, we present a theoretical basis of a secondary charge method and results of application of this method for several partially inhomogeneous models. This method is used for calculating electric field of the generating electrodes in a medium with given specific conductivity distribution and electric field of the secondary charge electrics at inhomogeneous points of the medium. The method enables us to derive directly the expressions for secondary charge density, electric field, electric potential at measuring points. From these results, the apparent resistivity curves of the different electrodes arrays are drawn. Test of this method by several different models and some of which are demonstrated in this paper, shows that the method gives reliable information

1. Introduction

The methods of integral equations or differential equations using of calculating for the 2-D and 3-D mediums require the disconnection of whole medium into volume elements and lead to very large system of algebraic equations. The increasing of effectivement of these methods is related to the improvement of algorithms for solution of system of equations and to the disconnected process. Beside, convergence of these methods strongly depends on physical distribution character of calculated model; convergence is very low for complicated conductivity distribution.

Apart from above mentioned two methods, there is a secondary charge method proposed by Alpin [1] for solution of two-dimensional and three-dimensional problems.

2. Theoretical outline of secondary charge method

When a current is generated in a medium with certain conductivity $\gamma = 1/\rho$, at points of medium, where inhomogeneity is caused by the generator then the secondary charge electrics are created. The aim of this method is to determine the secondary field E_{tc} as a sum of the fields dE_{tc} generated by secondary charge elements. From this quantity we are able to determine U_{tc} and derive values ρ at observation points.

At an observation point P, sum potential and sum electric field are:

$$U(P) = U_{tc}(P) + U_0(P)$$

$$\vec{E}(P) = \vec{E}_{tc}(P) + \vec{E}_0(P) \tag{2.1}$$

where: $U_0(P)$ is the potential of primary field; $\vec{E}_0(P)$ is the strength of the primary electric field, which are created by generating electrodes; $U_{tc}(P)$ and $\vec{E}_{tc}(P)$ are respectively the potential and the strength of the secondary electric field at the point P.

The potential and the strength of the primary electric field are calculated by:

$$U_0(P) = \sum_Q \frac{e_Q}{r_{QP}} \; ; \; \vec{E}_0(P) = \sum_Q \frac{e_Q \vec{r}_{QP}}{r_{QP}^3} \tag{2.2}$$

where: r_{QP} is the distance from the generating electrode A_Q at the point Q to observation point P; e_Q is charge of point generating electrode A_Q :

$$e_Q = \frac{I_Q}{4\pi\gamma_Q} = \frac{\rho_Q I_Q}{4\pi} \tag{2.3}$$

where: γ_Q is value for conductivity γ in vicinity of this point; ρ_Q is the real resistivity in vicinity of this point Q; I_Q is the current generated by generating electrode placed at point Q.

If given electrode A_Q placed at point Q goes through separated surface, the Eq. (2.3) should be replaced γ_Q by average angular conductivity:

$$\gamma_{gtb} = \frac{\sum W_i \gamma_i}{\sum W_i} \tag{2.4}$$

where: W_i is the volume angle with peak at vicivity of the electrode in the medium with conductivity γ_i (i = 1,2) and $W_1 = W_2 = 2\pi$. In case one separated surface is planar then $\gamma_{gtb} = \frac{\gamma_1 + \gamma_2}{2}$. If the medium on one side of the separated surface is non-conducting then $\gamma_{gtb} = \frac{\gamma}{2}$.

Using Eq. (2.3), Eq. (2.2) becomes:

$$U_0(P) = \sum_Q \frac{I_Q \rho_Q}{4\pi r_{QP}} \; ; \; E_0(P) = \frac{1}{4\pi} \sum_Q \frac{I_Q \rho_Q}{r_{QP}^3} \vec{r}_{QP} \tag{2.5}$$

In case the medium is of partial homogeneity, the source for the secondary field is the surface charges e_S appearing on separated surface where function γ is discrete. At point P on diviseve surface $S_{\alpha\beta}$ between the medium in which γ equals to γ_α and γ_β , charge density is determined by:

$$\sigma_p = \frac{1}{4\pi}\left[E_n^{\beta}(P) - E_n^{\alpha}(P)\right] = \frac{\rho_\beta - \rho_\alpha}{4\pi} J_n(P) = \frac{K_{\alpha\beta}(P)}{2\pi} E_n^{TB}(P) \tag{2.6}$$

where: $E_n^{\alpha}(P)$ and $E_n^{\beta}(P)$ are electric field strengths at point P along direction of two normal vectors of diviseve surface.

