CONTROL PROBLEM ON TIMED PLACE/TRANSITION NETS

Hoang Chi Thanh

College of Science, VNU

Abstract: Design, analysis and control of large systems are complicated because of their state space explosion and behaviours. In this paper we solve the control problem on Timed Place/Transition nets by proposing a concurrent composition, which is a good solution for systems design. We also show that the safety and aliveness of Timed Place/Transition nets are preserved by the controller. Increase of concurrency in composed Timed Place/Transition nets is also considered.

Keywords: Petri net, control, composition, concurrency, safety, aliveness and concurrent step.

1. Introduction

Control on concurrent systems and its application are a problem concentrating much of interest. The general control problem on systems was formalized by L. Alfaro, T. A. Henzinger and F. Y. C. Mang in [1]. This problem is meaningful to concurrent systems in the compositional level. The main foundation of design, analysis and control problem is a composition operation. The operation depends so much on the kind of systems. Therefore, systems properties preserved by controller may be different[1].

To date we have some good mathematical models for representing concurrent systems. One of them, which was proposed earliest and vigorously investigated is the net model introduced by C.A. Petri. The model, we will use for the control problem is Timed Place/Transition net. We recall some notations concerning Petri nets, which were defined in [2,3,6].

A Petri net is a triple $N = (P, T, F)$, where P, T are disjoint sets and $F \subseteq (P \times T) \cup (T \times P)$ is a relation, so-called *the flow relation* of the net N.

Let $N = (P, T, F)$ be a Petri net. $X_N = P \cup T$, is the set of all elements of the net N. For an element $x \in X_N$, we denote:

$^\bullet x = \{ y \in X_N \mid y F x \}$ and it is called the pre-set of x,

$x^\bullet = \{ y \in X_N \mid x F y \}$ and it is called the post-set of x,

and they are similar for a subset of X_N.

A net is *simple* if and only if its two different elements have no common pre-set and post-set.

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A simple net has been being used to represent statistical structure of a system. From a simple net one can construct different net models by adding some aspects. The Timed Place/Transition net is such a net. It is the Place/Transition net introduced in [2], added a duration function and is defined as follows:

Definition 1.1. The 7-tuple $\Sigma = (P, T, F, K, M^0, W, D)$ is called *a Timed Place / Transition net* (Timed T/P net, for short) iff:

1) $N = (P, T, F)$ is a simple net, whereas an element of P is called *a place* and an element of T is called *a transition*.

2) $K : P \rightarrow N \cup \{\infty\}$ is a function showing *a capacity* on each place.

3) $W : F \rightarrow N \setminus \{\infty\}$ is a function assigning *a weight* on each arc of the flow relation F.

4) $M^0 : P \rightarrow N \cup \{\infty\}$ is *an initial marking*, which is not greater than capacity on places, i.e.: $\forall p \in P$, $M^0(p) \leq K(p)$.

5) $D : T \rightarrow \{[a, b] \mid 0 \leq a < b\}$ is a function pointing out *a duration,* in which the transition will be performed only.

The initial marking represents given tokens on each place of a net. The tokens are not greater than the capacity of the corresponding place. If tokens on each place belonging to the pre-set of some transition are greater than or equal to weight of the arc connecting this place to the transition, i.e. it is enough for "paying", then the initial marking can activate the corresponding transition. After performing the transition, tokens on each place belonging to the pre-set of this transition are decreased by weight of the arc connecting the corresponding place to this transition and tokens on each place belonging to the post-set of this transition are increased by weight of the arc connecting this transition to the corresponding place. It must be ensured that new tokens are not greater than the capacity of that place.

When the initial marking activates some transition in suitable time, the transition is performed and then we get a new marking, the new marking can activate another transition and the process repeatedly continues in such a way. Therefore, the activities happened on a Timed P/T net will be mathematically formalized as follows:

The marking $M : P \rightarrow N \cup \{\infty\}$ can activate a transition t iff:

1) $\forall p \in {}^{\bullet}t$, $M(p) \geq W(p, t)$ and

2) $\forall p \in t^{\bullet}$, $M(p) \leq K(p) - W(t, p)$.

In such a case, the marking M is so-called *t-activating*. After performance of the transition t, we get the following new marking:

$$M'(p) = \begin{cases} M(p) - W(p,t) & \text{if } p \in {}^{\bullet}t \setminus t^{\bullet} \\ M(p) + W(t,p) & \text{if } p \in t^{\bullet} \setminus {}^{\bullet}t \\ M(p) - W(p,t) + W(t,p) & \text{if } p \in t^{\bullet} \setminus t^{\bullet} \\ M(p) & \text{otherwise} \end{cases}$$

and we often write that: $M[\, t > M'$.

