Using the Directional Analytic Signals of Magnetic Gradient Tensor to Determine Boundaries of Source

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Abstract: The analytic signals of the magnetic tensor gradient within two- and three-dimensional space domain can be applied as a useful tool to estimate the depth and position of magnetic sources because their values only depend on location but not on magnetization direction of the sources of the magnetic anomaly. In this paper, we present results of the study for application of the combination of derivatives of directional analytic signals of the magnetic tensor gradient and maximum horizontal gradient to determine the edges of the sources through the Edge-Detector function (|ED|). Algorithms and programs written in the Matlab language have been used for testing the calculation on 3D models in correlative comparison with the method using the amplitude function of analytic signals. The calculation results showed the advantages of the |ED| function and its applicability in determing the boundaries of sources of magnetic anomaly.

Keywords: Analytic signal, magnetic tensor gradient, Edge-detector, |ED|.

1. Introduction

In magnetic exploration, the quantitative interpretation or solving of an inverse problem to determine the position, shape, depth, magnetization of geological objects causing observed anomalies always plays an important role. It was performed by many different methods: Euler deconvolution method ([1-3]), 2D and 3D selection method ([4, 5]), etc. With its advantages in recent years, the use of analytic signal to interpret data has attracted the attention of many geophysicists in the world as well as in the country. In the world, the application of analytic signal to interpret magnetic data is given by Nabighian ([1, 6, 7]) for the 2D case as a tool for assessing the source depth and location. Recently, the method has been extended to the 3D case ([8]) to estimate the characteristics and the depth to the source. In the country, Vo Thanh Son et al. have also begun to use the 3D analytic signals of the magnetic field [9] and the higher derivatives of the magnetic field in the interpretation of the aeromagnetic anomaly maps [10].

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In this article, we attempt the application of a method to determine the boundaries of the sources by calculating the combination of derivatives of directional analytic signals of tensor gradient of total magnetic anomalies, a method recently has been successfully applied by Beiki [11] when interpreting data of gravity anomalies. The test calculation was performed on numerical models showed the advantage of the method.

2. Method

2.1. Analytic signal

The analytic signal of the potential field $\phi(x)$ caused by a two-dimensional source along the Ox-axis perpendicular to the trend of object is defined by Nabighian [6]:

$$A(\mathrm{x}) = \frac{\partial \phi(\mathrm{x})}{\partial x} + i\frac{\partial \phi(\mathrm{x})}{\partial z} \tag{1}$$

in which $\frac{\partial \phi(x)}{\partial x}$ and $\frac{\partial \phi(\mathrm{x})}{\partial z}$ is a Hilbert transform pair, i is a complex number, $i^2 = -1$. The amplitude of the two-dimensional analytic signal is:

$$|A(\mathrm{x})| = \sqrt{\left[\frac{\partial \phi}{\partial x}\right]^2 + \left[\frac{\partial \phi}{\partial z}\right]^2} \tag{2}$$

Nabighian generalized the analytic signals from two dimensions to three dimensions [3] and indicated that any Hilbert transform of any field satisfies the Cauchy-Riemann relations. Roest et al. [8] expanded the concept of analytic signals of the potential field $\phi(x, y)$ measured on a horizontal plane to three-dimensional space:

$$A(\mathrm{x,y}) = \frac{\partial \phi(x,y)}{\partial x} + \frac{\partial \phi(x,y)}{\partial y} + i\frac{\partial \phi(x,y)}{\partial z} \tag{3}$$

and indicated that the amplitude of the analytic signal $A(x,y)$ is given by the formula:

$$|A(\mathrm{x,y})| = \sqrt{\left[\frac{\partial \phi(x,y)}{\partial x}\right]^2 + \left[\frac{\partial \phi(x,y)}{\partial y}\right]^2 + \left[\frac{\partial \phi(x,y)}{\partial z}\right]^2} \tag{4}$$

2.2. The combination of derivatives of directional analytic signals amplitude of the tensor gradient of total magnetic anomalies

