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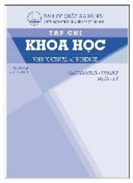

Original Article

The Ettingshausen Effect in Doped Semiconductor Superlattice under the Influence of Confined Optical Phonon

Cao Thi Vi Ba, Nguyen Thi Lam Quynh*, Nguyen Quang Bau

VNU University of Science, 334 Nguyen Trai, Thanh Xuan, Hanoi, Vietnam

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Abstract. The electron – optical phonon scattering is considered in detail to studying the Ettingshausen effect in doped semiconductor superlattice under the influence of phonon confinement and laser radiation. The analytical expressions for tensors and the Ettingshausen coefficient are obtained by using the kinetic equation method. The Ettingshausen coefficient depends on temperature of the sample, amplitude and frequency of laser radiation, magnetic field and the quantum number m specific for the confinement of phonon. The dependences are clearly displayed in the numerical results for GaAs:Be/GaAs:Si doped semiconductor superlattice. The magnetic field makes the Ettingshausen coefficient change in quantitative under the influence of temperature or laser amplitude and change the resonance condition. The numerical results show that both resonance condition and resonance peaks position are affected by the increase of quantum number m. We also get the result corresponding to the unconfined optical phonon case when m is set to zero. Due to the change of the wave function and energy spectrum of electrons, most of results for the Ettingshausen effect in doped semiconductor superlattice obtained are different from the case of bulk semiconductor. Moreover, in comparison with the case of unconfined optical phonon, under the influence of phonon confinement effect, the Ettingshausen coefficient changes in magnitude, the number and position of resonance peaks.

Keywords: Doped semiconductor superlattice, Ettingshausen effect, Quantum kinetic equation, confined optical phonons.

1. Introduction

It's well known that the confinement of electron as well as phonon is the main cause of changes in the properties of kinetic effects in two-dimensional semiconductor systems (2DS) and doped

*Corresponding author.
Email address: lamquynhnt@outlook.com

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semiconductor superlattice (DSS) in particular. Many papers dealing with this issue have been published [1-3]. Due to the emission of confined longitudinal optical (LO) phonon in GaAs-$Al_{0.45}Ga_{0.55}As$ superlattice, the dispersion of the relaxation time depends strongly on the total energy [1]. The electron – phonon scattering strength is strongly influenced by the polar optical phonon confinement in quantum well [2]. In comparison with the case of bulk phonons, due to the confinement of LO-phonon in doped semiconductor superlattices, the magnitude of magnetoresistance and the Hall coefficient are decreased [3].

The Ettingshausen effect is known as an unexpected result when Ettingshausen and his PhD student studied Hall effect in Bismuth. That has opened up a new field of research on magneto-thermoelectric phenomena. This effect has been studied in various materials such as metals [4], graphene [5], bulk semiconductor [6] and especially low-dimensional semiconductor systems (LDS) [7-11]. In parabolic quantum well with in-plane magnetic field, Ettingshausen effect change sign with temperature even if the mechanism of scattering remains unchanged [7]. Due to the influence of the dimension effects and laser radiation (LR), the Ettingshausen coefficient (EC) in quantum well is 10^2 times bigger than that in bulk semiconductor [8]. When the magnetic field increases, the curve representing the dependence of longitudinal magneto-thermoelectric coefficient on two-dimensional concentration in a quantum well shifts downwards [9]. The oscillations of absorbed microwave power due to Landau quantization causes the appearance of a peak of the longitudinal diffusive thermopower in quantum wells at cyclotron absorption frequency modified by heating mechanism [10]. The Shubnikov-de Haas oscillation appeared when surveying the dependence of EC on the magnetic field in DSS [11]. However, Ref. [7-11] have not taken interested in the confinement of phonon. So, the Ettingshausen effect in DSS under the influence of confined phonon is opened to study.

In this work, the influence of LR is taken into account for the Ettingshausen effect in DSS and the confined electron – confined optical phonon (COP) scattering is considered in detail. This article is organized as follows: the effect of COP on the EC in DSS is outlined in Sec.2; the numerical results and discussion for GaAs:Be/GaAs:Si DSS are given by Sec.3; Sec.4 shows conclusions.