The marking M' can activate some other transition and then we get another marking M"... The set of all markings reachable from the marking M is denoted by [M].

In [5] we applied the control problem on Condition/Event systems. In this paper, we solve the control problem on Timed Place/Transition nets, which are one of models usually used to represent real systems. The problem is defined as follows:

Given a Timed Place/Transition net Σ_A (a plant). Find a Timed Place/Transition net Σ_B (a controller) such that the composed net Σ_A # Σ_B meets the prior defined properties.

We show that the safety and aliveness of Timed Place/Transition nets are preserved by the controller. Increase of concurrency in composed Timed Place/Transition nets is also considered.

This paper is organized as follows, In section 2, we define a composition on Timed Place/Transition nets and show that the safety and aliveness are preserved by the composition. Section 3 proposes the notation of a concurrent step and considers increasing of concurrency in composed Timed Place/Transition nets. Finally, some conclusions and directions for future research are given in Section 4.

2. Composition of timed P/T nets

Given two Timed P/T nets $\Sigma_i = (P_i, T_i, F_i, K_i, M_i^0, W_i, D_i)$, $i = 1, 2$, we compose two these nets by the following way.

Definition 2.1. The Timed P/T net $\Sigma = (P, T, F, K, M^0, W, D)$, where:

1) $P = P_1 \cup P_2$
2) $T = T_1 \cup T_2$
3) $F = F_1 \cup F_2$
4) The weight function W on arcs of the flow relation F is determined as follows

$$W(e) = \begin{cases} W_1(e) & ,\text{if } e \in F_1 \setminus F_2 \\ \min(W_1(e), W_2(e)) & ,\text{if } e \in F_1 \cap F_2 \\ W_2(e) & ,\text{if } e \in F_2 \setminus F_1 \end{cases}$$

5) And the capacity function on places is defined as follows

$$K(p) = \begin{cases} K_1(p) & ,\text{if } p \in P_1 \setminus P_2 \\ \max(K_1(p), K_2(p)) & ,\text{if } p \in P_1 \cap P_2 \\ K_2(e) & ,\text{if } p \in P_2 \setminus P_1 \end{cases}$$

6) The initial marking is combined like that

$$M^0(p) = \begin{cases} M_1^0(p) & ,\text{if } p \in P_1 \setminus P_2 \\ \max(M_1^0(p), M_2^0(p)) & ,\text{if } p \in P_1 \cap P_2 \\ M_2^0(p) & ,\text{if } p \in P_2 \setminus P_1 \end{cases} \qquad *$$

7) At the end, the duration function for performance of each transition i determined in the following way

$$D(t)=\begin{cases} D_1(t) & ,\text{if } t\in T_1\setminus T_2 \\ D_1(t)\cap D_2(t) & ,\text{if } t\in T_1\cap T_2 \\ D_2(t) & ,\text{if } t\in T_2\setminus T_1 \end{cases}$$

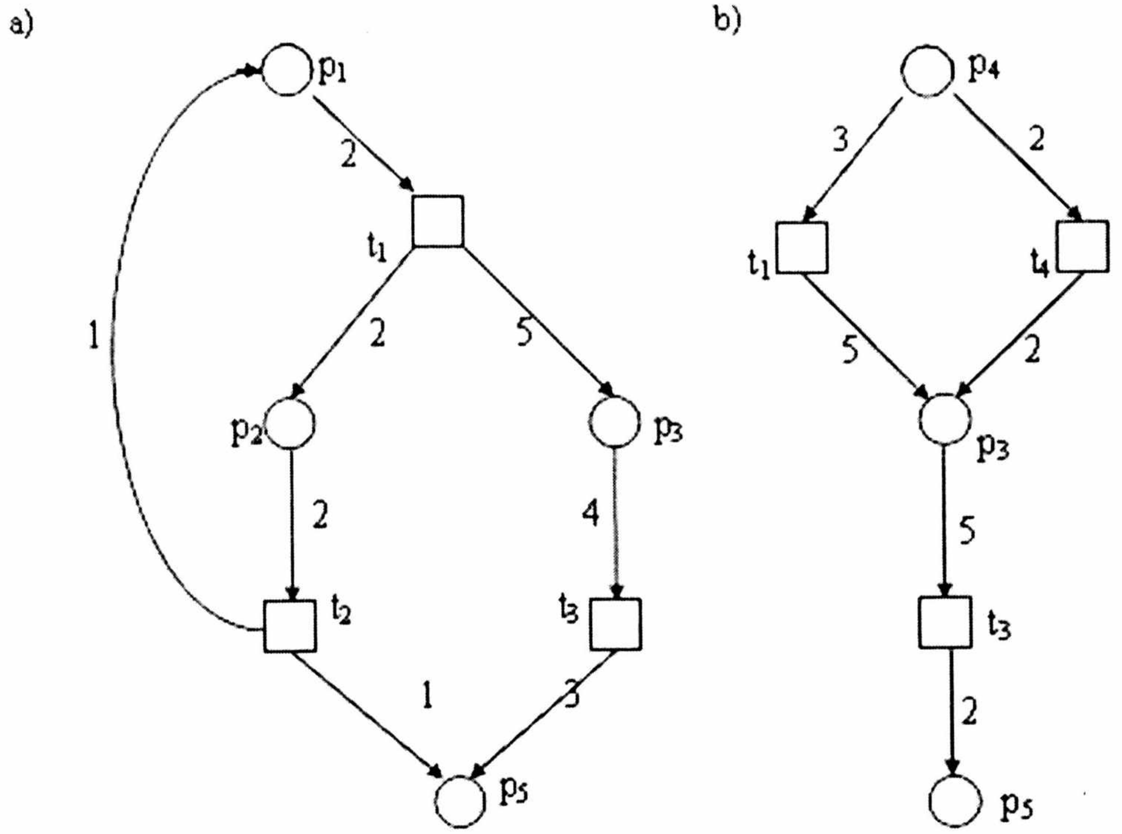
is called *a composition* of two nets Σ_1 and Σ_2. And then we denote

