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Stability of Arbitrarily Switched Discrete-time Linear Singular Systems of Index-1

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Abstract: In this paper, the index-1 notion for arbitrarily switched discrete-time linear singular systems (SDLS) has been introduced. Based on the Bohl exponents of SDLS as well as properties of associated positive switched systems, some necessary and sufficient conditions have been established for exponential stability.

Keywords: Switched system, linear discrete-time singular system, positive system, index-1 system.

1. Introduction

Recently there has been a great interest in arbitrarily switched discrete-time linear singular systems due to their importance in both theoretical and practical aspects, see [1- 4], and the references therein. Consider a switched system consisting of a set of subsystems and a rule that describes switching among them. It is well known that, even if all linear descriptor subsystems are stable but inappropriate switching may make the whole system unstable. On the other hand, since abrupt changes in system dynamics may be caused by unpredictable environmental factors or component failures, it is important to require the stability for some real-life switched systems under arbitrary switching. It should be noted that although there are a few works devoted to stability analysis of SDLS, see [1, 3-5], to our best of knowledge, the problem of investigating the stability for such switched systems via their Bohl exponents or properties of associated positive switched systems has not yet been studied before. Thus, this work was intended as an attempt to fill this gap.

2. Switched discrete-time linear singular systems of index-1

Consider the following autonomous SDLS of the form:

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$$E_{\sigma(k+1)}x(k+1) = A_{\sigma(k)}x(k) \quad (1)$$

where $\sigma : \mathbb{N} \cup \{0\} \to I_N := \{1,2,..,N\}$, $N \in \mathbb{N}$, is a switching signal taking values in the finite set I_N; $E_i, A_i \in \mathbb{R}^{n\times n}$ are given matrices, and $x(k) \in \mathbb{R}^n$ are unknown vector for al $k \in \mathbb{N}$. Suppose that the matrices E_i are singular for all $i = 1,2,..,N$.

We remark that in some works on SDLS [4, 6], instead of (1), a simpler system of the form

$$E_{\sigma(k)}x(k+1) = A_{\sigma(k)}x(k),$$

can be considered. Moreover, all the techniques developed in this paper can easily be applied to the above mentioned SDLS.

Definition 1 System (1) is called an arbitrarily switched singular system of index-1 (shortly, index-1 SDLS) if it satisfies the following conditions

(i) $\operatorname{rank} E_i = r < n$;

(ii) $S_{ij} \cap \ker E_i = \{0\}\ \forall\, i, j$, where $S_{ij} = A_i^{-1}(\operatorname{Im} E_j) = \{\xi : A_i\xi \in \operatorname{Im} E_j\}$.

From condition (ii) in Definition 1 we show that

$$S_{ij} \oplus \ker E_i = \mathbb{R}^n\ \forall\, i, j \in \{1,2,..,N\}.$$

Indeed, put $W_{ij} = \operatorname{Im} A_i \cap \operatorname{Im} E_j$. Then consider linear operators $T_{ij} : S_{ij} \to W_{ij}$, defined by $T_{ij}x = A_{ij}x$, we can easily show that $\ker T_{ij} = \ker A_i$. According to [7] we have

$$\dim S_{ij} = \dim W_{ij} + \dim \ker T_{ij} = \dim W_{ij} + \dim \ker A_i.$$

On the other hand

$$\begin{aligned}\dim W_{ij} &= \dim(\operatorname{Im} A_i \cap \operatorname{Im} E_j)\\ &= \dim \operatorname{Im} A_i + \dim \operatorname{Im} E_j - \dim(\operatorname{Im} A_i \cup \operatorname{Im} E_j).\end{aligned}$$

From last the relation we get

$$\begin{aligned}\dim S_{ij} &= \dim \operatorname{Im} A_i + \dim \operatorname{Im} E_j - \dim(\operatorname{Im} A_i \cup \operatorname{Im} E_j) + \dim(\ker A_i)\\ &= n + r - \dim(\operatorname{Im} A_i \cup \operatorname{Im} E_j).\end{aligned}$$

This relation shows that $\dim S_{ij} \geq r.$ Moreover, from condition (ii) in Definition 1 we have $\dim S_{ij} \leq r$. Hence $\dim S_{ij} = r$, i.e., $S_{ij} \oplus \ker E_i = \mathbb{R}^n$.

