INFLUENCE OF GAIN MEDIUM IN THE DYE RING LASER

Giang Manh Khoi

National Center For Technical Progress

Trinh Đinh Chien

Department of Physics, College of Science, VNU

Abstract : There are some factors that are influential in pulse-formation and pulses width of output laser pulses. In this report, we present influence of gain medium in the ring dye Laser, by using the semi – classical method for investigating interaction of Gauss-form input pulses is calculated in detail.

1. Introduction

There are many methods for generation of ultrashort laser pulses, one of the most popular methods in almost laser laboratories and laser reseach centers is the method of passive mode-locking using the saturable absorber inside the cavity of a dye ring Laser. We consider , one of factors influences output laser pulse is gain medium.

2. Equation of density matrix

Atomic systems in the electromagnetic field obey the Schrodinger equation:

$$\hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t}, \tag{1}$$

where, the total Hamiltonian $\hat{H}$ for the system is the sum of the (isolated) atomic Hamiltonian $\hat{H}^0$, the field Hamiltonian $\hat{H}^c$ and the atom-field interaction Hamiltonian $\hat{H}^1$ ($\hat{H}^1 = -\vec{\mu}\vec{E}$), with the electric-dipole momen $\vec{\mu} = -e.\vec{r}$)[2]:

$$\hat{H} = \hat{H}^0 + \hat{H}^c + \hat{H}^1 . \tag{2}$$

If the atomic system has the state vector $|\psi\rangle_n$ with the corresponding probability ω_n, from (1) the definition of the density operator will be:

$$\hat{\rho} = \sum_n \omega_n |\Psi_n\rangle\langle\Psi_n| . \tag{3}$$

We can easily determine the equation of motion of the density operator:

$$i\hbar\frac{d}{dt}\hat{\rho} = [\hat{H},\hat{\rho}] . \tag{4}$$

Hence, the equations for the density operator's matrix elements are:

$$\frac{d}{dt}\rho_{kl} + i\omega_{kl}\rho_{kl} + \frac{1\rho}{\tau_{kl}} = \frac{1}{i\hbar}\sum_{m}(H^{i}_{km}\rho_{ml} - \rho_{km}H^{i}_{ml}), \tag{5}$$

$$\frac{d}{dt}\rho_{pm} + \sum_{m}(K^{1}_{nm}\rho_{mm} - K_{mn}\rho_{mm}) = \frac{1}{i\hbar}\sum_{m}(H^{i}_{nm}\rho_{mn} - \rho_{nm}H^{i}_{mn})$$

where: $\omega_{kl} = (E^{0}_{k} - E^{0}_{l})/\hbar$ và τ^{-1}_{kl}, K_{mn} is specific bindingprobability of the system.

3. Influence of gain medium in cavity

3.1. Method

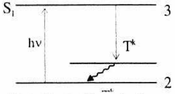

Fig.1. Sketch of three- level systems

Suppose that, the gain medium is atomic system has the three- level diagram (Fig.1) and the following approximations were used for simplification of the analysis:

$$T^{k}_{21} << T^{k} \quad \text{and} \quad \rho_{22} = 0 \tag{6}$$

So that, we have equations describes interation of pulses with gain medium from the equations for the density operator's matrix:

$$\frac{\partial n^{k}}{\partial t} + \frac{i}{\hbar}\left[\mu^{k}_{22}\rho^{k}_{22} - \mu^{k}_{32}\rho^{k}_{23}\right]\bar{n}^{k}E = -\frac{1}{T^{k}}\left[n^{k} - n^{k}_{0}\right],$$

$$\frac{\partial\rho^{k}_{23}}{\partial t} + \left(\frac{1}{\tau^{k}_{23}} - i\omega_{23}\right)\rho^{k}_{23} = \frac{i}{\hbar}\mu^{k}_{23}\rho^{k}_{23}E, \tag{7}$$

$$-\frac{\partial^{2}E}{\partial z^{2}} + \mu_{0}\varepsilon_{0}\varepsilon\frac{\partial^{2}E}{\partial t^{2}} = -\mu_{0}\bar{n}_{k}\left(\mu^{k}_{32}\frac{\partial^{2}\rho^{k}_{23}}{\partial t^{2}} + \mu^{k}_{23}\frac{\partial^{2}\rho^{k}_{32}}{\partial t^{2}}\right)$$

where: $n^{k} = \bar{n}^{k}\rho_{33}$: population of level k.

$\bar{n}^{k}$: population density.

n^{k}_{0} : Population in the balance of thermo- dynamics state.