2. The Calculation of EC in DSS

We apply the quantum kinetic equation method to study the Ettingshausen effect in a simple model of DSS in which the electron can move freely in the x-y plane. A magnetic field with $\vec{B}=(0,0,B)$, a dc electric field with $\vec{E_1}=(E_1,0,0)$ and the LR $\vec{E}=\vec{E_0}\sin(\omega t)$ were applied for our model [3]. From Hamiltonian of electron - COP system in DSS, the quantum kinetic equation for electrons is obtained. After some analytic transformations, we found out the expression for the Ettingshausen coefficient:

$$EC=\frac{1}{H}\frac{\sigma_{xx}(m)\varphi_{xy}(m)-\sigma_{xy}(m)\varphi_{xx}(m)}{\sigma_{xx}(m)\left\{\theta_{xx}(m)\varphi_{xx}(m)-\sigma_{xx}(m)\left[\alpha_{xx}(m)-K_L\right]\right\}} \tag{1}$$

In which $\sigma_{xx}(m)$, $\varphi_{xy}(m)$, θ_{xx}, $\alpha_{xx}(m)$ are respectively components of tensors:

$$\begin{aligned}\sigma_{ip}(m)=a\frac{e\tau(\varepsilon_F)}{1+\omega_c^2\tau^2(\varepsilon_F)}\delta_{kp}\delta_{01}+\frac{e}{m_e}\Big\{&\left[A_1(m)+A_3(m)\right]\tau_{13}(\varepsilon)\delta_{13}+\left[A_2(m)+A_4(m)\right]\tau_{24}(\varepsilon)\delta_{24}\\&+A_5(m)\tau_5(\varepsilon)\delta_5+A_6(m)\tau_6(\varepsilon)\delta_6+A_7(m)\tau_7(\varepsilon)\delta_7+A_8(m)\tau_8(\varepsilon)\delta_8\Big\}\end{aligned} \tag{2}$$

$$\theta_{ip}(m) = -\frac{1}{m_e T}\{[A_1(m)+A_3(m)](\hbar\omega_m)\tau_{13}(\varepsilon)\delta_{13} + [A_2(m)+A_4(m)](-\hbar\omega_m)\tau_{24}(\varepsilon)\delta_{24} + A_5(m)(-\hbar\omega_m+\hbar\Omega)\tau_5(\varepsilon)\delta_5 + A_6(m)(-\hbar\omega_m-\hbar\Omega)\tau_6(\varepsilon)\delta_6 + A_7(m)(\hbar\omega_m+\hbar\Omega)\tau_7(\varepsilon)\delta_7 + A_8(m)(\hbar\omega_m-\hbar\Omega)\tau_8(\varepsilon)\delta_8\} \quad (3)$$

$$\varphi_{ip}(m) = \frac{1}{m_e}\{[A_1(m)+A_3(m)](\hbar\omega_m)\tau_{13}(\varepsilon)\delta_{13} + [A_2(m)+A_4(m)](-\hbar\omega_m)\tau_{24}(\varepsilon)\delta_{24} + A_5(m)(-\hbar\omega_m+\hbar\Omega)\tau_5(\varepsilon)\delta_5 + A_6(m)(-\hbar\omega_m-\hbar\Omega)\tau_6(\varepsilon)\delta_6 + A_7(m)(\hbar\omega_m+\hbar\Omega)\tau_7(\varepsilon)\delta_7 + A_8(m)(\hbar\omega_m-\hbar\Omega)\tau_8(\varepsilon)\delta_8\} \quad (4)$$

$$\alpha_{ip}(m) = -\frac{1}{e m_e T}\{[A_1(m)+A_3(m)](\hbar\omega_m)^2\tau_{13}(\varepsilon)\delta_{13} + [A_2(m)+A_4(m)](-\hbar\omega_m)^2\tau_{24}(\varepsilon)\delta_{24} + A_5(m)(-\hbar\omega_m+\hbar\Omega)^2\tau_5(\varepsilon)\delta_5 + A_6(m)(-\hbar\omega_m-\hbar\Omega)^2\tau_6(\varepsilon)\delta_6 + A_7(m)(\hbar\omega_m+\hbar\Omega)^2\tau_7(\varepsilon)\delta_7 + A_8(m)(\hbar\omega_m-\hbar\Omega)^2\tau_8(\varepsilon)\delta_8\} \quad (5)$$

