VNU Journal of Science: Mathematics – Physics, Vol. 34, No. 3 (2018) 43-47

VNU Journal of Science: Mathematics - Physics

Journal homepage: https://js.vnu.edu.vn/MaP

# Influence of the Finite Size Effect on Properties of a Weakly Interacting Bose G as in Improved Hatree-fock Approximation

Nguyen Van Thu*, Luong Thi Theu

*Department of Physics, Hanoi Pedagogical University 2, Nguyen Van Linh, Phuc Yen, Vinh Phuc, Vietnam*

Received 28 July 2018
Revised 11 August 2018; Accepted 11 August 2018

**Abstract:** The finite size effect causes many interesting behaviors in properties of a weakly interacting Bose gas. These behaviors were considered in one-loop approximation of quantum field theory. In this paper the influence is investigated in improved Hatree-Fock approximation, which gives more accurate results.

*Keywords:* Finite size effect, improved Hatree-Fock approximation, Bose-Einstein condensate.

## 1. Introduction

The finite size effect is one of the most interesting effects in quantum physics, which takes place in all of real systems and has been considered thoroughly. It is a hot topic in magnetic material [1], superconductivity [2], nuclear matter [3] and so on.

In Bose-Einstein condensate (BEC) field, the finite size effect causes the quantum fluctuation on top of the ground state, which leads to Casimir effect [4]. For two-component Bose-Einstein condensates, this effect was investigated in [5], in which two essential results are that the Casimir force is not simple superposition of the one of two single component BEC due to the interaction between two species and one of the most important result is that this force is vanishing in limit of strong segregation. In a dilute BEC, using Euler–Maclaurin formula, author of Ref. [6] calculated the Casimir force corresponding to Dirichlet and Robin boundary conditions. The result shows that the Casimir force is attractive and divergent when distance between two slabs approaches to zero.

One common thing of these papers is that the finite size effect is studied in one-loop approximation. In this respect, the effective mass and order parameter do not depend on the distance between two slabs. In this paper, we consider the influence of finite size effect in a weakly interacting Bose gas in improved Hatree-Fock (IHF) approximation.

*Corresponding author. Tel.: 84-912924226.
Email: nvthu@live.com

https//doi.org/ 10.25073/2588-1124/vnumap.4278

## 2. Research content

To begin, let us start from Lagrangianof a weakly interacting Bose gas [7],

$$L=\psi^*\left(-i\hbar\frac{\partial}{\partial t}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\right)\psi-\mu\psi^*\psi+\frac{g}{2}(\psi^*\psi)^2, \tag{1}$$

where $m$ is atomic mass, $\hbar$ is Plack's constant, the coupling constant $g$ is determined through $s$-wave scattering length $a_s$ as $g=4\pi\hbar^2 a_s/m$; $\mu$ is the chemical potential and in case of dilute gas one has $\mu=gn_0$ with $n_0$ being bulk density of the condensate; $\psi$ is field operator and its mean value plays the role of order parameter. We limit our attention to order parameters that are translationally invariant in the $x$ and $y$ directions.

In order to obtain the Hatree-Fock approximation, we shift the field operator as follows

$$\psi=\psi_0+\frac{1}{\sqrt{2}}\left(\psi_1+i\psi_2\right), \tag{2}$$

Substituting (2) into (1), among others, we obtain the interaction Lagrangian

$$L_{\text{int}}=\frac{g}{\sqrt{2}}\psi_0\psi_1(\psi_1^2+\psi_2^2)+\frac{g}{8}(\psi_1^2+\psi_2^2)^2. \tag{3}$$

