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Original Article

Influence of Confined Acoustic Phonons on the Nonlinear Acousto-electric Effect in Doped Semiconductor Superlattices

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Abstract: By using a quantum kinetic equation for electrons, we have studied the Acousto-electric effects in doped semiconductor superlattice (DSSL) under the influence of confined phonon. Considering the case of the electron - acoustic phonon interaction, we have found the expressions of the nonlinear quantum acousto-electric current. From these expression, the acousto-electric current (AEC) depends nonlinearly on temperature, acoustic wave frequency and the characteristic parameters of DSSL (For example: the doped concentration n_D). Moreover, the expression of the AEC under the influence of confined phonons fairly different from the case of unconfined phonons. The results are numerically calculated for the GaAs:Be/ GaAs:Si DSSL; therefore, it can be easily seen that the dependence of the acousto-electric current on the characteristic parameters of the acoustic wave, temperature, the characteristic parameters of DSSL and the quantum number *m* characterizing the phonons confinement. The results have showed that the appearance of phonons confinement make the AEC value changes remarkably. The AEC is almost stable in low acoustic wave frequency condition and changes as a parabolic curve when ω_q move up. On the other hand, in case of low doped concentration number the AEC surges as a parabolic function in the dependence on n_D, then it remains stability at just below zero in high n_D value.

Keywords: Acousto-electric field, Quantum kinetic equation, Doped superlattices, Electron - phonon interaction.

1. Introduction

In recent years, the semiconductor materials have been widely used in electronics. The development of semiconductor electronics is mainly based on the phenomenon of contact p-n and the doped ability

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to alter the physical properties of crystals. Consequently, the acousto-electric effects in bulk materials and mesoscopic structures has become interests of many scientists [1-8]. In recent time, the acousto-electric effect was studied in both one-dimensional system [9] and finite-length ballistic channel [10-11]. In addition, the acousto-electric effect was measured by an experiment in a submicron-seperated quantum wire [12], in a carbon nanotube [13] and was also studied in the infinite potential cylindrical wire [14]. However, these studies only considered the electrons - unconfined phonons interaction while recent works show the vital contribution of phonons confinement in these kinetic properties [15]. So, in this work, we study the influence of confined acoustic phonons on the nonlinear acousto-electric effect in a DSSL

In literature, there are some methods to solve this problem such as: the Kubo-Mori method, the Boltzmann kinetic equation. However, the limitation of both the Kubo-Mori method and the Boltzmann equation are used in high temperature conditions only. In order to eliminate this limitation and to have more accurate results, we have used a quantum kinetic equation for electrons to study the acousto-electric effect in doped superlattice under the influence of confined phonons.

We have discovered some differences between the results obtained in this case and those in the case of unconfined phonons. Numerical calculations are carried out with a specifically doped superlattice GaAs:Be/ GaAs:Si.

2. Calculation of acousto-electric current in doped semiconductor superlattice A

In this report, we consider a simple model of a DSSL (n-i-p-i superlattice) in which electron gas is confined by an additional potential along the z-direction and free in the (x-y) plane. The DSSL is subjected to a crossed electric field $\vec{E} = (E, 0, 0)$, magnetic field $\vec{B} = (0, 0, B)$.

$$\psi_{n,\vec{p}}(\vec{r}) = e^{i\vec{p}_\perp \vec{r}_\perp} U_n(\vec{r}) \sum_{m=1}^{N_d} e^{ip_z md} \psi_n(z - md)$$

$$\varepsilon_n(\vec{p}_\perp) = \frac{\hbar^2 \vec{p}_\perp^2}{2m^*} + \hbar\omega_p \left(n + \frac{1}{2}\right)$$

Here: $n = 0,1,2, \ldots$ is the quantum number of the energy spectrum in z-direction, $U_n(\vec{r})$ is the matrix operator form of $U = \exp(ig_y - k_e z)$; $k_e = \left(q^2 - \omega_q^2 / C_e^2\right)^{1/2}$, m^* is the electron effective mass, d is the superlattices period length and N_d is superlattices period number. $\vec{p} = (\vec{p}_\perp, \vec{p}_z)$ is the electron momentum vector. And ω_p is the plasma frequency characterizing for the DSSL confinement in the z-direction, given by $\omega_p = \left(\frac{4\pi e^2 n_p}{X_0 m^*}\right)^2$, where X_0 is the electronic constant (vacuum permittivity),

