# PARAMETRIC INTERACTIONS OF ACOUSTIC AND OPTICAL PHONONS IN CYLINDRICAL QUANTUM WIRES

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**Abstract**: The parametric interactions of acoustic and optical phonons in cylindrical quantum wires in the presence of an external electromagnetic field is theoretically studied using a set of quantum kinetic equations for the phonons. Dispersions of the resonant phonon frequency and the threshold amplitude of the field for parametric amplification of the phonons in cylindrical quantum wires are obtained. Numerical computations are performed for the threshold amplitude of the field in a GaAs/GaAsAl quantum wire. The results have been compared with bulk semiconductor and quantum wells.

## 1. Introduction

It is well known that in the presence of an external electromagnetic field and when the conditions of parametric resonance are satisfied the parametric interactions and transformations of the same kinds of excitations such as phonon-phonon, plasmon-plasmon, or of different kinds of excitations, such as plasmon-phonon will arise [1]; i.e. the process of energy exchange between these excitations will occur. The parametric interaction and transformation of acoustic and optical phonon has been considered in detail in [2]. For semiconductor nanostructures, several works on the generation and the amplification of acoustic phonon [3,4,5] have been published.

In this paper we are study the parametric resonance of acoustic and optical phonons in cylindrical quantum wires in the presence of an external electromagnetic field. We estimate the numerical values for a GaAs/GaAsAl quantum wire.

## 2. Quantum kinetic equation for phonons

A cylindrical quantum wire: the radius R, the length L, the infinite confined potential:

$V(\vec{r}) = 0$ inside the wire and $V(\vec{r}) = \infty$ elsewhere [6]. Using bulk phonon assumption, from the Frohlich Hamiltonian H(t) for electron - acoustic phonon - optical phonon interacting system in external electromagnetic field ($E_0 \cos(\Omega t)$)for the cylindrical quantum wire, we obtain quantum kinetic equations for phonons: $x$ .

$$i\frac{\partial}{\partial t}\langle b_{\vec{q}}\rangle_t = \left\langle \left[b_{\vec{q}}, H(t)\right]\right\rangle_t , \tag{1}$$

where the symbol $\langle x\rangle_t$ means the usual thermodynamic average of operator x.

## 3. Acoustic phonon dispersion and condition for parametric amplification

We limit our calculation to the case of the first order resonance, $\omega_{\vec{q}} \pm \varpi_{\vec{q}} = \Omega$, and assume that the electron-phonon interactions satisfy the condition $\left|C_{n,l,n',l'}(\vec{q})\right|^2 \left|D_{n,l,n',l'}(\vec{q})\right|^2 << 1$ ($C_{n,l,n',l'}(\vec{q})$ and $D_{n,l,n',l'}(\vec{q})$ is the electron–acoustic phonon interaction coefficient and electron–optical phonon interaction coefficient). In these limitations, if we write the dispersion relations for acoustic and optical phonons as $\omega_{ac}(\vec{q}) = \omega_a + i\tau_a$ and $\omega_{oc}(\vec{q}) = \omega_a + i\tau_o$, we obtain the resonant acoustic phonon modes (using Fourier transformation and from the general dispersion equation for the parametric interactions and transformation of the acoustic and optical phonons):

$$\omega_{\pm}^{(\pm)} = \omega_a + \frac{1}{2}\left[(v_a \pm v_o)\Delta(q) - i(\tau_a + \tau_o) \pm \sqrt{\left[(v_a \pm v_o)\Delta(q) - i(\tau_a - \tau_o)\right]^2 \pm \Lambda^2}\,\right], \tag{2}$$

where $v_a$ and $\omega_a$ ($v_o$ and $\omega_o$) are the group velocity and the renormalization (by the electron-phonon interaction) frequency of the acoustic (optical) phonon, respectively, $\Delta(q) = q - q_0$, $q_0$ being the wave number for which the resonance is maximal, and:

$$\Lambda = 2\sum_{n,l,n',l'}\left|C_{n,l,n',l'}(\vec{q})\right|\left|D_{n,l,n',l'}(\vec{q})\right|\,\Pi_1(\vec{q},\omega_{\vec{q}}). \tag{3}$$