Using: $\rho^{k}_{23} = \bar{\rho}^{k}_{23}e^{i\omega^{k}_{23}} = \bar{\rho}_{23}e^{i\omega_{L}t}$ and

$$E(z,t) = \frac{1}{2}A_{L}e^{i(kz+\omega_{L}t)} + cc \tag{8}$$

and replacing (8) into a system of equations (7), one obtain:

$$\frac{\partial F_{L}}{\partial z} = \sigma^{k}n^{k}F_{L}$$ (σ^k is absorption cross-section in the gain medium),

$$\frac{\partial n^{k}}{\partial t} = -\frac{1}{T^{k}}\left(n^{k} - n^{k}_{0}\right) - \sigma^{k}n^{k}F_{L}. \tag{9}$$

where: $F_{L} = \frac{\left[\varepsilon_{0}\varepsilon/\mu_{0}\right]^{1/2}}{2\hbar\omega_{L}}\left|A_{L}\right|^{2}$ (photon density).

The expressions (9) with $T^k_{21} << T^k$ can integrated over the length of the gain medium (L) (with: $F_L(0,\tau)$ is initial photon density, $F_L(l,\tau)$ final photon density of the gain medium):

$$F_L(1,\tau) = F_L(0,\tau)\frac{e^{\sigma^k E_L(0,\tau)}}{e^{-\sigma^k n_k L} - 1 + e^{\sigma^k E_L(0,\tau)}}, \tag{10}$$

where: $F_L(0,\tau) = \int_{-\infty}^{\infty} F_L(0,t')dt'$

If the energy of input pulse are small, then we can be rewritten as :

$$A_L(1,\tau) = A_L(0,\tau)\left[1 + \frac{1}{2}\alpha_k\left(1 - \sigma^k E_L(0,\tau)\right)\right] \tag{11}$$

where: $\alpha_k = \alpha_0 [1 - \delta\, \sigma^k \varepsilon_L]$; $\delta = \frac{1}{e^{U/2T_k} - 1}$ and $\alpha_0 = \sigma\, n_0\, L$.

3.2 Gauss - form input pulses

Gauss-form input pulse is given by [1]:

$$A(z,\tau) = a_0 e^{\frac{n^2}{2\tau_L^2}}. \tag{12}$$

The ratio of output pulse amplitude to input pulse amplitude:

$$\frac{A_L(1,\tau)}{A_L(0,\tau)} = \left[\sqrt{\pi} + \int_0^{\tau} e^{-\tau^2} d\tau + \frac{1}{2L}\int_0^{L} d\xi e^{-\tau^2} dL\right] \times \left[1 + \frac{1}{2}\alpha_i\left(1 - \sigma^2 E_L(0,\tau)\right)\right] A. \tag{13}$$

Fig.3 shows the result of the amplitude of output pulse depend on the length of the gain medium (the equation 13). The result of the relative pulse duration and the relative intensity of the output pulse in the case of Gauss-form are input pulse compared as follow:

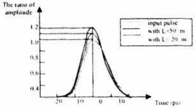

Fig. 3. ***The output Gauss-form pulse when passing gain medium with L=50 μm and L=20μm.***

L μm	$A_{L(1,\tau)}/A_{L\,0,\tau}$	$_{(L)}/\ _{(0)}$
50	1,1968	0,9326
20	1,1437	0,9564

4. Conclusion

The output pulse amplitude from gain medium is proportional to input pulse amplitude.

The output pulse enegy from gain medium depends on α_0 (ie, depends on gain medium with identical thermo- dynamics state), depend on n^0_k, depends on gain medium (σ^k :cross-section of the gain medium, L: length of gain medium).

Because of the complicated dependence of pulse time on a lot of parameters (special influence of chirp in cavity), because the limitatiol of paper, we don't concern that dependence in this paper. However, we can see the pulse duration is reduced when pulses traveling through the gain medium.

References

1. Peter W.Milonni, Joseph H. Eberly, "*Lasers*", A Wiley-Interscience Publication, New York, (1992), PP.139-408.
2. P.W.Smith, M.A. Duguay and E.P. Ippen, "*Mode- Locking of Lasers*", Pergamon Press, Oxford, New York, (1994).
3. Claude Rulliere, "*Femtosecond Laser Pulses,*" Springer, (1998).
4. G.M.Khôi and Trinh Dinh Chien, " Micro- Cavity Fluidic Dye Laser", *International Workshop on: Contemporary Appli. of Opto-Electronic & Infrared Tech.*, Dec., 2003.
5. S. De Silvestri, P. Laporta and O. Svelto, "*Effects of Cavity Dispersion on Femtosecond Mode- Locked Dye Lasers,*" Milano, Italy. 1988.