The Hamiltonian of the electron-acoustic phonon system in DSSL in the second quantization presentation can be written as:

$$H = \sum_{n',\vec{p}'_\perp} \varepsilon_{n'}(\vec{p}'_\perp) a^+_{n',\vec{p}'_\perp} a_{n',\vec{p}'_\perp} + \sum_{m,\vec{q}_\perp} \hbar\omega_{m,\vec{q}_\perp} b^+_{m,\vec{q}_\perp} b_{m,\vec{q}_\perp} + \sum_{n',\vec{p}'_\perp,n'_1,\vec{k}} C_{\vec{k}} U_{n',n'_1} a^+_{n',\vec{p}'_\perp+\vec{k}} a_{n',\vec{p}'_\perp} exp\left(-i\omega_{\vec{k}} t\right) + \sum_{n',\vec{p}'_\perp,m,\vec{q}_\perp} C_{m,\vec{q}_\perp} I^m_{n',n'_1,\vec{q}_\perp} a^+_{n',\vec{p}'_\perp+\vec{q}_\perp} a_{n',\vec{p}'_\perp} (b_{m,\vec{q}_\perp} + b^+_{m,-\vec{q}_\perp}) \quad (1)$$

Where $\vec{p}_\perp$ is the electron momentum tensor in perpendicular plane with the DSSL axis, $\omega_{m,\vec{q}_\perp} \approx \nu\sqrt{\vec{q}_\perp^2 + \vec{q}_m^2}$ is the acoustic phonon frequency. $a^+_{n',\vec{p}'_\perp}$ and $a_{n',\vec{p}'_\perp}$ ($b^+_{m,\vec{q}_\perp}$ and $b_{m,\vec{q}_\perp}$) are the creation and annihilation operators of electron (phonon), respectively. $C_{m,\vec{q}_\perp}$ is the electron – acoustic phonon interaction constant which depends on the scattering mechanism, $I^m_{n',n_1',\vec{q}_\perp}$ is the form factor of electron, given by:

$$\left|C_{m,\vec{q}_\perp}\right|^2 = \frac{\hbar \mathcal{E}^2}{2V_0\rho v_s}\sqrt{\vec{q}_\perp^2 + \vec{q}_m^2}$$

$$I^m_{n,n',\vec{q}_\perp} = \sum_{\wp}^{s_0}\int_0^d e^{\pm i\vec{q}_\perp d}\phi_n(z-\wp d)\,\phi_{n'}(z-\wp d)\mathrm{dz}$$

and

$$|C_k|^2 = \frac{|\wedge|^2 C_k^4 \hbar\omega^3_{\vec{q}_\perp}}{2\rho F s}$$

With $\mathcal{E}$ is the deformation potential, ρ is the mass density, V_0 is the normalization volume and v_s is the sound wave velocity. d is the period and s_0 is the number of periods of the DSSL.

The quantum kinetic equation of average number of electron $\mathcal{F}_{n,\vec{p}_\perp}(t) = \langle a^+_{n',\vec{p}_\perp} a_{n',\vec{p}_\perp}\rangle_t$ in DSSL is:

$$i\hbar\frac{\partial \mathcal{F}_{n,\vec{p}_\perp}(t)}{\partial t} = \left\langle\left[a^+_{n',\vec{p}_\perp} a_{n',\vec{p}_\perp}, H\right]\right\rangle_t \quad (2)$$

By replacing Eq.(1) on Eq.(2) we get the quantum kinetic equation:

