



Original Article

# Determine Dispersion Coefficient of $^{85}\text{Rb}$ Atom in the Y-Configuration

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**Abstract:** In this work, we derive analytical expression for the dispersion coefficient of  $^{85}\text{Rb}$  atom for a weak probe laser beam induced by a strong coupling laser beams. Our results show possible ways to control dispersion coefficient by frequency detuning and of the coupling lasers. The results show that a Y-configuration appears two transparent window of the dispersion coefficient for the probe laser beam. The depth and width or position of these windows can be altered by changing the intensity or frequency detuning of the coupling laser fields.

**Keywords:** Electromagnetically induced transparency, dispersion coefficient.

## 1. Introduction

The manipulation of subluminal and superluminal light propagation in optical medium has attracted many attentions due to its potential applications during the last decades, such as controllable optical delay lines, optical switching [1], telecommunication [2], interferometry, optical data storage and optical memories quantum information processing, and so on [3]. The most important key to manipulate subluminal and superluminal light propagations lies in its ability to control the absorption and dispersion properties of a medium by a laser field [4, 5].

As we know that coherent interaction between atom and light field can lead to interesting quantum interference effects such as electromagnetically induced transparency (EIT) [6]. The EIT is a quantum interference effect between the probability amplitudes that leads to a reduction of resonant absorption for a weak probe light field propagating through a medium induced by a strong coupling light field. Basic configurations of the EIT effect are three-level atomic systems including the  $\Lambda$ -Ladder and V-

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type configurations. In each configuration, the EIT efficiency is different, in which the  $\Lambda$ -type configuration is the best, whereas the V-type configuration is the worst [7], therefore, the manipulation of light in each configuration are also different. To increase the applicability of this effect, scientists have paid attention to creating many transparent windows. One proposed option is to add coupling laser fields to further stimulate the states involved in the interference process. This suggests that we choose to use the analytical model to determine the dispersion coefficient for the Y configuration of the  $^{85}\text{Rb}$  atomic system [8].

## 2. The density matrix equation

We consider a Y-configuration of  $^{85}\text{Rb}$  atom as shown in Fig. 1. State  $|1\rangle$  is the ground states of the level  $5S_{1/2}$  ( $F=3$ ). The  $|2\rangle$ ,  $|3\rangle$  and  $|4\rangle$  states are excited states of the levels  $5P_{3/2}$  ( $F'=3$ ),  $5D_{5/2}$  ( $F''=4$ ) and  $5D_{5/2}$  ( $F''=3$ ) [8].

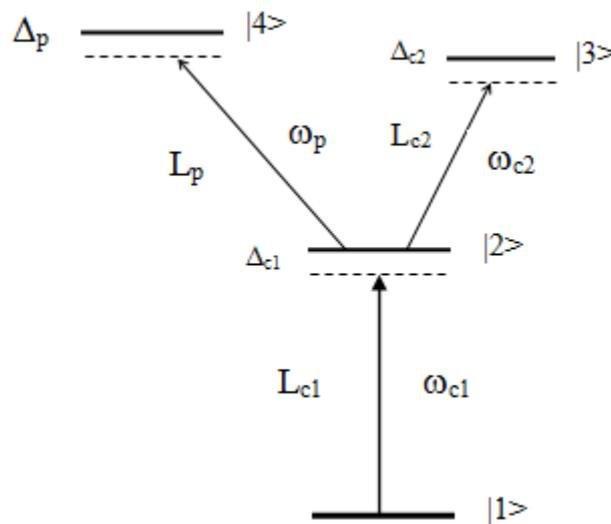


Fig 1. Four-level excitation of the Y- configuration.