The magnetic tensor gradient comprises the first derivatives in the x, y, z directions of the magnetic vector components. The magnetic field B of a magnetization distribution M with volume V is given by Blakely [12]:

$$B(\mathrm{r}) = -C_m \Delta \nabla_{r_0} \Phi(\mathrm{r}) = C_m \nabla_{r_0} \int_{\upsilon} M(\mathrm{r}_0) \nabla_{r_0} \frac{1}{|r - r_0|} d\upsilon \tag{5}$$

in which Φ is the magnetic scalar potential, r and r_0 are respectively observation point and integral point, $C_m = 10^{-7} Henry/m$. Then, the magnetic tensor gradient is defined by Beiki et al. [13]:

$$\Gamma = \begin{bmatrix} \frac{\partial^2\phi}{\partial x^2} & \frac{\partial^2\phi}{\partial x\partial y} & \frac{\partial^2\phi}{\partial x\partial z} \\ \frac{\partial^2\phi}{\partial y\partial x} & \frac{\partial^2\phi}{\partial y^2} & \frac{\partial^2\phi}{\partial y\partial z} \\ \frac{\partial^2\phi}{\partial z\partial x} & \frac{\partial^2\phi}{\partial z\partial y} & \frac{\partial^2\phi}{\partial z^2} \end{bmatrix} = \begin{bmatrix} B_{xx} & B_{xy} & B_{xz} \\ B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \end{bmatrix} \tag{6}$$

The components of the third column of the magnetic tensor gradient are the Hilbert transforms of the components in the first and second columns. So we can determine the analytic signals for every single row, called the analytic signals in the *x,y,z* directions. The directional analysis signal can be written in matrix form as Beiki [11]:

$$\begin{bmatrix} A_x(x,y,z) \\ A_y(x,y,z) \\ A_z(x,y,z) \end{bmatrix} = \begin{bmatrix} B_{xx} & B_{xy} & B_{xz} \\ B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ i \end{bmatrix} \tag{7}$$

The amplitude of the directional analytic signals are:

$$\begin{aligned} |A_x(x,y,z)| &= \sqrt{(B_{xx})^2 + (B_{xy})^2 + (B_{xz})^2} \\ |A_y(x,y,z)| &= \sqrt{(B_{yx})^2 + (B_{yy})^2 + (B_{yz})^2} \\ |A_z(x,y,z)| &= \sqrt{(B_{zx})^2 + (B_{zy})^2 + (B_{zz})^2} \end{aligned} \tag{8}$$

Debeglia & Corpel [14] shown that the derivatives of the analytic signals amplitude give a more efficient separation of anomalies caused by interfering structure than the analytic signals amplitude. Derivatives of directional analytic signals in *x* and *z* directions can be expressed as Beiki [11]:

$$\begin{aligned} |A_{xz}| &= \frac{\frac{\partial B_x}{\partial x}\left(\frac{\partial^2 B_x}{\partial x\partial z}\right) + \frac{\partial B_x}{\partial y}\left(\frac{\partial^2 B_x}{\partial y\partial z}\right) + \frac{\partial B_x}{\partial z}\left(\frac{\partial^2 B_x}{\partial z^2}\right)}{|A_x(x,y,z)|} = \frac{B_{xx}B_{xxz} + B_{xy}B_{xyz} + B_{xz}B_{xzz}}{\sqrt{(B_{xx})^2 + (B_{xy})^2 + (B_{xz})^2}} \\ |A_{yz}| &= \frac{\frac{\partial B_y}{\partial x}\left(\frac{\partial^2 B_y}{\partial x\partial z}\right) + \frac{\partial B_y}{\partial y}\left(\frac{\partial^2 B_y}{\partial y\partial z}\right) + \frac{\partial B_y}{\partial z}\left(\frac{\partial^2 B_y}{\partial z^2}\right)}{|A_y(x,y,z)|} = \frac{B_{xy}B_{xyz} + B_{yy}B_{yyz} + B_{yz}B_{yzz}}{\sqrt{(B_{xy})^2 + (B_{yy})^2 + (B_{yz})^2}} \end{aligned} \tag{9}$$

with α is *x,y,z*.

The combination of the first derivatives in the vertical direction $|A_{xz}|$và $|A_{yz}|$ of the amplitudes of directional analytic signals $|A_x|$ và $|A_y|$ is defined by:

$$|ED| = \sqrt{|A_{xz}|^2 + |A_{yz}|^2}$$

Compared to the amplitude function of the analytic signals, a function is often used quite a lot to detect the boudaries of the sources then the function |ED| can detect these boudaries better because the maximum value of |ED| occurs approximately on the boudaries of the sources and in particular, |ED| does not depend on the magnetization direction of the sources. The maximum value of the function |ED| can be determined by algorithm introduced by Blakely and Simpson [2].

