



Original Article

Quantum Statistics in Cylindrical Time-evolution of Electrons and Physical Reality

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Abstract: Due to helical cylindrical time-evolution of electrons the mankind observation at a quantum mechanical scale depends on synchronization between observers and their surrounding cosmological medium by collective dynamics. From one side, the synchronization leads to linearization of an embedded 4D space-time reminiscent of the flat Minkowski space-time. From another side, variation of the synchronization due to independent proper plane wave oscillations of each electron being constrained in a short time quantized period, implies that there only statistical averaged physical quantities are observable, which is in consistency with statistical indeterministic concept of traditional quantum mechanics.

Keywords: Quantum statistics, time synchronization, helical cylindrical evolution.

1. Introduction

Being an old conceptual problem set up since the famous Einstein-Bohr debate at the 5th Solvay conference-1927, physical reality of quantum mechanics (QM) is never solved. The Copenhagen scholars did not accept the ontological physical reality without measurement process, while Einstein insisted to fight for an objective physical reality of the individual microscopic substance. Nowadays, nano-material technology is the widest application field of quantum physics, even few people of the nano-community are dealing with quantum physical reality. However, for going toward the frontiers of material sciences, perhaps, they would be interested in the ways to understand this puzzle. For

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example, an analysis of Schrodinger's cat or EPR paradoxes leads to new understanding of non-locality with the quantum entanglement, which serves a basis for emerging applications of quantum information.

One of the most effective ways to deal with the problem is development of the Kaluza-Klein theory (K-K) with extra-dimensions. Recently, the modern K-K theories with large extra-dimensions (ED) are proven to be able to link with QM, e.g. Randall-Sundrum 5D-AdS theory [1] deals with the huge mass hierarchy of Planck scale and TeV-physics. Another model of modern K-K theories is the space-time-matter theory (5D-STM) [2,3]. The latter, being an effective matter-induced approach, could make a qualitative interpretations of QM based on the solution of 5D-general relativity (GR)-Ricci vacuum equation [4,5]. By adding a time-like extra-dimension Wesson shows how the 5D-STM theory can link higher-dimensional GR with QM to describe microscopic particles in the flat 4D-Mincowski space-time. The higher-dimensional GR interpretation of QM may be considered as a development of the old de Broglie-Bohm (dBB) theory of hidden parameters, which demonstrates a hidden causality of statistical randomness in QM [6-8].

Following the matter-induced general relativity, we proposed a time-space symmetry based cylindrical dynamical model for description of microscopic substances, such as an electron [9,10]. This model offers more quantitative quantum interpretations including: derivation of Klein-Gordon-Fock equation (KGF) as a dual sub-solution accompanied with the geodesic sub-solution of the general relativity Ricci vacuum equation; the origin of the Heisenberg inequalities; insights of the wave-particle duality and the physical meaning of energy-momentum operators, etc. Moreover, the higher-dimensions of time-like subspace allow to deal with the mass hierarchy of charged leptons as one of the beyond standard model (SM) problems of quantum field theories (QFT) [10,11]. The present study focuses on two questions: firstly, how to avoid microscopic curved time evolution against the macroscopic flatness of the Universe? and secondly, is the dual deterministic geodesic equation in contradiction against statistical randomness of quantum indeterminism? The plan of this article is as follows: the time-space symmetry based geometry is briefly introduced in Section 2; Section 3 deals with fine-tuning of mass hierarchy of charged leptons; in Section 4 one discusses on the problem of cosmological flatness; in Section 5 one can ensure conservation of statistical randomness in quantum mechanics.