Put this Y-configuration into three laser beams atomic frequency and intensity appropriate: a weak probe laser  $L_p$  has intensity  $E_p$  with frequency  $\omega_p$  applies the transition  $|2\rangle \rightarrow |4\rangle$  and the Rabi frequencies of the probe  $\Omega_p = \frac{\mu_{42}E_p}{\hbar}$ ; two strong coupling laser  $L_{c1}$  and  $L_{c2}$  couple the transition  $|1\rangle \rightarrow |2\rangle$  and  $|2\rangle \rightarrow |3\rangle$  the Rabi frequencies of the two coupling fields  $\Omega_{c1} = \frac{\mu_{21}E_{c1}}{\hbar}$  and  $\Omega_{c2} = \frac{\mu_{32}E_{c2}}{\hbar}$ , where  $\mu_{ij}$  is the electric dipole matrix element  $|i\rangle \leftrightarrow |j\rangle$ .

The evolution of the system, which is represented the density operator  $\rho$  is determined by the following Liouville equation [2]:

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H, \rho] + \Lambda \rho \tag{1}$$

where,  $H$  represents the total Hamiltonian and  $\Lambda \rho$  represents the decay part. Hamilton of the system can be written by matrix form:

$$H_0 = \hbar \omega_1 |1\rangle\langle 1| + \hbar \omega_2 |2\rangle\langle 2| + \hbar \omega_3 |3\rangle\langle 3| + \hbar \omega_4 |4\rangle\langle 4| \tag{2.2}$$

$$H_I = \frac{\hbar \Omega_p}{2} (|4\rangle\langle 2| e^{i\omega_p t} + |2\rangle\langle 4| e^{-i\omega_p t}) + \frac{\hbar \Omega_{c1}}{2} (|2\rangle\langle 1| e^{i\omega_{c1} t} + |1\rangle\langle 2| e^{-i\omega_{c1} t}) + \frac{\hbar \Omega_{c2}}{2} (|3\rangle\langle 2| e^{i\omega_{c2} t} + |2\rangle\langle 3| e^{-i\omega_{c2} t}) \tag{2.3}$$

In the framework of the semiclassical theory, the density matrix equations can be written as:

$$[\rho, H]_{44} = \frac{\hbar \Omega_p}{2} (e^{i\omega_p t} \rho_{42} - e^{-i\omega_p t} \rho_{24}) - \Gamma_{43} \rho_{44} \tag{3.1}$$

$$[\rho, H]_{41} = \frac{\hbar \Omega_{c1}}{2} e^{-i\omega_{c1} t} \rho_{42} - \frac{\hbar \Omega_p}{2} e^{-i\omega_p t} \rho_{21} + \hbar(\omega_1 - \omega_4) \rho_{41} - \gamma_{41} \rho_{41} \tag{3.2}$$

$$[\rho, H]_{42} = \frac{\hbar \Omega_{c1}}{2} e^{i\omega_{c1} t} \rho_{41} + \frac{\hbar \Omega_{c2}}{2} e^{-i\omega_{c2} t} \rho_{43} + \frac{\hbar \Omega_p}{2} e^{-i\omega_p t} (\rho_{44} - \rho_{22}) + \hbar(\omega_2 - \omega_4) \rho_{42} - \gamma_{42} \rho_{42} \tag{3.3}$$

$$[\rho, H]_{43} = \frac{\hbar \Omega_{c2}}{2} e^{i\omega_{c2} t} \rho_{42} - \frac{\hbar \Omega_p}{2} e^{-i\omega_p t} \rho_{23} + \hbar(\omega_3 - \omega_4) \rho_{43} - \gamma_{43} \rho_{43} \tag{3.4}$$

$$[\rho, H]_{33} = \frac{\hbar \Omega_{c2}}{2} (e^{i\omega_{c2} t} \rho_{32} - e^{-i\omega_{c2} t} \rho_{23}) + \Gamma_{43} \rho_{44} - \Gamma_{32} \rho_{33} \tag{3.5}$$

$$[\rho, H]_{31} = \frac{\hbar \Omega_{c1}}{2} e^{-i\omega_{c1} t} \rho_{32} - \frac{\hbar \Omega_{c2}}{2} e^{-i\omega_{c2} t} \rho_{21} + \hbar(\omega_1 - \omega_3) \rho_{31} - \gamma_{31} \rho_{31} \tag{3.6}$$