At finite temperature in the Hatree-Fock approximation, the interaction Lagrangian (3) gives Cornwall-Jackiw-Tomboulis (CJT) effective potential is defined as [8],

$$\begin{aligned} V_\beta^{CJT}\left(\psi_0,D\right)=&-\mu\psi_0^2+\frac{g}{2}\psi_0^4+\frac{1}{2}\int_\beta \text{Tr}\left[\ln D^{-1}(k)+D_0^{-1}(k,\psi_0)D(k)-1\right]\\ &+\frac{3g}{8}\left[\int_\beta D_{11}(k)\right]^2+\frac{3g}{8}\left[\int_\beta D_{22}(k)\right]^2+\frac{g}{4}\left[\int_\beta D_{11}(k)\right]\left[\int_\beta D_{22}(k)\right], \end{aligned} \tag{4}$$

in which $D_0$ and $D$ are propagators at tree and Hatree-Fock approximation, respectively. Here we use the symbol

$$\int_\beta f(k)=T\sum_{n=-\infty}^{+\infty}\frac{d^3\vec{k}}{(2\pi)^3}f(\omega_n,\vec{k}),$$

where $\vec{k}$ is wave vector and $\omega_n$ is Matsubara frequency at temperature $T$. We realized that Goldstone's theorem is not satisfied in this approximation. To satisfy Goldstone's theorem, the effective potential (4) need a quantity

$$\Delta V=-\frac{g}{4}\left[P_{11}^2+P_{22}^2-2P_{11}P_{22}\right], \tag{5}$$

Combining (4) and (5) we get CJT effective potential, which restores Goldstone boson

$$V_{\beta}^{CJT}\left(\psi_0, D\right) = -\mu\psi_0^2 + \frac{g}{2}\psi_0^4 + \frac{1}{2}\int_{\beta} tr\left[\ln D^{-1}\left(k\right) + D_0^{-1}\left(k,\psi_0\right)D\left(k\right) - 1\right]$$
$$+\frac{g}{8}P_{11}^2 + \frac{g}{8}P_{22}^2 + \frac{3g}{4}P_{11}P_{22}. \tag{6}$$

The dispersionrelationcan be obtained by request $\det D^{-1} = 0$ and

$$E(\vec{k}) = \sqrt{\frac{\vec{k}^2}{2m}\left(\frac{\vec{k}^2}{2m} + M\right)}, \tag{7}$$

with $M$ being the effective mass. Minimizing effective potential (6) one gets Schwinger–Dyson (SD) and gap equations

$$M = -\mu + 3g\psi_0^2 + \Sigma_1, \tag{8}$$

$$-\mu + g\psi_0^2 + \Sigma_2 = 0, \tag{9}$$

with

$$\Sigma_1 = \frac{g}{2}P_{11} + \frac{3g}{2}P_{22},$$
$$\Sigma_2 = \frac{3g}{2}P_{11} + \frac{g}{2}P_{22}. \tag{10}$$

We consider the effect from the compactified space along *z*-direction for the time being. Our system is confined between two parallel plates perpendicular to *z*-axis and separated by a distance $\ell$. Because of the confinement along *z*-axis, the wave vector is quantized as $k^2 \to k_\perp^2 + k_j^2$, in which the wave vector component $k_\perp$ is perpendicular to*0z*-axis and $k_j$ is parallel with*0z*-axis. For boson system the periodic boundary condition is employed, which has the form after combining to Dirichlet boundary condition at two plates

$$k_j = \frac{j\pi}{\ell},\ j = 1,2,3,...$$

To seek the simplicity, we introduce dimensionless distance $L = \frac{\ell}{\xi}$ with $\xi = \frac{\hbar}{\sqrt{2mgn_0}}$ being healing length, $n_0$ is density in bulk. By this way, the dimensionless wave vector $\kappa = k\xi$ becomes

$$\kappa^2 \to \kappa_\perp^2 + \kappa_j^2, \tag{11}$$

with $\kappa_j = \frac{\pi j}{L} = \frac{j}{\tilde{L}}, \tilde{L} = \frac{L}{\pi}$ .Using Euler–Maclaurin formula [9] one has

$$P_{11}=0, P_{22}=\frac{gn_0 m\tilde{M}^{1/2}}{12\hbar^2\ell},$$
$$\tilde{M}=\frac{M}{gn_0}. \tag{12}$$