2. A Cylindrical Geometrical Model of Time Evolution against the Flat Spacetime of Special Relativity

In [10] a bi-cylindrical geometrical description of 6D time-space is proposed:

$$d\Sigma^2 = ds^2 - d\sigma^2 = dt_i^2 - dx_j^2, \tag{1}$$

Where $i, j = 1 \div 3$ are summation indices of curved time-space; unless for a few exceptions, natural units are used generally, i.e. $\hbar = c = 1$. The external curvatures of cylindrical dynamics leads to a general relativity Ricci vacuum equation:

$$R_q^m = 0; \{m, q\} = (1 \div 6). \tag{2}$$

The bi-cylindrical geometry fits easily for description of microscopic particles with spin \vec{s} and pseudo-isospin $\vec{\tau}$, e.g. electrons. After spontaneous symmetry-breaking by a Higgs-like potential, the bi-cylindrical geometry (1) becomes asymmetrical, $ds^2 \gg d\sigma^2$, and when both vectors \vec{s} and $\vec{\tau}$ are polarized along cylindrical axes, this is simplified as shown in [10]:

$$d\Sigma^2 = ds^2 - d\sigma^2 = (ds_0^2 + ds_{ev}^2) - (d\sigma_{ev}^2 + d\sigma_L^2) = dt^2 - dz^2 > 0, \tag{3}$$

where: $dt^2 = d\psi(t_0, t_3)^2 + \psi^2(t_0, t_3) d\varphi(t_0, t_3)^2 + dt_3^2$

$$dz^2 = d\psi(x_n, x_3)^2 + \psi(x_n, x_3)^2 d\varphi(x_n, x_3)^2 + dx_3^2,$$

with functionals $\psi(t_i, x_j)$ and $\varphi(t_i, x_j)$ are expressed in each 3D-subspaces with only their corresponding effective time-like or space-like variables. The sub-intervals in Geometry (3) consist of odd terms (ds_0 and $d\sigma_L$) and/or even terms (ds_{ev} and $d\sigma_{ev}$), implying that the spinning cannot flip (for odd-term) or can flip (for even-term) regarding the helical axis. Their values correspond to realistic dynamical interactions of different intensities, including Higgs-like, electro-weak and CPV-potentials. Hereafter, the weak PNC-odd (parity non-conserving) and superweak CPV-even terms are ignored for simplification. When intervals are even, e.g. σ_{ev} , one can also observe from outside, i.e. in a laboratory frame with explicit 3D-axes, i.e. $\{x_j\}$, $j = 1 \div 3$.

A solution of Equation (2) being a geodesic equation in 6D-time-space, corresponding to Geometry (3) reads:

$$\frac{\partial^2 \psi}{\partial t_i^2} - \frac{\partial^2 \psi}{\partial x_j^2} \equiv \frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial x_j^2} = \left[\Lambda_T - \left(\frac{\partial \varphi}{\partial x_n} \right)_{even}^2 \right] \psi, \quad (4)$$

where $\Lambda_T \equiv \left(\frac{\partial \varphi}{\partial t_0} \right)^2$. Equation (4) describes a monotone microscopic cosmological geodesic evolution of time-space curvatures $\psi = \psi_0 e^{\pm \varphi} = \psi_0 e^{\pm(\Omega t - k_j x_j)}$. As a cylindrical composition, Geodesic (4) combines the two 3D-local balancing circulations in time-like and space-like subspaces with the linear translation. Moreover, there is another solution of Equation (2) with hidden plane isotropic oscillations:

$$\frac{\partial^2 \psi}{\partial t_k^2} - \frac{\partial^2 \psi}{\partial x_l^2}, \quad (5)$$

which would be added in (4). However, the plane oscillations (5) being much faster than cylindrical circulation, are adiabatically excluded and the radial oscillations are replaced by function $\psi(t_i, x_j) \equiv \langle \psi(t_1, t_2 | x_1 x_2) \rangle$, being averaged over the radial variations. It is found that the covariant transformations of space-time coordinates, such as:

$$\begin{aligned} t_i &\rightarrow i \cdot t_i \quad \text{and} \quad x_j \rightarrow -i \cdot x_j, \\ \frac{\partial}{\partial t_i} &\rightarrow i \frac{\partial}{\partial t_i} \quad \text{and} \quad \frac{\partial}{\partial x_j} \rightarrow -i \frac{\partial}{\partial x_j}, \end{aligned} \quad (6)$$

can turn the classical geodesic solution (4) in to a wave-like equation with $\psi_w \equiv \psi(y \rightarrow iy) \sim e^{i\varphi} =$