3. Test calculation on the models

Based on the theory in terms of the analytic signal method of the magnetic gradient tensor, we have built a program calculating the function |ED|, then according to the algorithm of Blakely and Simpson [2], determine the maximum positions of the function |ED| ($|ED|_{max}$) using Matlab programming language to define the boundaries of source on some specific models. For all models, the total magnetic anomalies caused by the objects are determined on the xOy plane with the origin *O* is placed on the obsevation plane, *Ox*-axis orients north pole, *Oy*-axis orients the east, *Oz*-axis orients the downward vertically. The point grid located parallel to the axes *Ox* and *Oy* has:

- Number of observation points along the Ox-axis: 316 points
- Number of observation points along the Oy-axis: 316 points
- Distance between observation points: $\Delta x = \Delta y = 0,2\,km$

By choosing the coordinate system as above, the magnetic anomalies at any *P(x,y,0)* point of the vertical prismatic object with sides parallel to the coordinate is calculated by algorithm of Rao and Babu [15].

In order to assess the effectiveness of the method, in each model we also have:

- Calculated and compared the results of determing the edges of the object for both magnetic anomalies without noise and anomalies with noise in accordance with Gaussian distribution rule.
- Calculated and compared the results of determing the edges of the object according to the maximum positions of the function |ED| ($|ED|_{max}$) and according to the maximum positions of the analytic signal amplitude function |A| ($|A|_{max}$).

3.1. Model of a magnetic prism

Parameters related to coordinates, geometric dimensions and prismatic magnetization are given in the table 1:

Table 1. Parameters of a magnetic prism model

Parameters	Center coordinate (km)	Declination (°)	Magnetization (A/m)	Edge length (km)	Depth to the top (km)	Depth to the bottom (km)	Rotation angle (°)
Value	31.5 ; 31.5	0	4	10	0.5	5.0	45

To investigate the effect of the inclination I of magnetization vector on the accuracy of the method, both cases of vertical magnetization and inclined magnetization were calculated.

Case 1: Vertical magnetization, inclination I = 90°. The calculation results are shown on the figure 1.