$$\begin{aligned}\frac{\partial \mathcal{F}_{n,\vec{p}_\perp}(t)}{\partial t} = &-\frac{\pi}{\hbar^2}\sum_{n',m,\vec{p}_\perp}\left|C_{m,\vec{q}_\perp}\right|^2\left|I^m_{n,n',\vec{q}_\perp}\right|^2 N_{\vec{q}_\perp}\{(n_{n,\vec{p}_\perp} - n_{n',\vec{p}_\perp+\vec{q}_\perp})\delta(\varepsilon_{n',\vec{p}_\perp} - \varepsilon_{n,\vec{p}_\perp} - \hbar\omega_{m,\vec{q}_\perp})\\ &+ (n_{n,\vec{p}_\perp} - n_{n',\vec{p}_\perp+\vec{q}_\perp})\delta(\varepsilon_{n',\vec{p}_\perp} + \varepsilon_{n,\vec{p}_\perp} + \hbar\omega_{m,\vec{q}_\perp})\\ &- (n_{n',\vec{p}_\perp+\vec{q}_\perp} - n_{n,\vec{p}_\perp})\delta(\varepsilon_{n,\vec{p}_\perp} - \varepsilon_{n',\vec{p}_\perp-\vec{q}_\perp} + \hbar\omega_{m,\vec{q}_\perp})\\ &- (n_{n',\vec{p}_\perp+\vec{q}_\perp} - n_{n,\vec{p}_\perp})\delta(\varepsilon_{n,\vec{p}_\perp} - \varepsilon_{n',\vec{p}_\perp-\vec{q}_\perp} - \hbar\omega_{m,\vec{q}_\perp})\}\\ &- \frac{\pi}{\hbar^2}\sum_{n',\vec{k}}\left|C_{\vec{k}}\right|^2\left|U_{n,n'}\right|^2 N_{\vec{k}}\{\left(n_{n,\vec{p}_\perp} - n_{n',\vec{p}_\perp+\vec{k}}\right)\delta\left(\varepsilon_{n',\vec{p}_\perp+\vec{k}} - \varepsilon_{n,\vec{p}_\perp} + \hbar\omega_{\vec{q}_\perp}\right.\\ &\left.- \hbar\omega_{\vec{k}}\right) - \left(n_{n',\vec{p}_\perp+\vec{k}} - n_{n,\vec{p}_\perp}\right)\delta\left(\varepsilon_{n,\vec{p}_\perp} - \varepsilon_{n',\vec{p}_\perp+\vec{k}} + \hbar\omega_{\vec{q}_\perp} - \hbar\omega_{\vec{k}}\right)\}\end{aligned}$$

We put t in the range from 0 to τ_p and calculate the integral value, we obtain:

$$\begin{aligned}\mathcal{F}_{n,\vec{p}_\perp}(t) = \mathcal{F}_1 = &\frac{\pi\tau_p}{\hbar^2}\sum_{n,n',m,\vec{p}_\perp}\left|C_{m,\vec{q}_\perp}\right|^2\left|I^m_{n,n',\vec{q}_\perp}\right|^2 N_{\vec{q}_\perp}\{(n_{n,\vec{p}_\perp} - n_{n',\vec{p}_\perp+\vec{q}_\perp})\delta(\varepsilon_{n',\vec{p}_\perp} - \varepsilon_{n,\vec{p}_\perp}\\ &- \hbar\omega_{m,\vec{q}_\perp}) + (n_{n,\vec{p}_\perp} - n_{n',\vec{p}_\perp+\vec{q}_\perp})\delta(\varepsilon_{n',\vec{p}_\perp} + \varepsilon_{n,\vec{p}_\perp} + \hbar\omega_{m,\vec{q}_\perp})\\ &- (n_{n',\vec{p}_\perp+\vec{q}_\perp} - n_{n,\vec{p}_\perp})\delta(\varepsilon_{n,\vec{p}_\perp} - \varepsilon_{n',\vec{p}_\perp-\vec{q}_\perp} + \hbar\omega_{m,\vec{q}_\perp})\\ &- (n_{n',\vec{p}_\perp+\vec{q}_\perp} - n_{n,\vec{p}_\perp})\delta(\varepsilon_{n,\vec{p}_\perp} - \varepsilon_{n',\vec{p}_\perp-\vec{q}_\perp} - \hbar\omega_{m,\vec{q}_\perp})\}\\ &- \frac{\pi}{\hbar^2}\sum_{n',\vec{k}}\left|C_{\vec{k}}\right|^2\left|U_{n,n'}\right|^2 N_{\vec{k}}\{\left(n_{n,\vec{p}_\perp} - n_{n',\vec{p}_\perp+\vec{k}}\right)\delta\left(\varepsilon_{n',\vec{p}_\perp+\vec{k}} - \varepsilon_{n,\vec{p}_\perp} + \hbar\omega_{\vec{q}_\perp}\right.\\ &\left.- \hbar\omega_{\vec{k}}\right) - \left(n_{n',\vec{p}_\perp+\vec{k}} - n_{n,\vec{p}_\perp}\right)\delta\left(\varepsilon_{n,\vec{p}_\perp} - \varepsilon_{n',\vec{p}_\perp+\vec{k}} + \hbar\omega_{\vec{q}_\perp} - \hbar\omega_{\vec{k}}\right)\}\end{aligned}$$