$= e^{i(\Omega t - k_j x_j)}$, $\{y\} \equiv \{t_i, x_j\}$. Namely, the wave-like equation reads:

$$-\frac{\partial^2 \psi}{\partial t^2} + \frac{\partial^2 \psi}{\partial x_j^2} = \left[\left(\frac{\partial \varphi}{\partial t_0} \right)^2 - B_e (k_n \cdot \mu_e)_{even}^2 \right] \psi \quad (7)$$

where μ_e is the P-even magnetic dipole moment of charged lepton, correlated with spin \vec{s} and B_e is a calibration factor. Actually, the covariant transformations (6) in scaling with the Planck constant is reminiscent quantum energy-momentum operators: $\frac{\partial}{\partial t} \rightarrow \hat{E} = i\hbar \frac{\partial}{\partial t}$; $\frac{\partial}{\partial x_j} \rightarrow \hat{p}_j = -i\hbar \frac{\partial}{\partial x_j}$. As a result, one obtains from Representation (7) a generalized Klein-Gordon-Fock equation:

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} + \hbar^2 \frac{\partial^2 \psi}{\partial x_j^2} - m^2 \psi = 0, \quad (8)$$

In the results, it has proven that the geodesic classical equation (4) is accompanied by its dual wave-like solution (7), which formulates the generalized Klein-Gordon-Fock equation (8). In the

latter, beside the rest mass m_0 , the square mass term $m^2 = m_0^2 - m_s^2$ contains a P-even contribution m_s linked with an external curvature due to spinning in 3D-space. At variance with the conventional Klein-Gordon-Fock equation of particle with spin zero, Equation (8) describes spinning particles and in particular, due to cylindrical specification it is reminiscent of the squared Dirac equation. Indeed, by factorization based on Dirac matrices, Equation (8) is linearized in to Dirac equation for each spinor component with its fixed spin projection ($s_n = +1/2$ along the momentum or $s_n = -1/2$ against the momentum). In particular, in case one has no polarization analysis, then $d\sigma_{ev}$ is vanished as well. The cylindrical curved geometry in 3D-spatial sub-space turns to linear one ($dx_j \rightarrow dx_l$) and the generalized bi-cylindrical geometry (3) is simplified:

$$ds_0^2 = dt^2 - dx_l^2. \tag{9}$$

In appearance, Geometry (9) recalls the special relativity geometry, in fact, dt is in origin a helical axis. Being 4D-Minkowski observers, we are naturally involved in this helical cylindrical evolution, because our biological sense-mechanism bases on the electro-magnetic interaction of electrons. In such a way, the cylindrical curvature is internal in our observation. However, the latter is the product of the two principal curvatures of the cylinder, which being vanished to be zero, that imitates to human observation instead the cylindrical evolution as evolution along a linear time-axis in the flat 4D space-time. Therefore, in appearance Geometry (9) with an originally curved dt is exacting the quadratic formula of special relativity.

In accordance with Geometry (9), one simplifies Equation (8) by formulation of the traditional KGF equation of scalar field ($dx_j \rightarrow dx_l, m \rightarrow m_0$):

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} + \hbar^2 \frac{\partial^2 \psi}{\partial x_l^2} - m_0^2 \psi = 0. \tag{10}$$

Therefore, the quantum energy-momentum operators serve transformers from general relativity to quantum mechanics. At variance with the conventional KGF equation in the flat 4D-Minkowski space-time, Equations (8) and (10) still work in a space-time curved geometry, in particular, at least the time axis originates of helical evolution in Geometry (9).