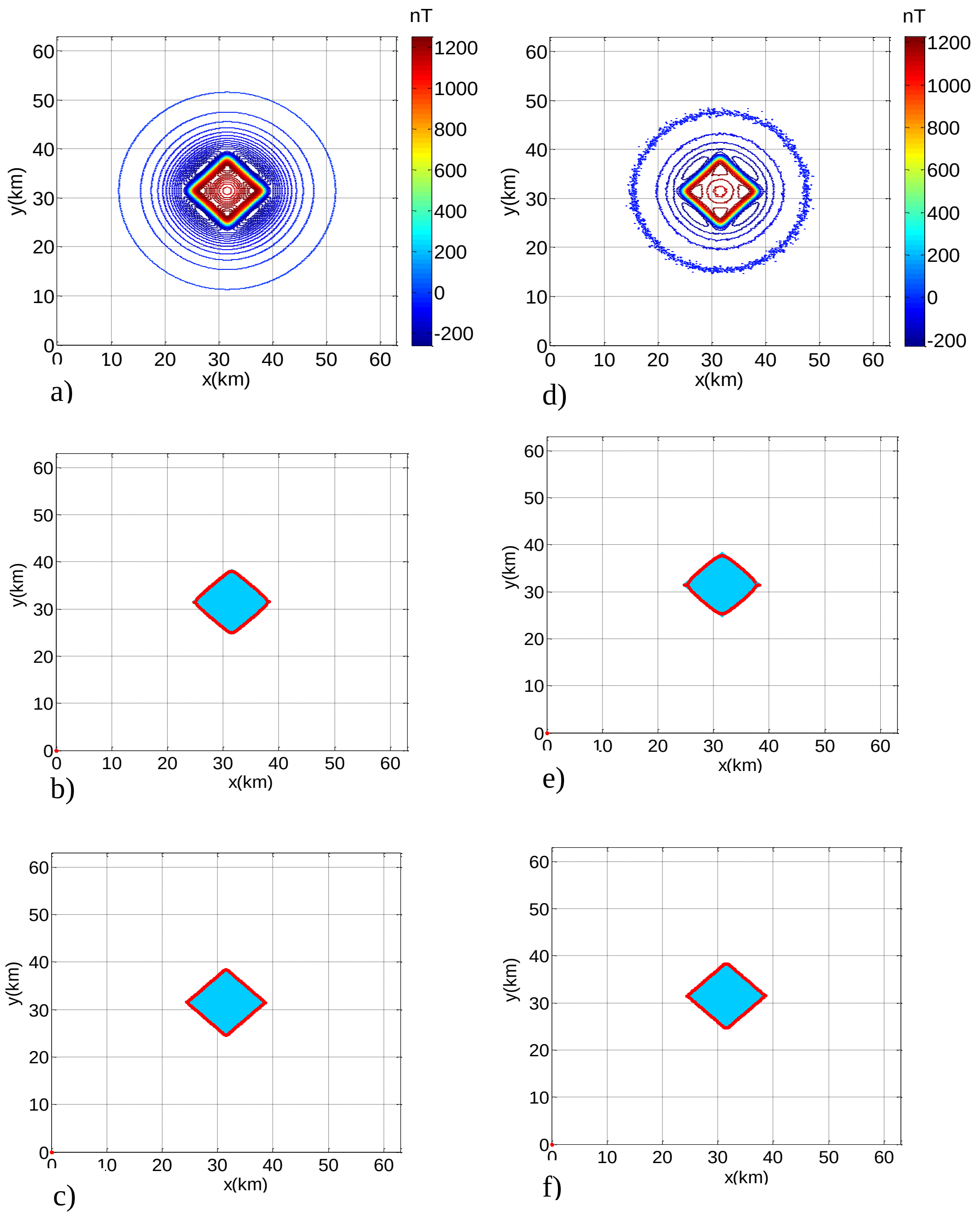

Figure 1. Determination of edges of a magnetized prisms with an inclination I = 90^0.
a) Theoretical anomalies; b) Edges of object determined by$|A|_{max}$;
c) Edges of object determined by$|ED|_{max}$;
d) Anomalies with noise 1%; e) Edges of object determined by$|A|_{max}$; f) Edges of object determined by$|ED|_{max}$

Object *Edges*

Case 2: Inclined magnetization, inclination I = 25°. The calculation results are shown on the figure 2