The nonlinear acousto-electric current density given by:

$$j = \frac{2e}{(2\pi\hbar)^2}\sum_n\int V_{\vec{p}_\perp}\mathcal{F}_1 d\vec{p}_\perp$$

$$= -\frac{e\tau_p \mathcal{E}^2 n_0}{2\pi\hbar^5 V_0 \rho v_s m\omega_{\vec{q}_\perp}}\left(\frac{2m}{\hbar^3\beta}\right)^3 e^{\varepsilon_F\beta}\sum\nolimits_{n,n',m,\vec{q}_\perp}\left|I^m_{n,n',\vec{q}_\perp}\right|^2 e^{\frac{-\hbar\beta}{2mR^2}B_n^2}\{\xi_1^3 e^{-\xi_1}\left[\left(\frac{2m}{\hbar^3\beta}\xi_1\right)^2 k_3(\xi_1)\right.$$
$$\left. + 3k_2(\xi_1) + 3k_1(\xi_1) + k_0(\xi_1)\right]$$
$$+ \xi_2^3 e^{-\xi_2}\left[\left(\frac{2m}{\hbar^3\beta}\xi_2\right)^2 k_3(\xi_2) + 3k_2(\xi_2) + 3k_1(\xi_2) + k_0(\xi_2)\right]\}$$
$$+ \frac{e\tau_p \Lambda^2 C_k^4 n_0 w(2\pi)^2}{\hbar^4 \rho F s v_s}\left(\frac{2m}{\beta}\right)^{\frac{3}{2}} e^{\varepsilon_F\beta}\sum\nolimits_{n,n'}\left|U_{n,n'}\right|^2 e^{\frac{-\hbar\beta}{2mR^2}B_n^2}\left\{e^{-\lambda_1}\lambda_1^{\frac{5}{2}}\left[k_{\frac{5}{2}}(\lambda_1)\right.\right.$$
$$\left. + 3k_{\frac{3}{2}}(\lambda_1) + 3k_{\frac{1}{2}}(\lambda_1) + k_{-\frac{1}{2}}(\lambda_1)\right]$$
$$\left. - e^{-\lambda_2}\lambda_2^{\frac{5}{2}}\left[k_{\frac{5}{2}}(\lambda_2) + 3k_{\frac{3}{2}}(\lambda_2) + 3k_{\frac{1}{2}}(\lambda_2) + k_{-\frac{1}{2}}(\lambda_2)\right]\right\} \quad (3)$$

Where:

$$\xi_1 = \frac{\hbar^3\beta}{2m}\left[\frac{\hbar(B_{n'}^2 - B_n^2)}{2R^2} - m\omega_{\vec{q}_\perp}\right];\ \xi_2 = \frac{\hbar^3\beta}{2m}\left[\frac{\hbar(B_{n'}^2 - B_n^2)}{2R^2} + m\omega_{\vec{q}_\perp}\right]$$

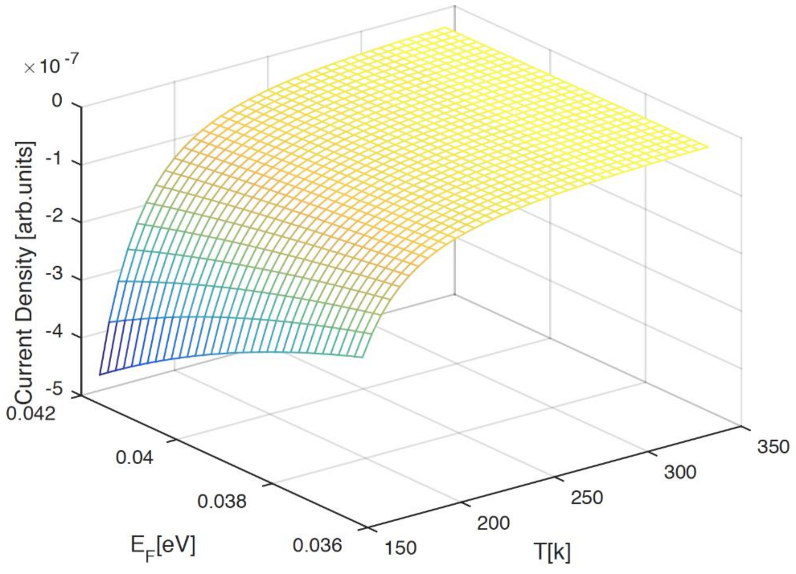
$$\lambda_1 = \xi_1 + \frac{\hbar^3\beta}{2}\omega_{\vec{k}}\ ;\ \lambda_2 = \xi_2 - \frac{\hbar^3\beta}{2}\omega_{\vec{k}}$$

With $k_a(x)$ is the modified Bessel function of the second kind. ε_F is the Fermi level, k_β is the Boltzmann constant and $\beta = \frac{1}{k_\beta T}$. From Eq (3), we see that the asousto-electric current expression in the doped superlattice is more complicated and depends nonlinearly on temperature, acoustic wave frequency, the characteristic parameters of DSSL and the quantum number m characterizing the phonons confinement.