After scaling to bulk density $n_0$the dimensionless order parameter is reduced to $\phi=\frac{\psi_0}{\sqrt{n_0}}$. Combining (8), (9), (10) and (12), we obtain SD and gap equations in dimensionless form

$$\tilde{M}=-1+3\phi^2+\frac{mg\tilde{M}^{1/2}}{8\hbar^2\ell},$$
$$-1+\phi^2+\frac{mg\tilde{M}^{1/2}}{24\hbar^2\ell}=0. \tag{13}$$

It is easily to find the solution for (13), which is read as

$$\tilde{M}=2, \tag{14}$$

$$\phi=\sqrt{1+\frac{mg\tilde{M}^{1/2}}{24\hbar^2\ell}}. \tag{15}$$

It is obviously that the effective mass is the same as that in one-loop approximation while the order parameter is different. In one-loop approximation, the order parameter is constant and equals to unity. Eq. (15) shows that in IHF approximation depends strongly on the distance between two plates, especially in small-$\ell$ region and it turns out to be divergent when the distance approaches to zero. Experimentally, consider for rubidi87 with parameters $m=1.44\times10^{-25}\,\text{kg}, a_s=5.05\times10^{-9}\,\text{m}$ and $\xi=400$ nm. *Fig. 1 is the evolution of order parameter versus the distance between two plates.* When the distance $\ell$ increases, the order parameter decays fast and tends to constant at large $\ell$.

Using the *s*-wave scattering length on can rewrite Eq. (15) in form

$$\phi=\sqrt{1+\frac{\sqrt{2}\pi^{3/2}\left(n_0a_s^3\right)^{1/2}\tilde{M}^{1/2}}{3L}}. \tag{16}$$

For a dilute Bose gas, $n_0a_s^3\ll1$, expanding (16) in power series one arrives

$$\phi=\phi_1+\frac{\pi^{3/2}\left(n_0a_s^3\right)^{1/2}}{3L}, \tag{17}$$

in which $\phi_1=1$ is order parameter in one-loop approximation.

## 3. Conclusions

By mean of CJT effective action method, in IHF approximation we consider the finite size effect on a weakly interacting Bose gas. Our main results are in order:

- The order parameter depends strongly on the distance between two plates, in which Bose gas is confined. For a dilute Bose gas, this parameter equals to its value in one-loop approximation after adding a term $\frac{\pi^{3/2}\left(n_0 a_s^3\right)^{1/2}}{3L}$. This term is significant in small region of distance $\ell$.

- Because of independence of the effective mass $M$ on distance between two plates, the finite size effect have no extra contribution on Casimir force in comparing to the one in one-loop approximation [6].


## Acknowledgements

This work is funded by the Vietnam National Foundation for Science andTechnology Development (NAFOSTED) under Grant No. 103.01-2018.02.


## References


[1] C. Mocutaet. al. (2017), Scientific Reports 7, 16970.
[2] X. Y. Lang, Q. Jiang (2005), Solid State Communications134, 797.
[3] Tran Huu Phat and Nguyen van Thu (2014), Int. J. Mod. Phys. A 29, 1450078.
[4] J. Schiefele, C. Henkel (2009), J. Phys. A 42, 045401.
[5] Nguyen Van Thu, Luong Thi Theu (2017), J. Stat. Phys. 168, 1.
[6] Nguyen Van Thu (2018), Phys. Lett. A 382, 1078.
[7] L. Pitaevskii, S. Stringari (2003), Bose–Einstein Condensation, Oxford University Press.
[8] T. H. Phat, L. V. Hoa, D. T. M. Hue (2014), Comm. Phys 24, 343.
[9] G. B. Arfken and H. J. Weber, Mathematical Methods for Physicists,6$^{th}$ edn (San Diego: Academic, 2005).