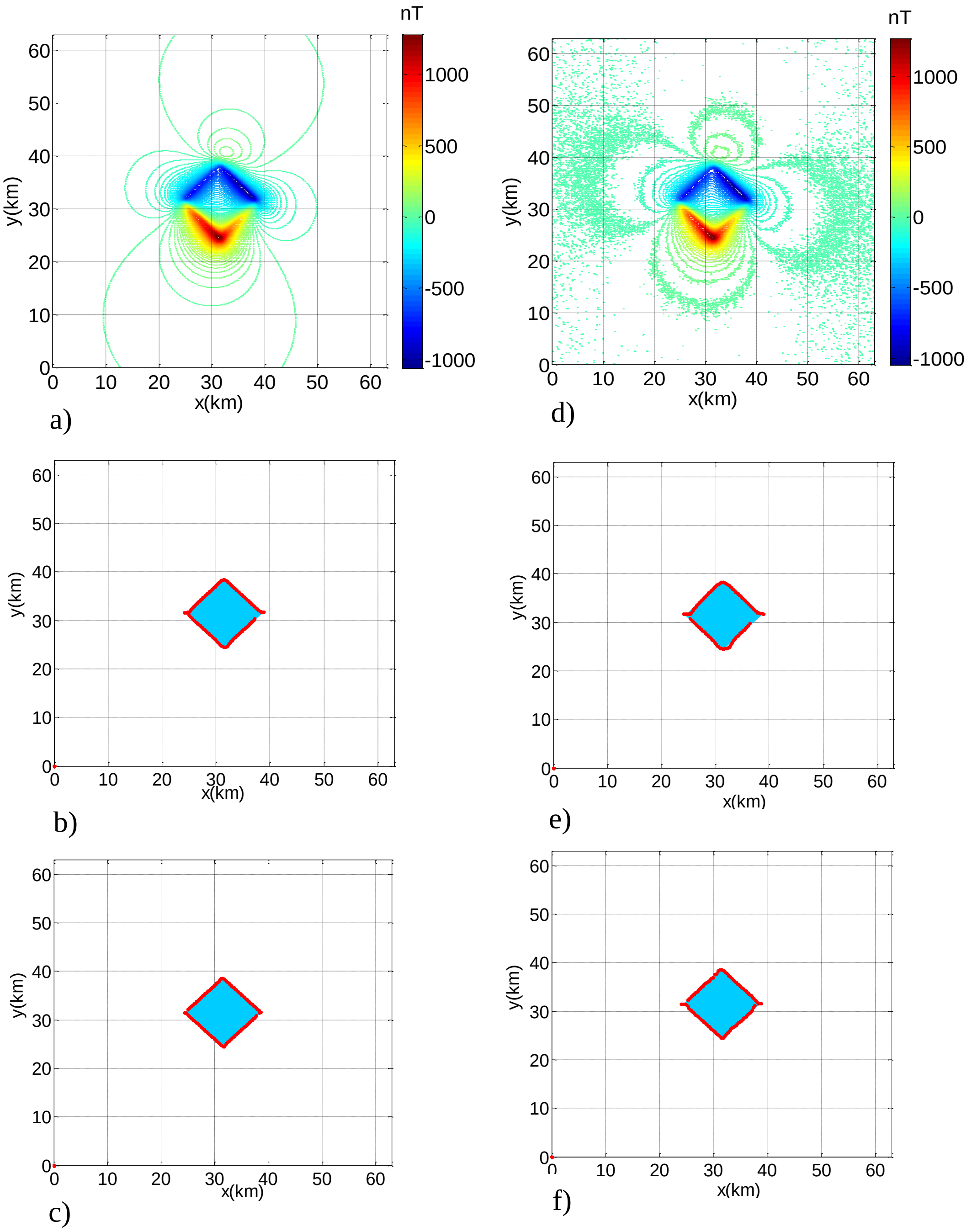

Figure 2. Determination of edges of a magnetized prisms with an inclination I = 25°.
a) Theoretical anomalies; b) Edges of object determined by$|A|_{max}$; c) Edges of object determined by$|ED|_{max}$;
d) Anomalies with noise 1%; e) Edges of object determined by$|A|_{max}$; f) Edges of object determined by$|ED|_{max}$

From the calculation results for the model of a magnetized prism, some following comments can be made in the correlative comparison between the two methods using the analytic signal amplitude function |A| and |ED| to determine bourdaries of the sources:

- According to maximum values of the function |A| and of the function |ED|, the determination of edges of the sources completely does not depend on the magnetized inclination of the sources, in both vertical magnetization and inclined magnetization.

- According to maximum values of the function |A| ($|A|_{max}$), the result of determining edges of the sources is not really clear at the corners of the sources. At these positions, the noise appears and the sources tend to be smooth and rounded. That is especially increased in case of noise in observed anomalies.

- According to maximum values of the function |ED| ($|ED|_{max}$), the bourdaries of the sources, including corners, are equally sharp and clear. On the other hand, it is also affected insignificantly by noise. Indeed, even if the random noise mixed in the anomalies has a maximum value of up to ±14nT, the bourdaries of the sources are determined with the nearly same sharpness as the anomalies without noise.

3.2. Model of two magnetic prisms

This model is designed to investigate the interference when using the function |ED| to determine edges of the sources in case they are distributed close together. Here the sources of magnetic anomalies are two vertical prisms whose sides are rotated by the 45^o angle with respect to the geographic north. Parameters related to coordinates, geometric dimensions and magnetization of prisms are given in the table 2.

Table 2. Parameters of the model of two magnetic prisms

Parameters	Center coordinate (km)	Declination (^{o})	Magnetization (A/m)	Edge length (km)	Depth to the top (km)	Depth to the bottom (km)	Rotation angle (^{o})
Prism 1	24.5;31.5	0	4	10	0.5	5.0	45
Prism 2	38.5;31.5	0	4	10	0.5	5.0	45

With the comments drawn through the calculation results for model 1 on the non-dependence on the inclination when using the function |ED| to determine edges of the sources, in this case, only the model with inclination $I = 25^o$ was investigated. The calculation results are shown on the figure 3.