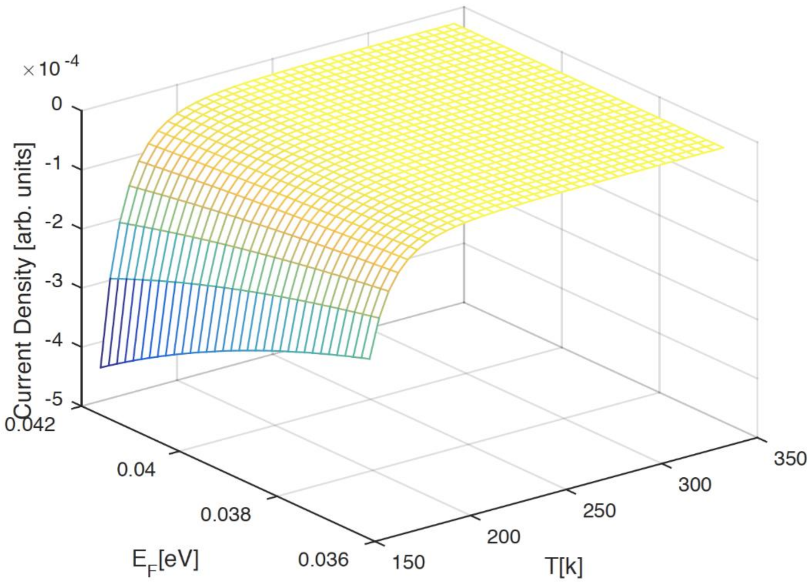
3. Numerical results and discussion

In this section, we present detailed numerical calculations of the AEC in a DSSL subjected to the system temperature, the frequency of the acoustic wave and the doped concentration n_D. Furthermore, we survey the influence of the AEC on the phonons confinement quantum number m and the doped concentration n_D. For the numerical evaluation, we consider the n-i-p-i superlattice of GaAs:Si/GaAs:Be with the parameters [7-8]: $m = 0067m_0$ (m_0 is the mass of free electron), $\hbar\omega_0 = 36.25\,MeV$ is the acoustic phonon energy, $n_0 = 10^{23}\,m^{-3}$ is the electron concentration and $d = 134.10^{-10}\,m$ is the superlattice period.

Figure 1 shows the dependence of AEC on the temperature and Fermi energy level in two cases: confined phonons and unconfined phonons. The results showed that AEC value increases nonlinearly and gradually before reach a peak of -0.13 $(arb.units)$ at $T = 325\,K$ and $\varepsilon_F = 0.038\,eV$ in case of $m = 0$ and at $T = 300\,K$ and $\varepsilon_F = 0.038\,eV$ in case of $m = 4.5$. That lead us to conclusion that the AEC goes up to the maximum value then reduces rapidly. Although, the AEC maximum position is almost the same at $T = 325\,K$, the influence of the unconfined phonons on the current density is remarkable at the point of $T = 200\,K$ and $\varepsilon_F = 0.042$, the value of the current density is higher. That means the orbit radius of the unconfined phonons case is smaller than the case of confined phonon. So that the impact of the phonons confinement on the AEC is considerable



m=0



m=4.5

Figure 1.The dependence of acousto-electric current on temperature $T = 150K \div 350K$ and Fermi energy level.