Where: $a = -\frac{e\beta\hbar v_d n_0}{m_e} I(p_y)\sum_{N,n} e^{\beta(\varepsilon_F-\varepsilon_{N,n})}$;

$$A_s(m) = A(m)\sum_{N',n',m}\sum_{N,n} I^m_{n,n'}\left(\frac{eE_1}{\hbar v_d}\bar{l}\right)e^{\beta(\varepsilon_F-\varepsilon_{N,n})} I(p_y) I(q_\perp)\delta\left(\Delta_{Nn} - eE_1\bar{l} \pm \hbar\omega_m\right)(s=1,2)$$

$$A_s(m) = A(m)\left(-\frac{\eta}{2}\right)\sum_{N',n',m}\sum_{N,n} I^m_{n,n'}\left(\frac{eE_1}{\hbar v_d}\bar{l}\right)^3 e^{\beta(\varepsilon_F-\varepsilon_{N,n})} I(p_y) I(q_\perp)\delta\left(\Delta_{Nn} - eE_1\bar{l} \pm \hbar\omega_m\right)(s=3,4)$$

$$A_s(m) = A(m)\left(\frac{\eta}{4}\right)\sum_{N',n',m}\sum_{N,n} I^m_{n,n'}\left(\frac{eE_1}{\hbar v_d}\bar{l}\right)^3 e^{\beta(\varepsilon_F-\varepsilon_{N,n})} I(p_y) I(q_\perp)\delta\left(\Delta_{Nn} - eE_1\bar{l} \pm \hbar\omega_m \pm \hbar\Omega\right)(s=5\div 8)$$

with definition: $\bar{l} = \left(\sqrt{N+\frac{1}{2}} + \sqrt{N+1+\frac{1}{2}}\right)\frac{l_B}{2}$ [3]; ε_F is the Fermi level; τ is the momentum relaxation time.

$$A(m) = \frac{4\pi^2 e^3\hbar\omega_m n_0\beta}{\varepsilon_0 m_e}\frac{1}{e^{\beta\hbar\omega_m}-1}\left(\frac{1}{\chi_\infty}-\frac{1}{\chi_0}\right); \Delta_{Nn} = (N'-N)\hbar\omega_c + (n'-n)\hbar\omega_p$$

$$\omega_m = \sqrt{\omega_o^2 - v_s^2(m\pi/L)^2}\,; \varepsilon_{N,n} = \left(N+\frac{1}{2}\right)\hbar\omega_c + \left(n+\frac{1}{2}\right)\hbar\omega_p + \frac{1}{2}mv_d^2$$

$$I(p_y) = \frac{L_y}{2\pi(\hbar v_d)^2}(\varepsilon_{N,n}-\varepsilon_F);\; I(q_\perp) = \frac{1}{2\pi l_B^2}\,;\; \eta = \left(\frac{eE_0}{m_e\omega}\right)^2$$

$$\tau_v(m) = \frac{\tau^2(\varepsilon_F \pm \hbar\omega_m)}{\left[1+\omega_c{}^2\tau^2(\varepsilon_F \pm \hbar\omega_m)\right]^2}(v=1\div 4);\tau_v(m) = \frac{\tau^2(\varepsilon_F \pm \hbar\omega_m \hbar\Omega)}{\left[1+\omega_c{}^2\tau^2(\varepsilon_F \pm \hbar\omega_m \pm \hbar\Omega)\right]^2}(v=5\div 8)$$

