PARAMETRIC RESONANCE OF ACOUSTIC AND OPTICAL PHONONS IN DOPED SUPERLATTICES

Luong Van Tung, Tran Cong Phong, Pham Thuy Vinh, Nguyen Quang Bau

Department of Physics, College of Science, VNU

Abstract: The parametric resonance of acoustic and optical phonons in Doped Superlattices in the presence of an external electromagnetic field is theoretically predicted using a set of quantum kinetic equations for the phonons. Dispersions of the resonant phonon frequency and the threshold amplitude of the field for parametric amplification of the acoustic phonons in Doped Superlattices are obtained. Numerical computations are performed for the threshold amplitude E_{th} in a Doped Superlattices n-i-p-i of GaAs:Si/GaAs:Be.

1. Introduction

It is well known that in the presence of an external electromagnetic field (EEF), an electron gas becomes non-stationary. When the conditions of parametric resonance (PR) are satisfied, parametric interactions and transformations (PIT) of same kinds of excitations, such as phonon-phonon, plasmon-plasmon, or of different kinds of excitations, such as plasmon-phonon will arise; i.e., energy exchange process between these excitations will occur [1]. The PIT of acoustic and optical phonon has been consider in bulk semiconductors [2]. The result of the study show that the PIT can speed up the damping process for one excitation and the amplification process for another excitation. For low-dimensional semiconductors, there have been serveral works on the generation and amplification of acoustic phonons [3]. However, in our opinion, the energy exchange processs between two different kinds of phonons in low-dimensional systems, which are driven by a PR of a two-phonon kind, have not yet been reported. It should be noted that the mechanism for PIT is different from that for phonon amplification under a laser field [4] and from PR of a defect mode[5].

In [6] we have studied the PIT in a quantum well with non-degenerative electron gas. In order to continue the ideas of [2, 6], the purpose of this paper is also study the parametric resonance of acoustic and optical phonons, but in a doped superlattice (DSL) in which the electron gas is non-degenerative.

2. Quantum equation

We consider a DSL,a vector potential $\vec{A}(t)=\vec{A}_0\cos(\Omega t)$. If the Frohlich electron-acoustic and optical phonon interaction potential is used, the Hamiltonian for the system of the electrons and the acoustic an d optical phonons in the laser field is H(t)[7]

Using Hamiltonian H(t) and realizing operator algebraic caculations as in [6,7], we obtain a set of couped quantum transport equations for the acoustic phonons:

$$i\frac{\partial}{\partial t}\left\langle b_{\vec{q}}\right\rangle_t = \left\langle \left[b_{\vec{q}}, H(t)\right]\right\rangle_t \tag{1}$$

where the symbol $\langle x\rangle_t$ means the usual thermodynamic average of operator x .

3. Acoustic phonons dispersion and condition for parametric amplification

We limit our calculation to the case of the first order resonance (h =1), in which $\omega_{\vec{q}} \pm v_{\vec{q}} = \Omega$. We also assume that the electron-phonon interactions satisfy the condition $\left|C_{n,n'}(\vec{q})\right|^2 \left|D_{n,n'}(\vec{q})\right|^2 << 1$ ($C_{n,l,n',l'}(\vec{q})$ and $D_{n,l,n',l'}(\vec{q})$ is the electron-acoustic phonon interaction coefficient and electron optical phonon interaction coeffcient)

In these limitations, if we write the dispersion relations for acoustic and optical phonons as $\omega_{ac}(\vec{q}) = \omega_a + i\tau_a$ and $\omega_{op}(\vec{q}) = \omega_o + i\tau_o$, we obtain the resonant acoustic phonon modes:

$$\omega_{\pm}^{(\pm)} = \omega_a + \frac{1}{2}\left[(v_a \pm v_o)\Delta(q) - i(\tau_a + \tau_o) \pm \sqrt{\left[(v_a \pm v_o)\Delta(q) - i(\tau_a - \tau_o)\right]^2 \pm \Lambda^2}\right] \tag{2}$$

In eq. (2), the signs $(\pm)$ in the subscript of $\omega_{\pm}^{(\pm)}$ correspond to the signs $(\pm)$ in front of the root and the signs $(\pm)$in the subscript of $\omega_{\pm}^{(\pm)}$ correspond to the other sign pairs. These signs depend on the resonance condition $\omega_{\vec{q}} \pm v_{\vec{q}} = \Omega$. For instance, the existence of a positive imaginary part of $\omega_{+}^{(-)}$ implies a parametric amplification of the acoustic phonon. In such cases that $\lambda << 1$, the maximal resonance, and $q = q_\perp$ ($q_z = 0$), where