$$[\rho, H]_{32} = \frac{\hbar \Omega_{c1}}{2} e^{i\omega_{c1} t} \rho_{31} + \frac{\hbar \Omega_{c2}}{2} e^{-i\omega_{c2} t} (\rho_{33} - \rho_{22}) + \frac{\hbar \Omega_p}{2} e^{-i\omega_p t} \rho_{34} + \hbar(\omega_2 - \omega_3) \rho_{32} - \gamma_{32} \rho_{32} \tag{3.7}$$

$$[\rho, H]_{34} = \frac{\hbar \Omega_p}{2} e^{i\omega_p t} \rho_{32} - \frac{\hbar \Omega_{c2}}{2} e^{-i\omega_{c2} t} \rho_{24} + \hbar(\omega_4 - \omega_3) \rho_{34} - \gamma_{43} \rho_{34} \tag{3.8}$$

$$[\rho, H]_{22} = \frac{\hbar \Omega_{c1}}{2} (e^{i\omega_{c1} t} \rho_{21} - e^{-i\omega_{c1} t} \rho_{12}) + \frac{\hbar \Omega_{c2}}{2} (e^{-i\omega_{c2} t} \rho_{23} - e^{i\omega_{c2} t} \rho_{32}) + \frac{\hbar \Omega_p}{2} (e^{-i\omega_p t} \rho_{24} - e^{i\omega_p t} \rho_{42}) + \Gamma_{32} \rho_{33} - \Gamma_{21} \rho_{22} \tag{3.9}$$

$$[\rho, H]_{21} = \frac{\hbar \Omega_{c1}}{2} e^{-i\omega_{c1} t} (\rho_{22} - \rho_{11}) - \frac{\hbar \Omega_{c2}}{2} e^{i\omega_{c2} t} \rho_{31} - \frac{\hbar \Omega_p}{2} e^{i\omega_p t} \rho_{41} + \hbar(\omega_1 - \omega_2) \rho_{21} - \gamma_{21} \rho_{21} \tag{3.10}$$

$$[\rho, H]_{23} = \frac{\hbar\Omega_{c2}}{2} e^{i\omega_c t} (\rho_{22} - \rho_{33}) - \frac{\hbar\Omega_{c1}}{2} e^{-i\omega_c t} \rho_{13} - \frac{\hbar\Omega_p}{2} e^{i\omega_p t} \rho_{43} + \hbar(\omega_3 - \omega_2) \rho_{23} - \gamma_{32} \rho_{23} \quad (3.11)$$

$$[\rho, H]_{24} = \frac{\hbar\Omega_p}{2} e^{i\omega_p t} (\rho_{22} - \rho_{44}) - \frac{\hbar\Omega_{c1}}{2} e^{-i\omega_c t} \rho_{14} - \frac{\hbar\Omega_{c2}}{2} e^{i\omega_c t} \rho_{34} + \hbar(\omega_4 - \omega_2) \rho_{24} - \gamma_{42} \rho_{24} \quad (3.12)$$

$$[\rho, H]_{11} = \frac{\hbar\Omega_{c1}}{2} e^{i\omega_c t} (\rho_{12} - \rho_{21}) + \Gamma_{21} \rho_{22} \quad (3.13)$$

$$[\rho, H]_{12} = \frac{\hbar\Omega_{c1}}{2} e^{i\omega_c t} (\rho_{11} - \rho_{22}) + \frac{\hbar\Omega_{c2}}{2} e^{-i\omega_c t} \rho_{13} + \frac{\hbar\Omega_p}{2} e^{-i\omega_p t} \rho_{14} + \hbar(\omega_2 - \omega_1) \rho_{12} - \gamma_{21} \rho_{12} \quad (3.14)$$

$$[\rho, H]_{13} = \frac{\hbar\Omega_{c2}}{2} e^{i\omega_c t} \rho_{12} - \frac{\hbar\Omega_{c1}}{2} e^{i\omega_c t} \rho_{23} + \hbar(\omega_3 - \omega_1) \rho_{13} - \gamma_{31} \rho_{13} \quad (3.15)$$

$$[\rho, H]_{14} = \frac{\hbar\Omega_p}{2} e^{i\omega_p t} \rho_{12} - \frac{\hbar\Omega_{c1}}{2} e^{i\omega_c t} \rho_{24} + \hbar(\omega_4 - \omega_1) \rho_{14} - \gamma_{41} \rho_{14} \quad (3.16)$$