Define the matrix $V_{ij} = \{s_{ij}^1,...,s_{ij}^r, h_i^{r+1},...,h_i^n\}$, whose columns form bases of S_{ij} and $\ker E_i$, respectively, and $Q = \operatorname{diag}(O_r, I_{n-r})$, $P = I_n - Q$. Here O_r is the $r\times r$ zero matrix and I_m stands for the $m\times m$ identity matrix.

Then the matrix $Q_{ij} = V_{ij}QV_{ij}^{-1}$ defines a projection onto $\ker E_i$ along S_{ij} and $P_{ij} = I_n - Q_{ij}$ is the projection onto S_{ij} along $\ker E_i$.

Using similar arguments as in [8-11] we can prove the following results.

Theorem 1 For index-1 SDLS (1), the following assertions hold.

(i) $G_{ijk} = E_j + A_iV_{ij}QV_{jk}^{-1}$ *is non-singular for all* $i, j, k \in \{1,2,..,N\}$;

(ii) $E_j P_{jk} = E_j$;

(iii) $P_{jk} = G_{ijk}^{-1} E_j$;

(iv) $V_{jk}^{-1} G_{ijk}^{-1} A_i V_{ij} Q = Q$.

Proof.

(i) Assume that $x \in \ker G_{ijk}$, we have $0 = G_{ijk} x = (E_j + A_i V_{ij} Q V_{jk}^{-1}) x = E_j x + A_i V_{ij} Q V_{jk}^{-1} x$ .Then $E_j x = -A_i V_{ij} Q V_{jk}^{-1} x$, thus $V_{ij} Q V_{jk}^{-1} x \in S_{i,j}$. Furthermore, $V_{ij} Q V_{jk}^{-1} x = V_{ij} Q V_{ij}^{-1} V_{ij} V_{jk}^{-1} x = Q_{ij} V_{ij} V_{jk}^{-1} x \in \ker E_i$. Since $S_{ij} \cap \ker E_i = \{0\}$ we get $V_{ij} Q V_{jk}^{-1} x = 0$, thus $E_j x = -A_i V_{ij} Q V_{jk}^{-1} x = 0$, hence $x \in \ker E_j = \operatorname{Im} Q_{jk}$, i.e., $x = Q_{jk} x$. On the other hand, from the relation $Q_{jk} x = V_{jk} V_{ij}^{-1} V_{ij} Q V_{jk}^{-1} x = 0$, we have $x = Q_{jk} x = 0$. It means that $\ker G_{ijk} = \{0\}$, i.e., the matrix G_{ijk} is non-singular.

(ii) Since Q_{jk} is the projection onto $\ker E_j$ then we have $E_j Q_{jk} = 0$, i.e.,

$$E_j = E_j (P_{jk} + Q_{jk}) = E_j P_{jk}.$$

(iii) From relation $G_{ijk} P_{jk} = (E_j + A_i V_{ij} Q V_{jk}^{-1}) V_{jk} P V_{jk}^{-1} = E_j P_{jk} + A_i V_{ij} Q P V_{jk}^{-1} = E_j$, we get $P_{jk} = G_{ijk}^{-1} E_j$.

(iv) From formula of $G_{ijk} = E_j + A_i V_{ij} Q V_{jk}^{-1}$ we have $G_{ijk} V_{jk} = E_j V_{jk} + A_i V_{ij} Q$, thus $A_i V_{ij} Q = G_{ijk} V_{jk} - E_j V_{jk}$.

The last assertion follows from relations:

$$\begin{aligned} V_{jk}^{-1} G_{ijk}^{-1} A_i V_{ij} Q &= V_{jk}^{-1} G_{ijk}^{-1} (G_{ijk} V_{jk} - E_j V_{jk}) \\ &= V_{jk}^{-1} V_{jk} - V_{jk}^{-1} G_{ijk}^{-1} E_j V_{jk} \\ &= I_n - V_{jk}^{-1} P_{jk} V_{jk} \\ &= Q. \end{aligned}$$

Theorem 1 is proved.