From the calculation results for the model of two magnetic prisms, some following comments can be made: When the environment has many sources of anomalies distributed close together then

- With the method of using the maximum values of the analytic signal amplitude function |A|, the east and west corners of the objects, especially the contacted corners between two bodies, are poorly determined. On the other hand, at the corners, as in the case of an single object, the edges also tend to be smooth and rounded. This is especially clear in case of anomalies with noise.

- With the method of using the maximum values of the function |ED|, the position and shape of the magnetizing objects are defined accurately and sharply even in corners of the objects, including anomalies with noise.

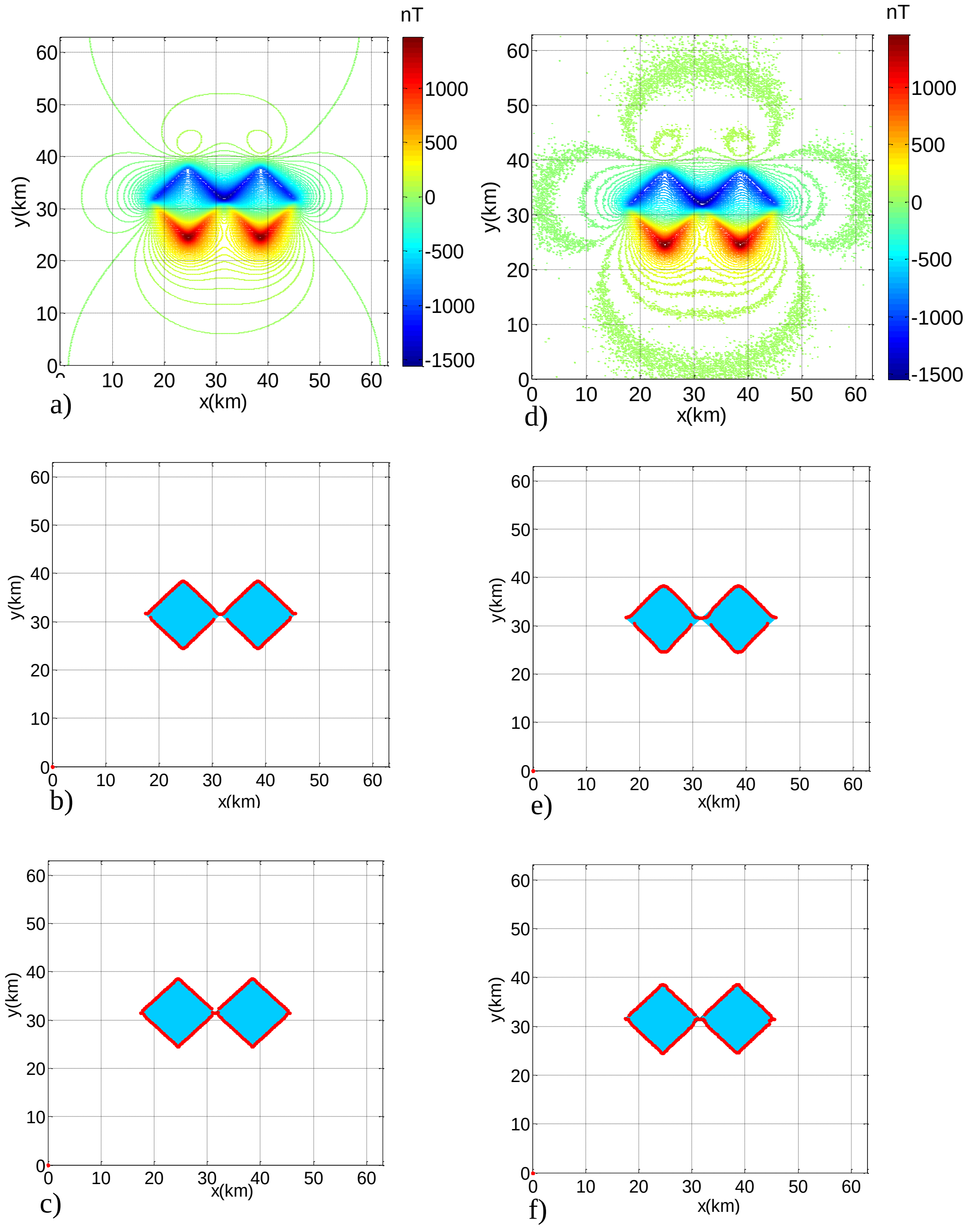

Figure 3. Determination of edges of two magnetized prisms with an inclination $I = 25^0$
a) Theoretical anomalies; b) Edges of object determined by$|A|_{max}$; c) Edges of object determined by$|ED|_{max}$;
d) Anomalies with noise 1%; e) Edges of object determined by$|A|_{max}$; f) Edges of object determined by$|ED|_{max}$