In addition, we suppose the initial atomic system is at a level  $|2\rangle$  therefore,  $\rho_{11} \approx \rho_{33} \approx \rho_{44} \approx 0, \rho_{22} = 1$  and solve the density matrix equations under the steady-state condition by setting the time derivatives to zero:

$$\frac{d\rho}{dt} = 0 \quad (4)$$

We consider the slow variation and put:  $\rho_{43} = \tilde{\rho}_{43} e^{-i(\omega_p - \omega_{c2})t}$ ,  $\rho_{42} = \tilde{\rho}_{42} e^{-i\omega_p t}$ ,  $\rho_{41} = \tilde{\rho}_{41} e^{-i(\omega_p + \omega_{c1})t}$ ,  $\rho_{32} = \tilde{\rho}_{32} e^{-i\omega_{c2}t}$ ,  $\rho_{31} = \tilde{\rho}_{31} e^{-i(\omega_{c1} + \omega_{c2})t}$ ,  $\rho_{21} = \tilde{\rho}_{21} e^{-i\omega_{c1}t}$ . Therefore, the equations (3.2), (3.3) and (3.4) are rewritten:

$$0 = \frac{i\Omega_{c1}}{2} \tilde{\rho}_{42} - \frac{i\Omega_p}{2} \rho_{21} + [i(\Delta_{c1} + \Delta_p) - \gamma_{41}] \tilde{\rho}_{41} \quad (5.1)$$

$$0 = \frac{i\Omega_{c1}}{2} \tilde{\rho}_{41} + \frac{i\Omega_{c2}}{2} \tilde{\rho}_{43} + \frac{i\Omega_p}{2} (\rho_{44} - \rho_{22}) + (i\Delta_p - \gamma_{42}) \tilde{\rho}_{42} \quad (5.2)$$

$$0 = \frac{i\Omega_{c2}}{2} \tilde{\rho}_{42} - \frac{i\Omega_p}{2} \tilde{\rho}_{23} + [i(\Delta_p - \Delta_{c2}) - \gamma_{43}] \tilde{\rho}_{43} \quad (5.3)$$

where, the frequency detuning of the probe and  $L_{c1}$ ,  $L_{c2}$  coupling lasers from the relevant atomic transitions are respectively determined by  $\Delta_p = \omega_p - \omega_{42}$ ,  $\Delta_{c1} = \omega_{c1} - \omega_{21}$ .

Because of  $\Omega_p \ll \Omega_{c1}$  and  $\Omega_{c2}$  so that we ignore the term  $\frac{i\Omega_p}{2} \tilde{\rho}_{21}$  and  $\frac{i\Omega_p}{2} \tilde{\rho}_{23}$  in the equations (4) and (5). We solve the equations (4) – (5):

$$\tilde{\rho}_{42} = \frac{-i\Omega_p / 2}{\gamma_{42} - i\Delta_p + \frac{\Omega_{c1}^2 / 4}{\gamma_{41} - i(\Delta_p + \Delta_{c1})} + \frac{\Omega_{c2}^2 / 4}{\gamma_{43} - i(\Delta_p - \Delta_{c2})}} \tag{6}$$

### 3. Dispersion coefficient

We start from the susceptibility of atomic medium for the probe light that is determined by the following relation:

$$\chi = -2 \frac{Nd_{21}}{\epsilon_0 E_p} \rho_{21} = \chi' + i\chi'' \tag{7}$$

The dispersion coefficient  $n$  of the atomic medium for the probe beam is determined through the imaginary part of the linear susceptibility (7):

$$n = 1 + \frac{1}{2} \chi' = 1 - \frac{N\mu_{42}^2}{\Omega_p \hbar \epsilon_0} \text{Re}(\tilde{\rho}_{42}) \tag{8}$$