Using items (iii), and (iv) of Theorem 1, we get

$$\bar{A}_{ijk} := V_{jk}^{-1} G_{ijk}^{-1} A_i V_{ij} = \begin{pmatrix} \bar{A}_{ijk}^1 & O \\ O & I_{n-r} \end{pmatrix}; \qquad (2)$$

$$\bar{E}_{ijk} := V_{jk}^{-1} G_{ijk}^{-1} E_j V_{jk} = \begin{pmatrix} I_r & O \\ O & O_{n-r} \end{pmatrix}.$$

Theorem 2 The index-1 SDLS (1) has a unique solution with $x(0) = x_0 \in \mathbb{R}$ if and only if $x_0 \in S_{\sigma(0)\sigma(1)}$, i.e., the initial condition x_0 is consistent. In this case, the following solution formula holds.

$$x(k) = V_{\sigma(k)\sigma(k+1)} \bar{A}_{\sigma(k-1)\sigma(k)\sigma(k+1)} \dots \bar{A}_{\sigma(0)\sigma(1)\sigma(2)} V_{\sigma(0)\sigma(1)}^{-1} x(0).$$

Proof.

Multiplying both sides of system (1) by $V_{\sigma(k+1)\sigma(k+2)}^{-1} G_{\sigma(k)\sigma(k+1)\sigma(k+2)}^{-1}$, and using the transformation $\bar{x}(k) = V_{\sigma(k)\sigma(k+1)}^{-1} x(k)$, we get

$$\bar{E}_{\sigma(k)\sigma(k+1)\sigma(k+2)}\bar{x}(k+1)=\bar{A}_{\sigma(k)\sigma(k+1)\sigma(k+2)}\bar{x}(k). \tag{3}$$

Putting $\bar{x}(k):=(v(k)^T,w(k)^T)^T$, where $v(k)\in\mathbb{R}^r$, $w(k)\in\mathbb{R}^{n-r}$, we can reduce system (3) to the following systems

$$\begin{cases} v(k+1)=\bar{A}^1_{\sigma(k)\sigma(k+1)\sigma(k+2)}v(k),\\ w(k)=0. \end{cases} \tag{4}$$

System (4) has the solution

$$v(k)=\bar{A}^1_{\sigma(k-1)\sigma(k)\sigma(k+1)}\ldots\bar{A}^1_{\sigma(0)\sigma(1)\sigma(2)}v(0),$$
$$w(k)=0,$$

hence the solution of system (1) can be written as

$$\begin{aligned} x(k)&=V_{\sigma(k)\sigma(k+1)}\bar{x}(k)\\ &=V_{\sigma(k)\sigma(k+1)}\begin{pmatrix} v(k)\\ w(k)\end{pmatrix}\\ &=V_{\sigma(k)\sigma(k+1)}\bar{A}_{\sigma(k-1)\sigma(k)\sigma(k+1)}\ldots\bar{A}_{\sigma(0)\sigma(1)\sigma(2)}\begin{pmatrix} v(0)\\ 0\end{pmatrix}\\ &=V_{\sigma(k)\sigma(k+1)}\bar{A}_{\sigma(k-1)\sigma(k)\sigma(k+1)}\ldots\bar{A}_{\sigma(0)\sigma(1)\sigma(2)}V^{-1}_{\sigma(0)\sigma(1)}x(0). \end{aligned}$$

3. Stability of linear switched singular systems of index-1

Suppose that system (1) is of index-1 and the initial condition x_0 is consistent.

Definition 2 System (1) is called exponentially stable if there exist a positive constant γ and a constant $0\le\lambda<1$such that such that for all switching signals and all solutions *x* of (1) the following inequality holds

$$\|x(k)\|\le\gamma\lambda^k\|x_0\|\quad\forall k\ge 0.$$

3.1. Bohl exponents and exponential stability

To define Bohl exponent for system (1), we first construct the so-called one-step solution operator $\Phi_\sigma(k,k-1)$ from $x(k-1)$ to *x*(*k*).

$$\begin{aligned} x(k)&=V_{\sigma(k)\sigma(k+1)}\bar{x}(k)\\ &=V_{\sigma(k)\sigma(k+1)}\begin{pmatrix} v(k)\\ w(k)\end{pmatrix}\\ &=V_{\sigma(k)\sigma(k+1)}\bar{A}_{\sigma(k-1)\sigma(k)\sigma(k+1)}\bar{x}(k-1)\\ &=V_{\sigma(k)\sigma(k+1)}\bar{A}_{\sigma(k-1)\sigma(k)\sigma(k+1)}V^{-1}_{\sigma(k-1)\sigma(k)}x(k-1). \end{aligned}$$