$$\tau_a = -\frac{1}{\hbar^2}\sum_{n,n'}\left|C_{n,n'}(\vec{q})\right|^2 \gamma(\omega_{\vec{q}})\,;\ \tau_o = -\frac{1}{\hbar^2}\sum_{n,n'}\left|D_{n,n'}(\vec{q})\right|^2 \gamma(v_{\vec{q}}) \tag{3}$$

$$|\Lambda| = \frac{\lambda}{\hbar^2}\sum_{n,n'}\left|C_{n,n'}(\vec{q})\right|\left|D_{n,n'}(\vec{q})\right|\left([\theta(\omega_{\vec{q}}) - \theta(\omega_{\vec{q}} - \Omega)]^2 + [\gamma(\omega_{\vec{q}}) - \gamma(\omega_{\vec{q}} - \Omega)]^2\right)^{1/2} \tag{4}$$

From eq. (4), the condition for the resonant acoustic phonon modes to have a positive imaginary part leads to: $|\Lambda|^2 > 4\tau_a\tau_o$. Using these condition and eqs. (3),(4), we obtained the threshold amplitude for the EEF for the degenerative electron gas:

$$E_0 > E_{th} = \frac{\sqrt{2\pi\beta}m^{3/2}\Omega}{\hbar^2 q^2 e}\frac{\varepsilon_{n',n}(\omega_{\vec{q}})[\varepsilon_{n',n}(\omega_{\vec{q}}) + \hbar\Omega]}{1 - \exp[-\beta(\varepsilon_n - \varepsilon_{n'})]}$$

$$\times \exp\left(\frac{\beta m}{4\hbar^2 q^2}[(\varepsilon_{n',n}(\omega_{\vec{q}}))^2 + (\varepsilon_{n',n}(v_{\vec{q}}))^2]\right)\sqrt{[1 - \exp(\beta\hbar\omega_{\vec{q}})][1 - \exp(\beta\hbar v_{\vec{q}})]} \tag{5}$$

where $\beta = \frac{1}{k_B T}$, k_B is the Boltzmann constant and T is the temperature of the system, ε_F is the Fermi level, and $\varepsilon_{n',n}(\varpi) = \varepsilon_0(n' - n) - \hbar\varpi - \hbar^2 q^2/(2m)$

Equation (5) means that the parametric amplification of the acoustic phonons is achieved when the amplitude of the EEF is higher than the threshold amplitude. To numerically estimate the threshold amplitude E_{th} for the parametric amplification of the acoustic phonons, we use the superlattice n-i-p-i of GaAs: Si/GaAs: Be with the parameters as folows [6,7]: ξ = 13.5eV , ρ = 5.32g/cm^{-3}, v_a = 5370ms^{-1}, ε_F=50 meV, s_0 =100, d=40 nm, n_D =10^{23} 1/m^3, χ_∞=10.9, χ_o = 12.9, m = 0.067m_0, m_0 is the mass of free electron, and $\hbar\nu_{\vec{q}} \approx \hbar\nu_0$=36.25 meV. In this case, the threshold amplitude must be from 10kV cm^{-1} to 25kV cm^{-1} for the wave number of phonon from 10^8 m^{-1} to 10^9 m^{-1}.

4. Conclusion

In this paper, we obtained a general dispersion equation for parametric amplification and transformation of phonons. However, an analytical solution to the equation can only be obtained within some limitations. Using these limitations for simplicity, we obtained dispersions of the resonant acoustic phonon modes and the threshold amplitude of the field for acoustic phonon parametric amplification. Similarly to the mechanism pointed out by several authors for bulk semiconductors and quanrtum wells, parametric amplification for acoustic phonons in a doped superlattice can occur under the condition that the amplitude of the external electromagnetic field is higher than some threshold amplitude. Numerical results for GaAs/GaAsAl quantum well clearly show the predicted mechanism. Parametric amplification for acoustic phonons and the threshold amplitude depend on the physical parameters of the system and are sensitive to the temperature.

Acknowledgments: This work is completed with financial support from the Program of Basic Research in Natural Science 411204.

References

1. V.P.Silin, *Parametric Action of the High Power Radiation on Plasma* (Moscow, 1973).
2. E. M. Epstein, *Sov. Phys. Semicond.* **10**(1976), 1164.
3. N. Nishiguchi, *Phys. Rev. B* **52**(1995) 5279.
4. P. Zhao, *Phys. Rev. B* **49**(1994), 13589; Feng Peng, *J. Phys.: Condens. Matter* **11**(1999) 4039.
5. V. Konotop and V. Kuzmiak, *Phys. Rev. B* **64**(2001), 125120.
6. T. C. Phong, N. Q. Bau, *J. Kor. Phys. Soc.* **42**2003) 647.
7. N. Q. Bau, N. V. Nhan, T. C. Phong, *J. Kor. Phys. Soc.* **41**(2002), 149.