We consider the case of  $^{85}\text{Rb}$  atom:  $\gamma_{42} = 3\text{MHz}$ ,  $\gamma_{41} = 0.3\text{MHz}$  and  $\gamma_{43} = 0.03\text{MHz}$ , the atomic density  $N = 10^{17}/\text{m}^3$ . The electric dipole matrix element is  $d_{42} = 2.54 \cdot 10^{-29} \text{Cm}$ , dielectric coefficient  $\epsilon_0 = 8.85 \cdot 10^{-12} \text{F/m}$ ,  $\hbar = 1.05 \cdot 10^{-34} \text{J.s}$ , and frequency of probe beam  $\omega_p = 3.84 \cdot 10^8 \text{MHz}$ .

Fixed frequency Rabi of coupling laser beam  $L_{c1}$  in value  $\Omega_{c1} = 16\text{MHz}$  (correspond to the value that when there is no laser  $L_{c2}$  then the transparency of the probe beam near 100%) and the frequency coincides with the frequency of the transition  $|1\rangle \leftrightarrow |2\rangle$ , it means  $\Delta_{c1} = 0$ . We consider the case of the frequency detuning of the coupling laser beam  $L_{c2}$  is  $\Delta_{c2} = 10\text{MHz}$  and plot a three-dimensional graph of the dispersion coefficient  $n$  at the intensity of the coupling laser beam  $L_{c2}$  (Rabi frequency  $\Omega_{c2}$ ) and the frequency detuning of the probe laser beam  $L_p$ , the result is shown in Fig 2.

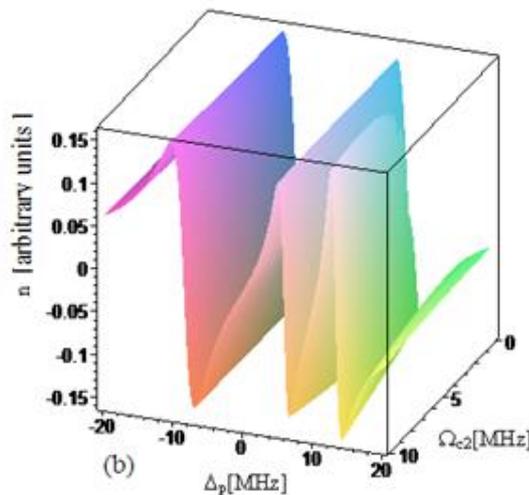


Fig 2. Three-dimensional graph of the dispersion coefficient  $n$  according to  $\Delta_p$  and  $\Omega_{c2}$  with  $\Delta_{c1} = 0 \text{ MHz}$

As shown in Fig 2, we see that when there is no coupling laser beam, it makes  $L_{c2}$  ( $\Omega_{c2} = 0$ ), only a normal dispersion domain (in anomalous dispersion domain for a two-level system) corresponds to the transparent window above absorbing curve. In the presence of the coupling laser beam  $L_{c2}$  (with the selected frequency detuning  $\Delta_{c2} = 10\text{MHz}$ ) and gradually increasing the Rabi frequency  $\Omega_{c2}$ , we see the second normal dispersion domain corresponding to the transparent window of the second on the absorption current, the spectral width of this region also increases with the increase of  $\Omega_{c2}$  but the slope of this curve is reduced. To be more specific, we plot a two-dimensional graph of Fig 3 with some specific values of Rabi frequency  $\Omega_{c2}$ .