Then put $\Phi_\sigma(k,k-1):=\Phi_{\sigma(k-1)\sigma(k)\sigma(k+1)}=V_{\sigma(k)\sigma(k+1)}\bar{A}_{\sigma(k-1)\sigma(k)\sigma(k+1)}V^{-1}_{\sigma(k-1)\sigma(k)}$ we get the following one-step solution operator

$x(k) = \Phi_\sigma(k, k-1)x(k-1).$

Hence we can define the state transition matrix as

$\Phi_\sigma(i, j) := \Phi_{\sigma(i-1)\sigma(i)\sigma(i+1)} \dots \Phi_{\sigma(j)\sigma(j+1)\sigma(j+2)}, \forall i \geq j \geq 0.$

Definition 3 Assume that system (1) is of index-1 and $\Phi_\sigma(i, j)$ is the state transition matrix. Then Bohl exponent for system (1) is defined as follows:

$\kappa_B = \inf\{w \in \mathbb{R} : \exists M_w : \| \Phi_\sigma(i, j)\| \leq M_w . w^{i-j}, \forall \sigma,\ i \geq j \geq 0\}.$

To show the existence of Bohl exponent κ_B for system (1) we will prove that the set

$S = \{w \in \mathbb{R} : \exists M_w : \| \Phi_\sigma(i, j)\| \leq M_w . w^{i-j}, \forall \sigma,\ i \geq j \geq 0\},$

is non-empty and bounded from below.

Indeed, from the formula $\Phi_{ijk} = V_{jk} \bar{A}_{ijk} V_{ij}^{-1},\ i, j, k \in \{1, 2, .., N\}$, we see that the set of matrices Φ_{ijk} is finite, then there exists a positive constant $\gamma > 0$ such that

$\gamma = \max_{i,j,k \in \{1,2,..,N\}} \| \Phi_{ijk} \| .$

Thus we obtain that

$\| \Phi_\sigma(i, j)\| \leq \gamma^{i-j}, \forall \sigma,\ i \geq j \geq 0,$

hence $\gamma \in S$. Besides, for all $w \in S$ we have $w \geq 0$. It follows that the set S is non-empty and bounded from below.

Lemma 1 Assume that system (1) is of index-1 and $\Phi_\sigma(i, j)$ is the state transition matrix. Then

$$\kappa_B = \lim_{i \to \infty} \max_\sigma \| \Phi_\sigma(i, 0)\|^{\frac{1}{i}} . \tag{5}$$

Proof.

We carry the proof of Lemma 1 in 3 steps.

Step 1.We show the existence of the limit in (5)

Put $a_i = \max_\sigma \| \Phi_\sigma(i, 0)\|$. Then we have $a_{i+j} \leq a_i a_j$ for all $i, j \geq 0$. According to Polya-Szego [12] we obtain that $\lim_{i \to \infty} a_i^{\frac{1}{i}}$ exists. It means that the limit in (5) exists.

Step 2.Put $\kappa_1 = \lim_{i \to \infty} \max_\sigma \| \Phi_\sigma(i, 0)\|^{\frac{1}{i}}$. We prove $\kappa_1 \leq \kappa_B$.

Since $\kappa_B = \inf S$ then for all $\epsilon > 0$ there exists $w_\epsilon \in S$ such that $w_\epsilon < \kappa_B + \epsilon$, i.e., there exists M_{w_ϵ} such that

$\| \Phi_\sigma(i, 0)\| < M_{w_\epsilon} (\kappa_B + \epsilon)^{i-j}, \forall \sigma, i \geq 0.$

It follows

$\lim_{i \to \infty} \max_\sigma \| \Phi_\sigma(i, 0)\|^{\frac{1}{i}} \leq \kappa_B + \epsilon.$

Then we have

Step 3. We prove $\kappa_B \le \kappa_1$.