4. Results of the calculation on the observation data

To evaluate the effectiveness of the directional analytical signal method in the analysis and processing of the real magnetic data, we apply this method to interprete the magnetic data from Boong Quang (Cao Bang). This is the area with predicted mineral resources. The used magnetic data is a ground-based measurement of 1:25,000 by the Federation of Geological Physics (General Department of Geology), established in 2004 [16].

Boong Quang area is about 9km to the south-east of Cao Bang Town, within the meridian from $106^o19'47"$E to $106^o20'43"$E and the latitude from $22^o36'21"$N to $21^o37'06"$N, with a survey area of about 2 km^2. Anomalous magnetic field in Boong Quang area has a high intensity. Outstanding on the general background it is the cluster of anomalies distributed in the center of the survey area. In this place, the anomalies have a clear positive and negative polarization, extending in the Northwest - Southeast direction about 600m, anomaly amplitude reaching about 6000-7000nT. Apart from the anomalous cluster in the center, there are small anomalies located to the northwest of the survey area of 350-500nT, which are smaller and have a clear positive and negative polarization.

In this paper we select the strongest anomalous region in the center of Boong Quang area (Figure 4) and apply the analytical signal in the direction of the magnetic gradient to interprete this data. Calculated results include the position of the maxima of the function |ED| ($|ED|_{max}$) and the positions of the anomalous objects from which their margins are determined by the position of the maxima $|ED|_{max}$ are shown respectively in Figures 5 and 6 below. In these figures, the geographical location of the study area is represented in the UTM coordinate

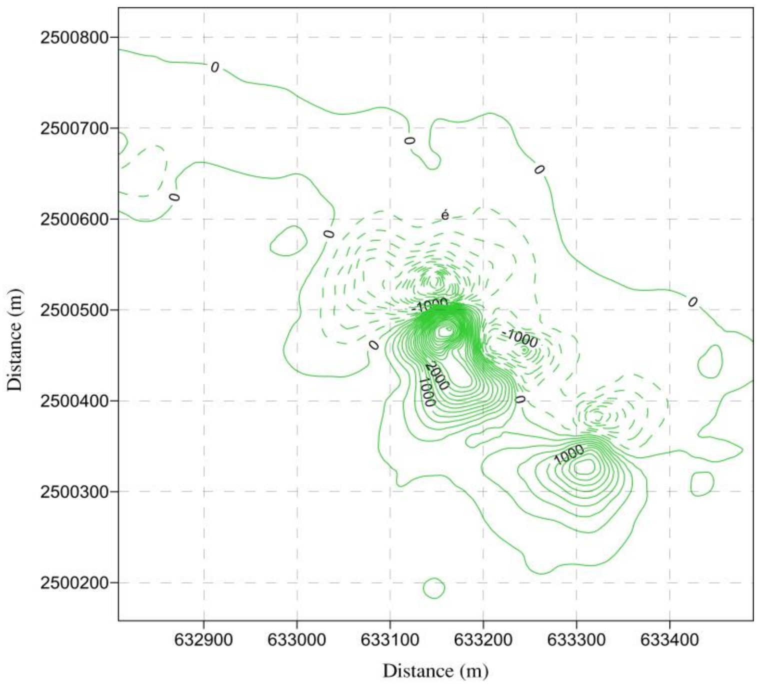

Figure 4. Total magnetic anomaly ΔT_a in Boong Quang center.