3. Charged Lepton Hierarchy as an Evidence for Triple Dimensions of Time-like Subspace

In [10] all higher-order curvatures of hyper-spherical surfaces in 3D time-like subspace are introduced in energy density of three corresponding charged lepton generations, i.e.: $m_1(T_\infty) = \epsilon_\infty T_\infty$; $m_2(T_\infty) = \epsilon_\infty 4\pi T_\infty^2$ and $m_3(T_\infty) = \epsilon_\infty 4\pi T_\infty^3$. Moreover, to those curvatures there are minor perturbative corrections added in [11] for fine-tuning the mass hierarchy calculations. In the results, the mass formulas of charged leptons depend on two free parameters, lepton energy factor ϵ_∞ and time-like Lagrange radius T_∞ , which are able to be determined by experimental masses of electron and muon in Equations $m_e = m_1(\infty) = F_1(\epsilon_\infty, T_\infty)$ and $m_\mu = m_2(\infty) = F_2(\epsilon_\infty, T_\infty)$. Then one can predict the mass of tauon by the following formula:

$$m_\tau = m_3(\infty) = m_3(T_\infty) + m_1(T_\infty) \frac{\rho_{21}}{\rho_{21}-1} \frac{\rho_{31}}{\rho_{31}-1} + \frac{1}{2} m_2(T_\infty) \frac{2\rho_{32}}{2\rho_{32}-1}, \tag{11}$$

where: $\rho_{ij} = \frac{m_i(T_\infty)}{m_j(T_\infty)} > 1$ for $i > j$.

For $\epsilon_\infty=31.20769729$ (keV) and $T_\infty=16.37413114$, the calculated mass of tauon is $m_\tau(theor) = m_3 = 1776.40$ MeV which reaches almost a precision of 3σ in comparison to the experimental tauon mass $m_\tau(exp) = 1776.86(12)$ MeV. Let's recall that the number of generations and the mass

hierarchy of charged leptons are classified as beyond standard model puzzles, being unsolved by traditional quantum field theories. Therefore, the new solution by the above-mentioned extra-dimensional GR approach can serve as positive argument for a preferable triple dimensional structure of the microscopic 3D-time in a symmetry with the macroscopic 3D-space.

4. Dealing with the Problem of Cosmological Flatness

In a physical observation human interaction with the detection device consist of a huge number of atoms and molecules with electronic shells, therefore the man-system is involved in the same helical evolution due to cylindrical geometry in KGF equation (8), which finally makes the external cylindrical curvature getting hidden to internal observation with a zero-curvature. The same appearance of linearization of time evolution has been proven for heavy charged leptons, because their internal time-like structures identically contain the same basic helical evolution through the mass term $m_1(T_\infty)$ of electrons. This is equivalent to a universal U(1)-local gauge transformation which turns the basic geometrical curvature in 3D-time into a universal proper mass term of the electron:

$$\psi_{SM} \equiv \psi(t_k, x_l) \rightarrow \psi = \psi(t_k, x_l)e^{i\varphi(t_0)} \equiv \psi_{SM}e^{-i\Omega_0 t_0} = \psi(t_i, x_l) \equiv \psi(t, x_l), \quad (12)$$