From the definition of κ_1, for all $\epsilon > 0$ there exists $T > 0$ such that

$|a_i^{\frac{1}{i}} - \kappa_1| < \epsilon, \ \forall i > T,$

i.e.,

$$\|\Phi_\sigma(i,0)\| < (\kappa_1 + \epsilon)^i, \ \forall i > T, \forall \sigma. \quad (6)$$

We will show that there exists $M > 0$ such that

$$\|\Phi_\sigma(i,j)\| < M(\kappa_1 + \epsilon)^{i-j}, \ \forall i > T, \forall \sigma. \quad (7)$$

Indeed, when $i - j > T$, for every σ we always have switching signal σ^* such that $\Phi_\sigma(i,j) = \Phi_{\sigma^*}(i-j,0)$. Hence we have

$\|\Phi_\sigma(i,j)\| = \|\Phi_{\sigma^*}(i-j,0)\| < (\kappa_1 + \epsilon)^{i-j}, \ \forall i - j > T, \forall \sigma.$

When $i - j \le T$, we have the following estimate

$\|\Phi_\sigma(i,j)\| < \gamma^{i-j} = \left(\frac{\gamma}{\kappa_1 + \epsilon}\right)^{i-j} (\kappa_1 + \epsilon)^{i-j}.$

Choosing $M = \max\{1, \left(\frac{\gamma}{\kappa_1 + \epsilon}\right)^T\}$, we get the inequality (7). It means that

$\kappa_B \le \kappa_1$.

Thus we obtain $\kappa_B = \kappa_1$.

Lemma 1 is proved.

Theorem 2 An index-1 SDLS (1) is exponentially stable if and only if $\kappa_B < 1$.

Proof.

Necessity.Assume that system (1) is exponentially stable. It follows that there exist a positive constant $M > 0$ and $0 < \omega < 1$ such that

$\Phi_\sigma(i,j) \le M\omega^{i-j} \ \forall \sigma, i \ge j \ge 0.$

Thus, $\kappa_B < 1$.

Sufficiency. Assume that $\kappa_B < 1$. Then there exist $\epsilon > 0$ and $M > 0$ such that $\omega = \kappa_B + \epsilon < 1$ and $\Phi_\sigma(i,j) \le M\omega^{i-j} \ \forall \sigma, i \ge j \ge 0.$ It shows that system (1) is exponentially stable.

Theorem 2 is proved.

3.2. Stability of positive linear switched singular systems of index-1

In this Subsection, we investigate the stability of index-1 SDLS satisfying some positivity condition. Let $\mathcal{K} := \{x = (x_1, x_2, ..., x_r)^T, x_i \ge 0\}$ be a positive octant in $\mathbb{R}^r$, $Int(\mathcal{K})$ be the interior of $\mathcal{K}$. Consider an order unit norm $\|.\|_u$, defined in [13], [14], and the corresponding order unit space $(\mathbb{R}^r, \mathcal{K}, \|\ \|_u)$.

Theorem 3 Assume that the matrices $\bar{A}^1_{ijk}$, determined by (2), are positive definite, and there exists a vector $\hat{v}\in Int(\mathcal{K})$ such that $\hat{v}-\bar{A}^1_{ijk}\hat{v}\in Int(P\mathcal{K})$ for all i,j,k. Then system (4) is exponentially stable, hence system (1) is also exponentially stable.

Proof.

Since $\hat{v}-\bar{A}^1_{ijk}\hat{v}\in Int(P\mathcal{K})$ then there exists a $\delta_{ijk}\in(0,\|\hat{v}\|_u)$ such that the closed ball $B[\hat{v}-\bar{A}^1_{ijk}\hat{v},\delta_{ijk}]\subseteq\mathcal{K}$. Since $\hat{v}-\bar{A}^1_{ijk}\hat{v}-\frac{\delta_{ijk}}{\|\hat{v}\|_u}\hat{v}\in B[\hat{v}-\bar{A}^1_{ijk}\hat{v},\delta_{ijk}]$ we get $\hat{v}-\bar{A}^1_{ijk}\hat{v}-\frac{\delta_{ijk}}{\|\hat{v}\|_u}\hat{v}\geq 0$. Let $\epsilon_{ijk}=\frac{\delta_{ijk}}{\|\hat{v}\|_u}\in(0,1)$, then $\bar{A}^1_{ijk}\hat{v}\leq(1-\epsilon_{ij})\hat{v}$.