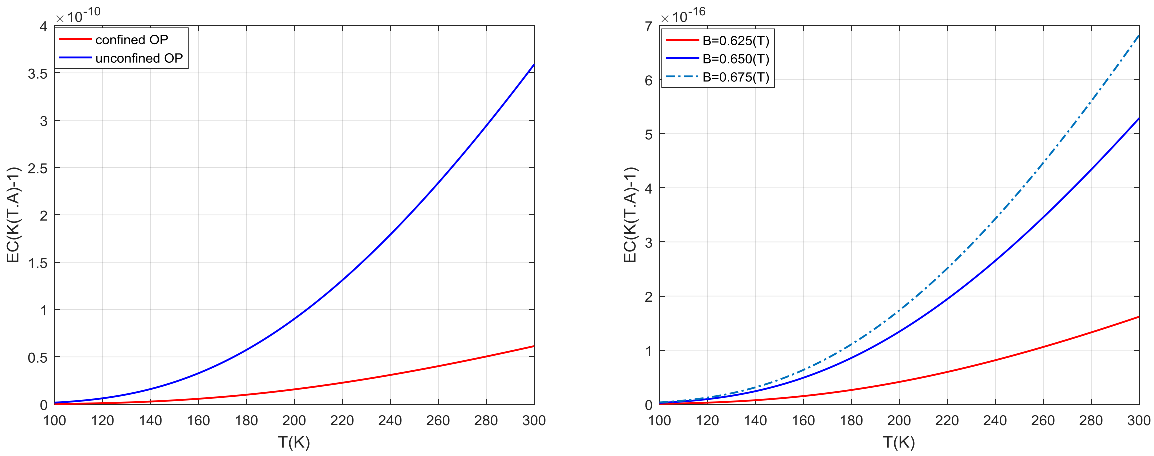
$$\delta_u = \left[\delta_{ik} + \omega_c\tau_v(\varepsilon)\kappa_{ijk}h_j - \omega_c{}^2\tau^2(\varepsilon)h_ik_k\right]\left[\delta_{kp} + \omega_c\tau_v(\varepsilon)\kappa_{klp}h_l - \omega_c{}^2\tau^2(\varepsilon)h_kk_p\right](u,v=1\div 8)$$

Here: δ_{ik} is the Kronecker delta; the Latin symbols i, j, k, l, p stand for the components x, y, z of the Cartesian coordinates; κ_{klp} is the anti-symmetrical Levi - Civita tensor; $\omega_c = \frac{eB}{m_e}$ and $\omega_p = \left(\frac{4\pi e^2 n_D}{\chi_0 m_e}\right)^2$ are cyclotron frequency and plasma frequency, respectively (n_D is the doping concentration of the DSS).

From above analytic expression, we can see clearly that the EC depends on many quantities such as amplitude and frequency of the LR, the magnetic induction, the temperature. Especially, the EC depend in complicated way on the *m*-quantum number specific for COP. We also obtained the expression that corresponds to un-COP when *m* is set to zero.