where ψ_{SM} is the standard model wave function of QFT and $\psi(t_k, x_l)$ is the plane wave in the flat time-space, while $\psi(t, x_l)$ is the wave solution of Equation (10). Indeed, extending the gauge invariant of electromagnetic potential $A_\mu \equiv \{A_0, \vec{A}\} \rightarrow A'_\mu = A_\mu + \frac{1}{e} \frac{\partial \varphi(y)}{\partial y}$, where $\{y\} \equiv \{t_0, x_n\}$, by shifting its time-like component $A_0 \rightarrow A'_0 = A_0 - \frac{1}{e} \Omega_0$, it is able to involve all charged leptons in the same time-like cylindrical circulation of the electron to muon and taon. In the results [11], the heavier charged leptons project their higher time-like curved configurations on the time-evolutional axis dt . Let's recall that the sign of integer electric charge was proposed to serve as an indicator of the direction of time-evolution and moreover, an absolute statistical electric potential $V_0 \equiv A_0$ being not meaningful as an difference of potentials, that is easily renormalized by the above U(1)-gauge transformation. For these reasons, the action of the above U(1)-gauge transformation is universal and can extend as well for inducing the basic time-like configuration of electron from charged leptons to atomic nuclei which coexist with their electronic shells in an electric charged balance. If this assumption is adapted, mass configurations of nuclei would be projected in a similar way on the linearized time-evolutional axis dt . Because the visible ordinary matter consisting of 4% in kinds of neutral hydrogen atoms or molecules in gas and dust of interstellar medium (ISM), the proposed assumption leads to a macroscopic flatness of 4D-Minkowski spacetime of the observable Universe. This is in agreement with the low experimental upper-limit of the cosmological constant Λ .

5. To Meet the Statistical Randomness of Quantum Mechanics

The action of the geodesic (4) seems to restore physical reality with a continuous causal evolution and motion of classical mechanics, which would violate the statistical randomness of quantum mechanics. However, it should not happen. The matter is that all human observation in a microscopic scale is made by some macroscopic detector system in combination with human sensing organs, which coexist with electrons in a time-synchronization. This synchronization is approximation with a time-period of the minimum quantized evolution.

According to the hidden time-like cylindrical curvature [10], a helical circulation is assumed to be quantized in a portion of the time evolutional period T_S as following:

$$\varphi_0 = \Omega_0 t_0 = 2n\pi, \quad (13)$$

where n is an integer and $\varphi_0 \rightarrow \varphi_{min} = 2\pi = \Omega_0 T_S$. Indeed, a particle detector is always a macroscopic system, containing a huge number of electrons $e^-(i)$ which synchronize with each other in their time evolution. However, each electron is involved not only in the monotone cylindrical evolution, but also in independent fluctuations with a microscopic plane isotropic harmonic mode (5). In the results, individual phases may shift randomly from each other within a period 2π . When some of those electrons interact with the registered object, e.g. another elementary particle, being distributed statistically in accordance with the phase distribution of the ensemble of electrons $e^-(i)$ in the detector-sensor, the moment of registration of an event is determined by the squared average:

$$dt^2 = \langle dt(i)^2 \rangle = dt_0^2 + \langle dt_3(i)^2 \rangle, \quad (14)$$

where statistical summation is extended over all electrons with index i , interacting with the registered particle. Those i -electrons are synchronized by cylindrical evolution in dt_0 , but shifting randomly from each other in $dt_3(i)$ within a period $T_S = \varphi_{min}/\Omega_0$. As far as a detector system is a macroscopic mechanism, its collective statistical randomness can not be avoided. Moreover, it is an objective principal concept, because the statistics is not only a collective behavior, but also caused by intrinsic fluctuations (5) of each electron as an individual microscopic substance.

6. Conclusions

The time-space symmetry (TSS) based bi-cylindrical geometrical model is quite effective for quantum mechanical interpretations. It links the classical higher-dimensional geodesic description of general relativity with quantum mechanical equation which is reminiscent of de Broglie-Bohm philosophy. Moreover, the unique solution of mass hierarchy of charged leptons as a beyond quantum field standard model problem based on the TSS-model adds a strong argument for the causal interpretation of quantum physical reality. Based on a universal U(1)-gauge invariance, the assumption of time evolution synchronization of all charged particles with electrons ensures a consistency between the time-like helical evolution and the macroscopic flatness of the visible Universe. By an analysis of the collective effect of human observation based on the universal role of coherent electron ensemble, one can reserve the statistical feature of quantum mechanics. Simultaneously, the TSS-general relativity description implies objectiveness of ontological physical reality of an individual quantum object, i.e. an interpretation without any intervention of human observation.

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