$$\Sigma = \Sigma_1 \# \Sigma_2.$$

Note that the weight function W, the capacity function K, the initial marking M^0 and the duration function D of the composed net have been being constructed from corresponding aspects of two component nets in the following way: In the partial domains we choose values of the corresponding function on all partial elements, whilst in the common domain we choose only minimum (maximum or intersection) of values of the corresponding function on common elements. This illustrates the general principle of composition proposed by the author in [4].

The control problem on Timed P/T nets is solvable and a Timed P/T net always can control another Timed P/T net.

Example 2.2. Consider two Timed P/T-nets given in Figure 1. a) and b).



	K1	M1^0
p1	3	3
p2	2	0
p3	8	0
p5	4	1

	D1
t1	[0,10]
t2	[2,8]
t3	[2,6]

	K2	M^{0}2
p4	5	5
p3	7	1
p5	5	0

	D2
t1	[1,12]
t4	[5,10]
t3	[3,8]

Figure 1: Two Timed P/T nets

In such a case, transitions in the step U can be performed concurrently and after their performance we get the following marking:

$$M'(p) = \begin{cases} M(p) - \sum_{t \in U} W(p,t) & ,\text{if } p \in {}^\bullet U \setminus U^\bullet \\ M(p) + \sum_{t \in U} W(t,p) & ,\text{if } p \in U^\bullet \setminus {}^\bullet U \\ M(p) - \sum_{t \in U} W(p,t) + \sum_{t \in U} W(t,p) & ,\text{if } p \in {}^\bullet U \cap U^\bullet \\ M(p) & ,\text{otherwise.} \end{cases}$$

We also denote that: M[U > M' and the marking M is called U-activating. Such as above, we can find steps sequences on the net. As big are the steps as high concurrency is.

Let $M^0[\ U_1 > M^1[\ U_2 > M^2 \ldots M^{k-1}[\ U_k > M^k$ be a steps sequence on the net Σ, illustrating a concurrent behaviour. If transitions of each step can be performed concurrently, then the total time for performance of the behaviour decreases remarkably. Therefore, we always expect to find sequences of maximally concurrent steps on the net and at that time, the performance of behaviours becomes optimal.

Let $\Sigma_i = (P_i, T_i, F_i, K_i, M_i^0, W_i, D_i)$, $i = 1, 2$ be two Timed P/T nets and the net $\Sigma = \Sigma_1 \# \Sigma_2$.

Theorem 3.2. For every step U of the composed net Σ, there are steps U_i of the corresponding net Σ_i, such that: $U_{|T_i} = U_i$, $i = 1, 2$.

Proof. Implies from the definition of the composition and the projection.

The theorem asserts that the composition enlarges concurrent steps in general.

4 Conclusion and future research

We have solved the control problem on Timed P/T nets by proposing a composition and proved two important properties preserved by this composition. Presented results may be applied to the control problem on some other models of concurrent systems and in system bottom-up design. We will construct concurrent steps of the composed Timed P/T net directly from concurrent steps of component Timed P/T nets in the future. Besides, the relationships between language behaviours as well as trace language behaviours of the composed Timed P/T net and ones of component Timed P/T nets will be investigated.

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