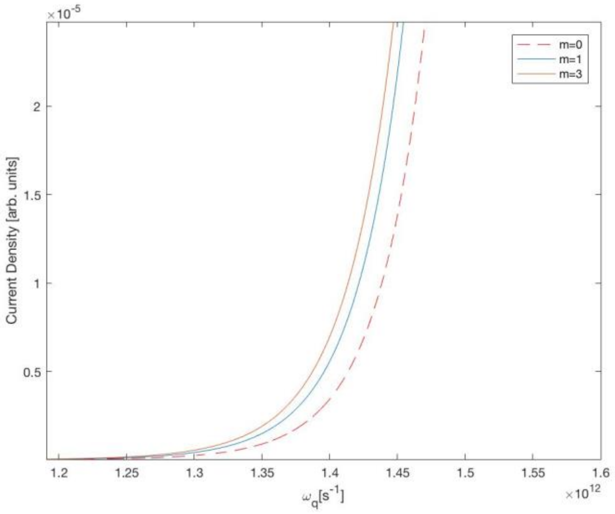


Figure 2. The dependence of the current density on the acoustic wave frequency

$$\omega_q = 1.2 \div 1.6 \times 10^{12}\, s^{-1}$$

Figure 2 shows the dependence of AEC on acoustic wave frequency in three cases: confined phonons (dashed line), unconfined phonons $m = 1$ (blue line) and unconfined phonons $m = 3$ (orange line). It is clearly that the current density depends on the acoustic wave frequency as parabolic curved. The curve shape in three cases look similar, the position of maximum and minimum density current value is concurrent. The difference we obtain in three cases is the value of the current density. As the quantum number m grows up, the current density surges more significant in the condition of high acoustic wave frequency (>1.25× $10^{12}\, s^{-1}$). For the case of $m = 0$, the value of the AEC is 7% and 10% stronger than the case of unconfined phonons $m = 1$ and $m = 3$, respectively. So, this is the influence of confinement phonons on the current in particular.

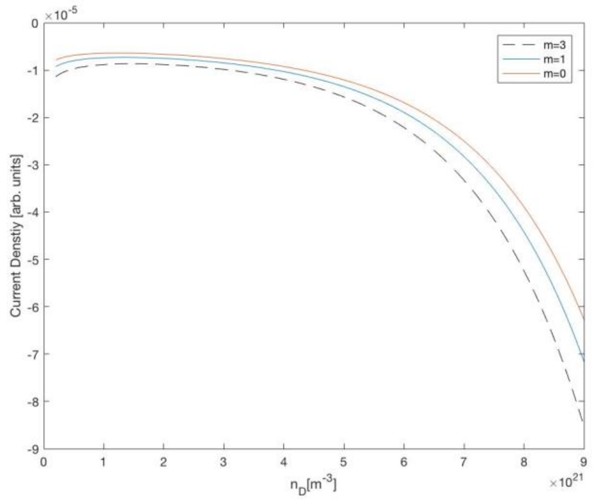


Figure 3. The dependence of current density on the doped concentration n_D.

The figure describes the dependence of current density on the doped concentration n_D also in three cases: confined phonons (orange line), unconfined phonons $m = 1$ (blue line) and unconfined phonons $m = 3$ (dashed line)). We found that the dependence of the current on n_D is nonlinear. The appearance of phonons confinement makes the current density value become lower than the cases of unconfined phonons. The changing of the quantum number m does not impact on the shape of the lines although the value of the current density decreases as m rises. Over the doped concentration period, the value of AEC with the appearance of confined phonons ($m = 0$) is about 7% and 15% stronger than the cases of $m = 1$ and $m = 3$, respectively. Moreover, we can just see clearly the impact of phonons confinement in low doped concentration condition, when the doped concentration n_D goes up ($n_D > 5m^{-3}$), the three cases of m vary together as a straight line. This is the new findings that we have studied.

4. Conclusion

In this paper, we have analytically investigated the acousto-electric field in doped superlattices. The electron-acoustic phonon interaction is taken into account at low temperatures. We expose the analytical expressions of the acousto-electric current AEC in DSSL. The results have been evaluated in GaAs: Be / GaAs: Si DSSL to see the AEC's dependence on acoustic wave, temperature, parameters characterizing superlattices and the phonons confinement quantum number m.

The results show that when the temperature rises up, the current density surges and then remain stability at the highest value position. The appearance of the phonons confinement boosts the current density value in low temperature and high fermi energy level condition but doesn't change the maximum condition of temperature.

Also, we have found the parabolic curved appears when we survey the dependence of the current density on the acoustic wave. The result we found is similar as above but more detailed. When the phonons confinement number m increases, the dependence of the current density on the acoustic wave frequency value increase slightly bigger in high frequency condition. So, this is the potential impact of phonons confinement on the AEC.

When we studied the dependence of the current density on the doped concentration n_D, we found that the current density in the doped superlattices decreases non-linearly and does not appear the maximum value or the oscillation as in the bulk semiconductor.

These are new results for acousto-electric effect in the doped semiconductor superlattices that we have studied.

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