Put $\epsilon=\inf\{\epsilon_{ijk}, i,j,k\in\{1,2,...,N\}\}$, we obtain $\bar{A}^1_{ijk}\hat{v}\leq(1-\epsilon)\hat{v}$ for all i,j,k. Using the positive definiteness of matrices $\bar{A}^1_{ijk}$ and the monotonicity of $\|\hat{v}\|_u$ we get

$$
\begin{aligned}
&\|\bar{A}^1_{\sigma(k-1)\sigma(k)\sigma(k+1)}...\bar{A}^1_{\sigma(0)\sigma(1)\sigma(2)}\|_{\mathcal{L}(\mathbb{R}^r,\|\cdot\|_{\hat{v}})}\\
&=\|\bar{A}^1_{\sigma(k-1)\sigma(k)\sigma(k+1)}...\bar{A}^1_{\sigma(0)\sigma(1)\sigma(2)}\hat{v}\|_{\hat{v}}\\
&\leq\|\bar{A}^1_{\sigma(k-1)\sigma(k)\sigma(k+1)}...\bar{A}^1_{\sigma(1)\sigma(2)\sigma(3)}(1-\epsilon)\hat{v}\|_{\hat{v}}\\
&=(1-\epsilon)\|\bar{A}^1_{\sigma(k-1)\sigma(k)\sigma(k+1)}...\bar{A}^1_{\sigma(1)\sigma(2)\sigma(3)}\hat{v}\|_{\hat{v}}\\
&\qquad\qquad ...\\
&\leq(1-\epsilon)^k\|\hat{v}\|_{\hat{v}}=(1-\epsilon)^k.
\end{aligned}
$$

According to [15], system (4) is exponentially stable. It follows that there exist finite positive constants $0<\lambda<1$ and $\gamma>0$ such that

$$\|v(k)\|\leq\gamma\lambda^k\|v(0)\|.$$

Furthermore since the corresponding solution of system (1) is $x(k)=V_{\sigma(k)\sigma(k+1)}(v(k)^T,0)^T$, we have

$$
\begin{aligned}
\|x(k)\| &\leq\|V_{\sigma(k)\sigma(k+1)}\|\,\|(v(k)^T,0)^T\|\\
&\leq\gamma\lambda^k\|V_{\sigma(k-1)}D_{\sigma(k)\sigma(k-1)}\|\,\|(V_{\sigma(k)\sigma(k+1)}v(0)^T,0)^T\|\\
&\leq\gamma\lambda^k\|V_{\sigma(k)\sigma(k+1)}\|\,\|V^{-1}_{\sigma(0)\sigma(1)}x(0)\|\\
&\leq\gamma\lambda^k\|V_{\sigma(k)\sigma(k+1)}\|\,\|V^{-1}_{\sigma(0)\sigma(1)}\|\,\|x(0)\|.
\end{aligned}
$$

Putting $\mu=\gamma\max_{i,j=1,2,..,N}\|V_{ij}\|\,\|V^{-1}_{ij}\|$, we have

The last relation shows that the solution of system (1) is exponentially stable.

Theorem 3 is proved.

Example 1 Put

$$E_1=\begin{pmatrix}2&-3&0\\0&2&0\\0&0&0\end{pmatrix},\quad E_2=\begin{pmatrix}3&2&0\\1&6&0\\0&0&0\end{pmatrix},$$

$$A_1 = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix},$$

$$V_{11} = V_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad V_{21} = V_{22} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

We calculate the matrices $\bar{A}_{ijk}, i, j, k \in \{1,2\}$ as

$$\bar{A}_{111} = \bar{A}_{112} = \begin{pmatrix} 1/2 & 1/12 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \bar{A}_{121} = \bar{A}_{122} = \begin{pmatrix} 3/16 & -1/12 & 0 \\ -1/4 & 1/6 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$\bar{A}_{211} = \bar{A}_{212} = \begin{pmatrix} 7/8 & 1/3 & 0 \\ 1/4 & 1/6 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \bar{A}_{221} = \bar{A}_{222} = \begin{pmatrix} 5/8 & 1/8 & 0 \\ 1/16 & 5/16 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Clearly all the matrices $\bar{A}_{ijk}$ are positive definite. We choose $\hat{v} = (9,3)^T \in Int(\mathcal{K})$ and find that $\hat{v} - \bar{A}^1_{ijk}\hat{v}$ are also inside $Int(\mathcal{K})$. It means that this system satisfies all the condition of Theorem 3, thus it is exponentially stable.

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