$E_n^{TB}(P)$ is the algebraic average of these two values:

$$E_n^{TB}(P) = \frac{1}{2}\left[E_n^{\alpha}(P) + E_n^{\beta}\right] = \frac{\rho_\alpha + \rho_\beta}{2} J_n(P) \tag{2.7}$$

$K_{\alpha\beta}(P)$ is the coefficient of contacting surface at P:

$$K_{\alpha\beta}(P) = \frac{\gamma_\alpha - \gamma_\beta}{\gamma_\alpha + \gamma_\beta} = \frac{\rho_\beta - \rho_\alpha}{\rho_\beta + \rho_\alpha} \tag{2.8}$$

In case the function γ is continuous (gradient medium), the source for the secondary field is the volume charges e_v generated in the medium limited by the volume V_{rg} having $grad\,\gamma$ not equal to zero, i.e. the continuous function γ is charged at different points. At point P in the medium V_{rg}, the density of these charges is:

$$\delta_P = \frac{1}{4\pi}\mathrm{div}\vec{E}(P) = -\frac{1}{4\pi\gamma_P}\left[\vec{E}(P)\nabla\gamma_P\right] \tag{2.9}$$

In general case, function γ can be charged discretely or continuously in medium at the same time, therefore source for the secondary field can be both surface charges e_S and volume charges e_V. Then potential U_{tc} and electric field strength $\vec{E}_{tc}$ are determined by:

$$\begin{cases} U_{tc}(P) = \int\limits_{S_{\alpha\beta}} \frac{\sigma_Q}{r_{QP}} dS_Q + \int\limits_{V_{rg}} \frac{\delta_Q}{r_{QP}} dV_Q \\ \vec{E}_{tc}(t) = \int\limits_{S_{\alpha\beta}} \frac{\sigma_Q \vec{r}_{QP}}{r_{QP}^3} dS_Q + \int\limits_{V_{rg}} \frac{\delta_Q \vec{r}_{QP}}{r_{QP}^3} dV_Q \end{cases} \tag{2.10}$$

where: σ_Q is the surface charge density in the medium at point Q of an element dS_Q of the diviseve surface;

δ_Q is the volume charge density in the medium at point Q of an element dV_Q at which $\nabla\gamma$ has a finite value;

r_{QP} is the distance from point Q to point P;

$de_s = \sigma_Q dS_Q$ and $de_v = \delta_Q dV_Q$ are respectively the charges of surface and volume elements at Q.

The surface integral in (2.10) is over all diviseve surface, and volume integral is over all volumes where γ is charged continuously.

For determination of (2.1) and (2.10) we need to know σ_Q and δ_Q (Q site). These functions are derived by integral equation:

$$\sigma_P = \frac{K_{\alpha\beta}(P)}{2\pi}\left[\int_{S_{\alpha\beta}} \frac{\sigma_Q}{r_{QP}^3}(\vec{r}_{QP}.\vec{n}_P)dS_Q + \int_{V_{rg}} \frac{\delta_Q}{r_{QP}^3}(\vec{r}_{QP}.\vec{n}_P)dV_Q + E_n^{sc}(P)\right] \quad (2.11)$$

$$\sigma_P = -\frac{1}{4\pi\gamma}\left[\int_{S_{\alpha\beta}} \frac{\sigma_Q}{r_{QP}^3}(\vec{r}_{QP}.\nabla\gamma_P)dS_Q + \int_{V_{rg}} \frac{\delta_Q}{r_{QP}^3}(\vec{r}_{QP}.\nabla\gamma_P)dV_Q + (E^{sc}(P)\nabla\gamma_p\right] \quad (2.11')$$

where:

$$E_n^{sc}(P) = \frac{1}{4\pi}\sum_Q \frac{I_Q}{\gamma r_{PQ}^3}(\vec{r}_{PQ}.\vec{n}_P) \quad (2.12)$$

$$E_n^{sc}(P)\nabla\gamma_p = \frac{1}{4\pi}\sum_Q \frac{I_Q}{\gamma r_{PQ}^3}(\vec{r}_{PQ}.\nabla\gamma_P) \quad (2.12')$$

Thus, determination in case of complicated medium is limited to deriving the functions σ_P and δ_P by solving of system of equations (2.11) and (2.11').

3. Applicating of secondary charge method for solving favourable problem in partially inhomogeneous models

In this paper, we apply the secondary charge method only for the partially inhomogeneous models. In this case, the source for the secondary field is the surface charges e_S, appearing on diviseve surface at which function γ is discrete.