Signs ($\pm$) depend on the resonance condition $\omega_{\vec{q}} \pm \varpi_{\vec{q}} = \Omega$. In such case that $\lambda << 1$, we obtain the threshold amplitude :

$$E_{th} = \frac{2m*\Omega}{eq\wp}\sqrt{\tau_a\tau_o}\ ; \tag{4}$$

$$\wp = \frac{L}{\hbar}\left(\frac{m}{2\pi\Omega\beta}\left[\sum_{n,l,n',l'}\left|C_{n,l,n',l'}(\vec{q})\right|\left|D_{n,l,n',l'}(\vec{q})\right|\left(e^{\frac{\beta\hbar^2A^2_{n,l}}{2m^*R^2}} - e^{\frac{\beta\hbar^2A^2_{n',l'}}{2m^*R^2}}\right)\left(\frac{1}{\gamma_0(\omega_{\vec{q}})+\hbar\Omega} - \frac{1}{\gamma_0(\omega_{\vec{q}})-\hbar\Omega}\right)\right]^2\right.$$
$$\left. + \frac{m^2}{4\hbar^2q^2\Omega}\left[\sum_{n,l,n',l'}\left|C_{n,l,n',l'}(\vec{q})\right|\left|D_{n,l,n',l'}(\vec{q})\left(\Xi_1(\omega_{\vec{q}}) - \Xi_{-1}(\omega_{\vec{q}})\right)\right|\right]^2\right)^{1/2} \tag{5}$$

$$\Xi_\nu(x) = \exp\left(-\beta\frac{\hbar^2A^2_{n,l}}{2m^*R^2} + \frac{m}{2\hbar^2q^2}\gamma_\nu(x)\right)\left(e^{\beta\hbar x + \nu\hbar\Omega} - 1\right) \tag{6}$$

## 4. Numerical results and discussions

A GaAs/GaAsAl quantum wire: $\xi$ = 13.5eV , $\rho$ = 5.32 gcm$^{-3}$, $\upsilon_s$ = 5370 ms$^{-1}$, $k_\infty$ =10.9, $k_o$ = 12.9, R = 5 nm, $m^*$ = 0.067 $m_0$, $m_0$ is the mass of free electron.

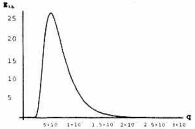


**Fig.1.** Threshold amplitude(kVcm $^{-1}$) As a function of q(m $^{-1}$) $\Omega$ =200THz;T=30K ;R=5nm

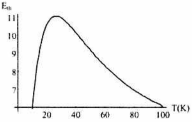


**Fig.2.** Threshold amplitude(kVcm $^{-1}$ ) As a function of T(K);R=8nm; q=4.10 $^{8}$ m $^{-1}$ ; $\Omega$ =250THz

This figure 1 shows that the curve has maximal value and is non-symmetric around the maximum. This is due to the fact that a fixed external electromagnetic field, with an amplitude greater than the corresponding threshold amplitude, can induce parametric amplification for acoustic phonon in the two regions of wave number corresponding to the two signs in $\omega_{\vec{q}} \pm \varpi_{\vec{q}} = \Omega$. We also demonstrate the threshold amplitude as a function of temperature using the above set of data in figure 2.

## 5. Conclusions

In this paper, We have obtained a general dispersion equation for parametric amplification and transformation of phonons. However, an analytical solution applying to this equation can only be obtained within some limitations. Using these limitations for simplicity, we obtained dispersions of the resonant acoustic phonon modes and the threshold amplitude of the field for acoustic phonon parametric amplification. Similarly to the mechanism pointed out by several authors for bulk semiconductors and quantum wells, parametric amplification for acoustic phonons in a quantum wire can occur under the condition that the amplitude of the external electromagnetic field is sometimes higher than that of threshold amplitude. Numerical results for GaAs/GaAsAl quantum wire clearly confirmed the predicted mechanism.

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## References

1. V.P.Silin., *Parametric Action of the High Power Radiation on Plasma* (National Press on Physics Theory Literature, Moscow, (1973).
2. E.M.Epstein, Sov. Phys. Semicond. 10, 1164(1976); M.V. Vyazovskii, V.A. Yakovlev, Sov. *Phys. Semicond.* 11, 809(1977); Vo Hong Anh, *Phys. Rep.* 1, 1(1980).
3. Tran Cong Phong, Nguyen Quang Bau, *J. Korean Phys. Soc.* 42, 647 (2003)
4. Peiji Zhao, Phys. Rev. B 49, 13589(1994); A.L. Troncoin, O.A.C. Nunes, *Phys. Rev.* B 33, 4125 (1986); Feng Peng, *Phys. Rev.* B 49, 4646 (1994) ;
5. N. Nishiguchi, *Phys. Rev.* B 52, 5279 (1995) ;
6. Gold, A. Ghazali, *Phys. Rev.* B41, 7626 (1990).