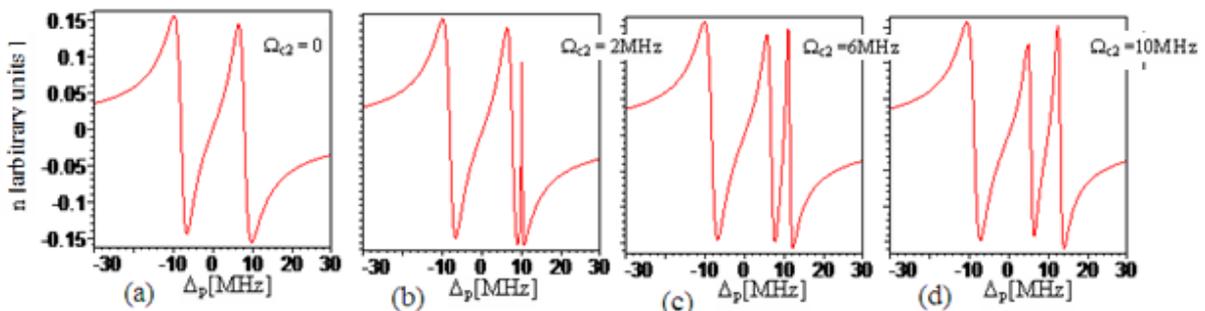


Fig 3. Two-dimensional graph of the dispersion coefficient  $n$  according to  $\Delta_p$  with  $\Omega_{c1} = 16\text{MHz}$ ,  $\Delta_{c1} = 0$  MHz and  $\Delta_{c2} = 10\text{MHz}$ .

Fig 3 is a graph of dispersion coefficient when there are no coupling laser fields, ie a two-level system. We found that the maximum absorbance at the resonance frequency and the anomaly dispersion region has not yet appeared the normal dispersion domain, the dispersion coefficient has a very small value at the adjacent resonant frequency of the probe beam.

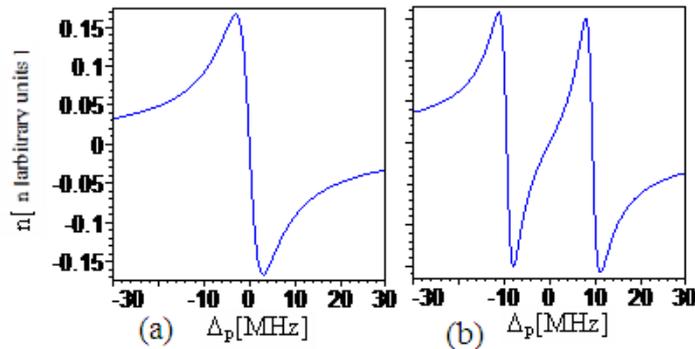


Fig 4. Two-dimensional graph of the dispersion coefficient  $n$  according to  $\Delta_p$  with  $\Omega_{c1} = 16$  MHz (a),  $\Omega_{c2} = 10\text{MHz}$  (b) when frequency detuning  $\Delta_{c1} = \Delta_{c2} = 0$ .

Fig 4 is a graph of the dispersion coefficients when there are simultaneous presence of three laser beams (a probe laser and two coupling lasers), in which the control laser beams are tuned to resonate with the corresponding shift, ie  $\Delta_{c1} = \Delta_{c2} = 0$ . We see two transparent windows overlap each other (ie only one transparent window on the absorption curve) and therefore only one normal dispersion often corresponds.

#### 4. Conclusions

In the framework of the semi-classical theory, we have cited the density matrix equation for the  $^{85}\text{Rb}$  atomic system in the Y-configuration under the simultaneous effects of two laser probe and coupling beams. Using approximate rotational waves and approximate electric dipoles, we have found solutions in the form of analytic for the dispersion coefficient of atoms when the probe beam has a small intensity compared to the coupling beams. Drawing the dispersion coefficient expression will facilitate future research applications. Consequently, we investigated the absorption of the detector beam according to the intensity of the coupling beam  $\Omega_{c1}$ ,  $\Omega_{c2}$  and the detuning of the probe beam  $\Delta_p$ . The results show that a Y-configuration appears two transparent window for the probe laser beam. The depth and width or position of these windows can be altered by changing the intensity or frequency detuning of the coupling laser fields.

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