As it is known, the field at point P is the field generated by the generating electrodes and the field of the secondary charges appearing on diviseve surface. Therefore:

$$E_n(P) = E_n^{T\grave{B}}(P) = E_n^{tc}(P) + E_n^0(P) \quad (3.1)$$

Where:

$$E_n^{tc}(P) = \int_{S_{\alpha\beta}} dE_n^{tc}(P) \quad (3.2)$$

with $dE_n^{tc}(P)$ is the field at P caused by a secondary charge element at a point Q on diviseve surface:

$$dE_n^{tc}(P) = \frac{dE_{tcQ}}{r_{QP}^2}.\left(\frac{\vec{r}_{QP}.\vec{n}_P}{r_{QP}}\right) \quad (3.3)$$

Where: $\vec{r}_{QP}$ is the distance from Q to P,

$\vec{n}_P$ is normal vector at point P of diviseve surface $S_{\alpha\beta}$,

dE_{tcQ} is the secondary charge of the surface element dS_Q at point Q on separated surface:

$$dE_{tcQ} = \sigma_Q.dS_Q \quad (3.4)$$

From (3.3) and (3.4) we have:

$$dE_{tc}(P)=\frac{\sigma_Q}{r_{QP}^3}.(\vec{r}_{QP}.\vec{n}_P)dS_Q \tag{3.5}$$

Using (3.5), (3.2) be comes:

$$E_n^{tc}(P)=\int_{S_{\alpha\beta}}\frac{\sigma_Q}{r_{QP}^3}(\vec{r}_{QP}.\vec{n}_P)dS_Q \tag{3.6}$$

The primary field $E_n^0(P)$ at point P is determined by:

$$E_n^0(P)=\frac{1}{4\pi}\sum_Q\frac{I_Q}{\gamma_Q r_{QP}^3}.(\vec{r}_{QP}.\vec{n}_P) \tag{3.7}$$

With (3.6), (3.7) and (3.1) become:

$$E_n(P)=\int_{S_{\alpha\beta}}\frac{\sigma_Q}{r_{QP}^3}(\vec{r}_{QP},\vec{n}_P)dS_Q+\frac{1}{4\pi}\sum_Q\frac{I_Q}{\gamma_Q \vec{r}_{QP}^3}(\vec{r}_{QP}.\vec{n}_P) \tag{3.8}$$

Combining (3.8) with expression (2.6), we determine the surface charge density at point P as:

$$\sigma_P=\frac{K_{\alpha\beta}(P)}{2\pi}\left[\int_{S_{\alpha\beta}}\frac{\sigma_Q}{r_{QP}^3}(\vec{r}_{QP}.\vec{n}_P)dS_Q+\frac{1}{4\pi}\sum\frac{I_Q\rho_Q}{r_{QP}^3}(\vec{r}_{QP}.\vec{n}_P)\right] \tag{3.9}$$

From these results we are able to derive the following expression for potential and strength of the secondary field at observation P:

$$U_{tc}(P)=\int_{S_{\alpha\beta}}\frac{\sigma_Q}{r_{QP}}dS_P\ ;\ \vec{E}_{tc}(P)=\int_{S_{\alpha\beta}}\frac{\sigma_Q}{r_{QP}^3}(\vec{r}_{QP}.\vec{n}_P)dS_P \tag{3.10}$$

In practical field, with installations of different electrode arrays, measuring generating current I and potential difference ΔU between receiving electrodes, we can calculate the apparent resistivity by the expression:

$$\rho_{bk}(r)=K(r)\frac{\Delta U}{I} \tag{3.11}$$

where: $K(r)$ is coefficient of electrode array.

4. Some results

We have developed a software by MATLAB language [2] for PC to solve favourable problem by using above - mentioned secondary charge method. We applied it to some models of partially inhomogeneous medium. Below, we show some obtained results.

The two-layered geo-electrical model

Figure 1 shows the apparent resistivity curves in the two-layered geo-electrical model ($\rho_1 = 1$, $h_1 = 10$, $\rho_2 = 10$) corresponding to two-electrode array, calculated by Rưgiôp algorithm (dashed line) and by secondary charge method (solid line). As can be seen in this figure 1, two curves are very close to each other.