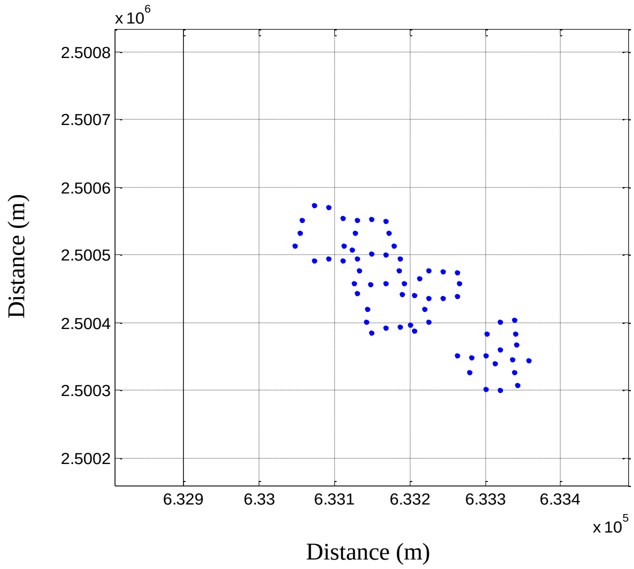

Figure 5. Maximum locations of |ED| function ($|ED|_{max}$) in Boong Quang center.

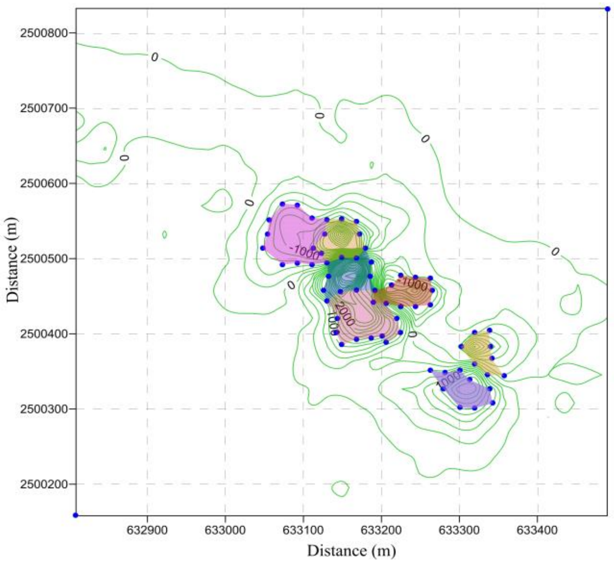

Figure 6. Magnetic sources determined by maximum locations of |ED| function ($|ED|_{max}$) in Boong Quang center (— magnetic anomaly; ● $|ED|_{max}$; sources).

From the results of calculations it can be seen that the maximum points (blue dots) of the function |ED| ($|ED|_{max}$) forms closed lines that clearly reflect the boundary of the anomalous objects (Fig. 5). From there, by connecting the points of these maxima, we will determine the positions of the anomalous objects. In this way, we find in the study area that many anomalous objects from close distribution form a major ore body distributed in the center of the area. The ore body has a longitudinal direction in the direction of northwest - southeast. This result is consistent with the interpretation results of the Federation of Geological Physics, it is said that the Boong Quang area also has a main ore body distributed in the center of the area including 8 vertical prism, top and bottom face lying horizontally, stacked against each other, extending in the direction of North West - Southeast, located in the contact zone between limestone with Bac Son formation and Nui Dien granophyr complex [16]. This result confirms that the directional analytic signal method can be able to be used in practice for analyzing or processing magnetic anomalies.

5. Conclusions

By using the directional analytic signal method of the magnetic tensor gradient to determine the location of the magnetizing object on the modeling and observation data, some following conclusions can be drawn:

- The boundaries of the sources of total magnetic anomalies can be well determined by the method of combination of directional analytic signals of the magnetic tensor gradient and maximum horizontal gradient. With this method, according to the maximum values of the function |ED| ($|ED|_{max}$), the determination of boundaries of the sources does not depend on the magnetized inclination of the sources, in both vertical magnetization and inclined magnetization, the boundaries of the sources, including the corners, are equally sharp and clear.

- With the method of using the maximum values of the function |ED|, the interference occurring in the case of the environment with multiple sources distributed close together was excluded. The position and shape of the sources are still defined accurately and sharply in positions where the objects are in contact.

- The method is affected very little by noise. The test results on the model show that even when the random noise mixed in the anomalies has a maximum value of up to ±14nT (± 1% ΔT_{max}), the bourdaries of the sources are determined with the same sharpness as the anomalies without noise.

- The results of the experimental calculation on the magnetic data of Bong Quang area show that the directional analytical signal method can be a useful tool in explaining magnetic anomaly data in Vietnam.

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