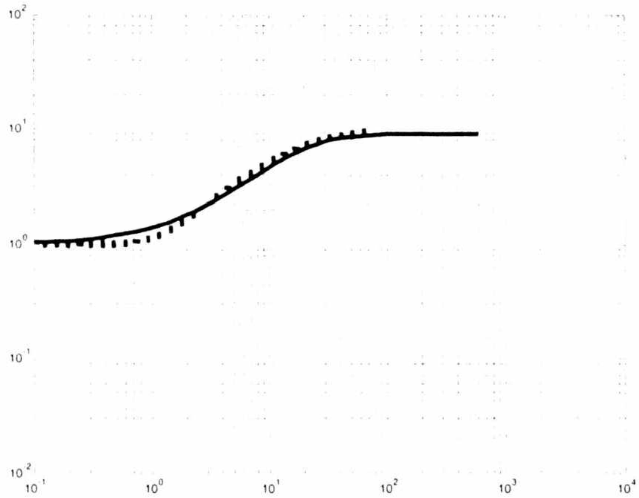

Fig. 1. The apparent resistivity curves calculated by secondary charge method *(solid line)* and by Rưgiôp algorithm *(dashed line)*

Model of one vertical separated surface

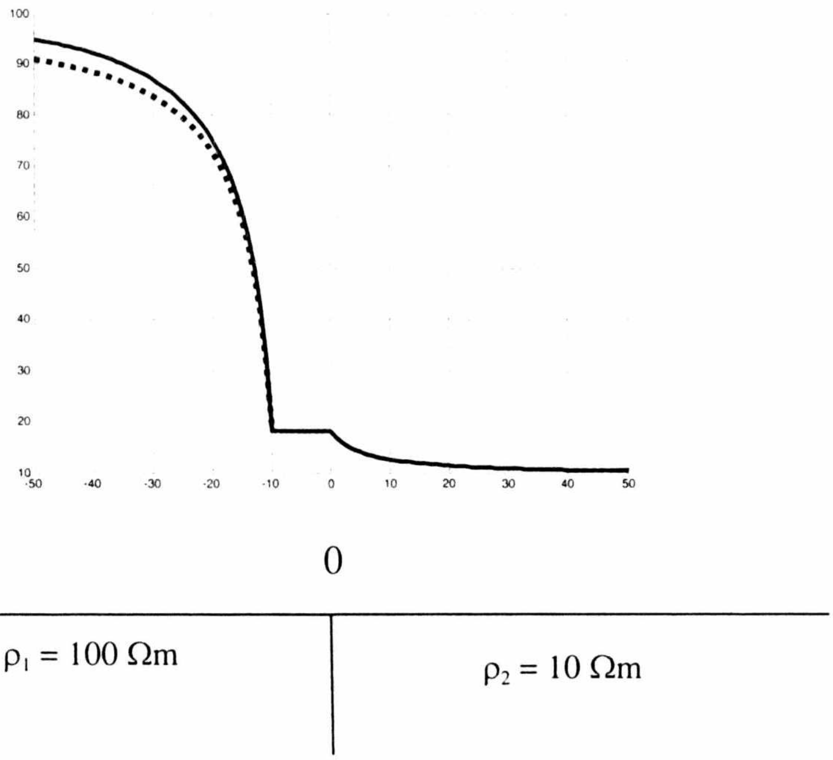

Fig. 2. The apparent resistivity curves calculated by secondary charge method *(solid line)* and by electric image method *(dashed line)* ($\rho_1 = 100\ \Omega m$, $\rho_2 = 10\ \Omega m$).

Model of one oblique 30^o boundary surface

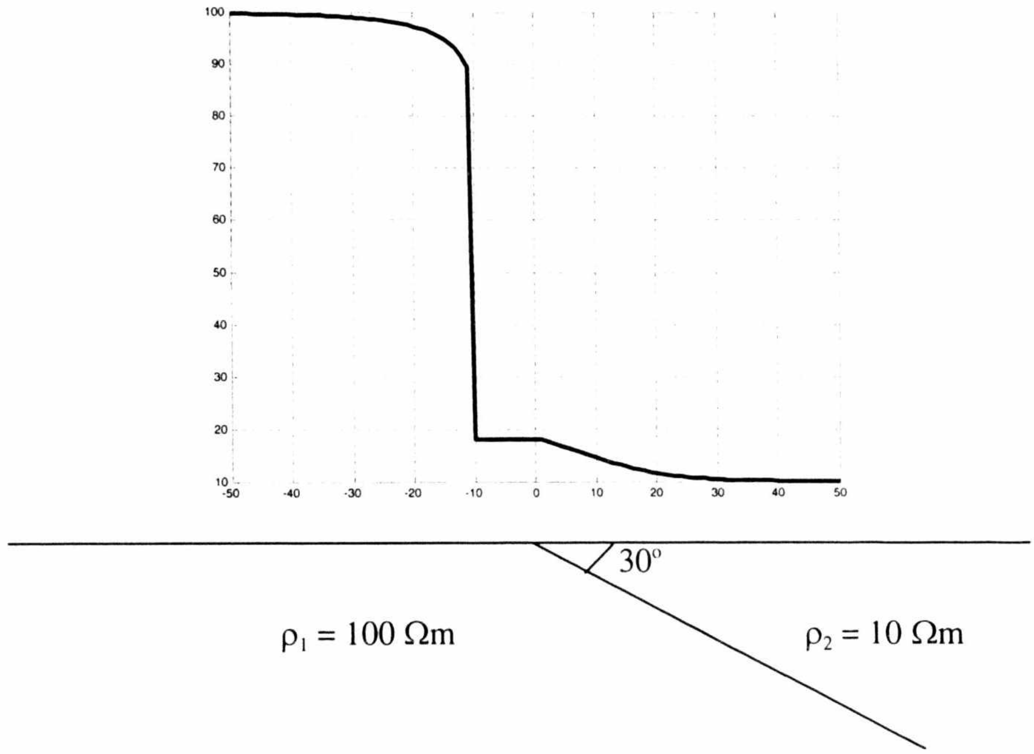

Fig. 3. The apparent resistivity curve in model of one oblique 30° boundary surface, two-electrodes array of 10 m spacing

Model of one oblique 150^o boundary surface

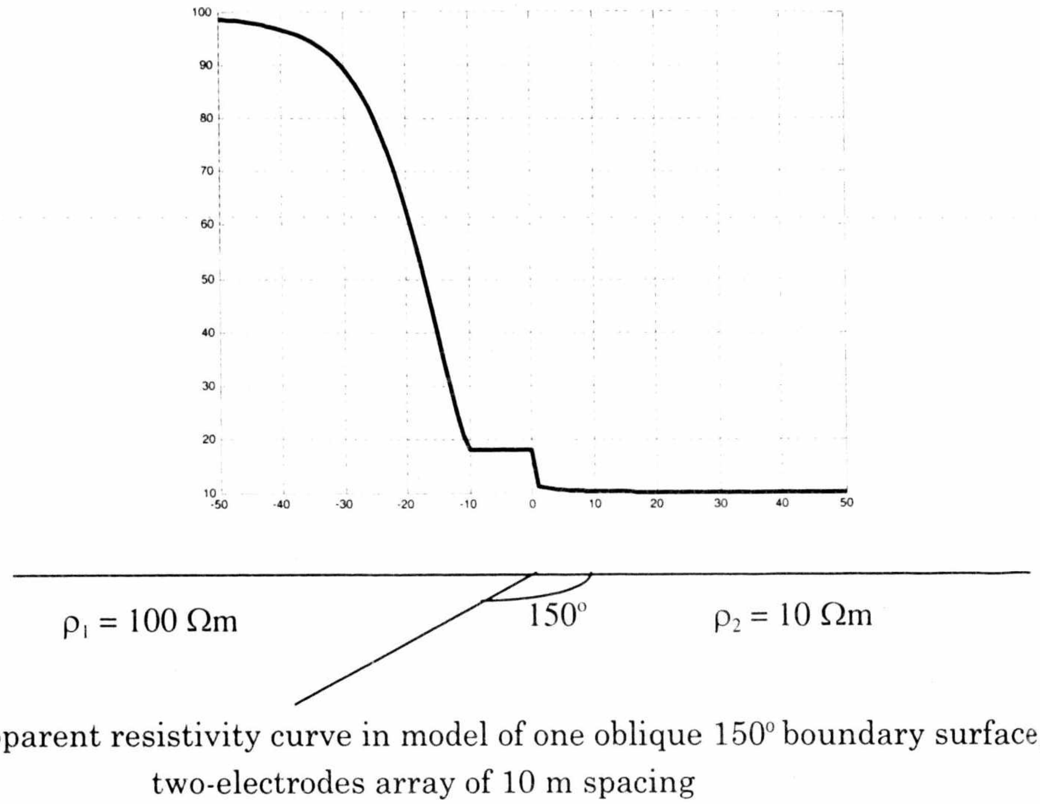

Fig. 4. The apparent resistivity curve in model of one oblique 150° boundary surface, two-electrodes array of 10 m spacing

Results shown in Fig.3 and Fig.4 fully correspond to standard palets for geo-electrical slices with vertical contacting boundary surface [3].

5. Conclusions

By investigating theoretically the secondary charge method and applying it for calculation in some models of partial homogeneity, we may draw the following remarks:

- This method in combination of modem calculation methods can well be applied for model of any medium.

- Test of this method by several different models and some of which are demonstrated above, shows that the method gives reliable information.

We will continue studying further for grid of the secondary charge elements corresponding to characteristics of each type of medium to have analysing results of geophysics document are in progress, as well as in order to have practical applications in the near future.

References

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