3. Numerical Results and Discussion

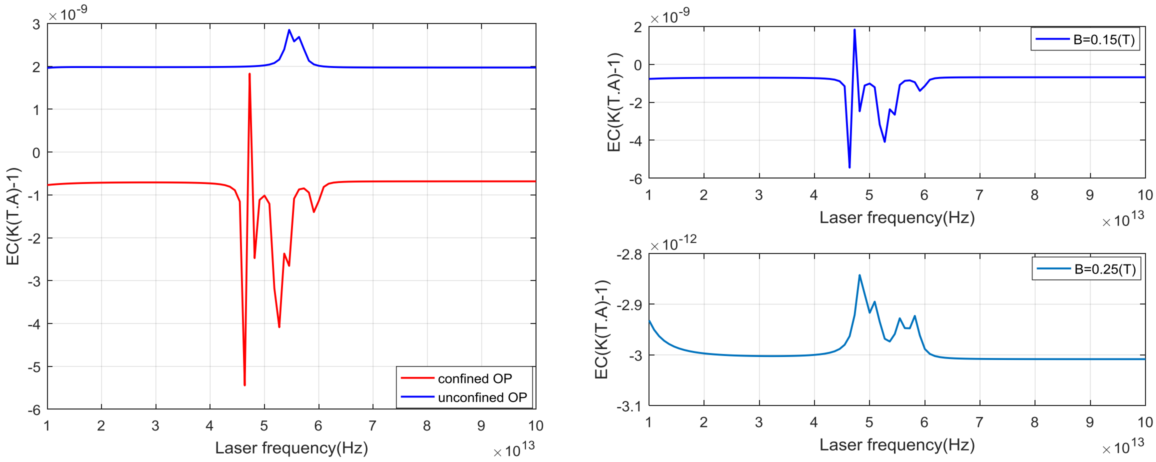
We have considered a GaAs:Be/GaAs:Si DSS with characteristic parameters: m_e = 0.067$m_{0,}$ e=2.07e_0, v_s = 87300ms^{-1}, ε_F= 50eV, τ (ε_F) = 10^{-12}s [3], N = 1, N' = 3; n and n' rate from 1 to 3.



(1a) (1b)

Figure 1. The denpendence of the EC on the temperture.

According to Figure 1, the EC exerts non-linear dependence on temperature. Figure (1a) shows that the phonon confinement reduces the EC in DSS. The difference between two cases (with and without phonon confinement) is obvious at high temperature. As we see in Figure (1b), the magnetic field also affects the EC in temperature range surveyed when optical phonons are confined. The EC increases when the magnetic induction increases. However, the magnetic field does not change the rule of the EC's dependence on temperature.



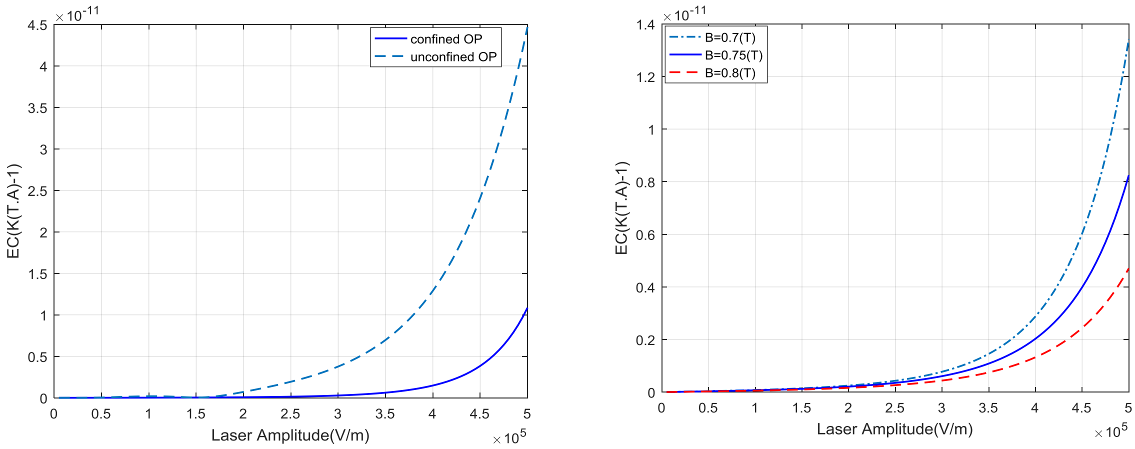
(2a) (2b)

Figure 2. The dependence of the EC on the laser frequency.

As we can see in Figure 2, the EC oscillates when the laser frequency changes. Figure (2a) describes the dependence of EC on laser frequency in two cases: with and without confinement of optical phonons. In COP case, the EC not only increases in quantitative but also enhances the number of resonance peaks. The resonance peaks correspond to the condition: $\hbar\Omega = (n-n')\hbar\omega_p + eE_1\bar{l} \pm \hbar\omega_m - 2\hbar\omega_c$ or $\hbar\Omega = (n'-n)\hbar\omega_p - eE_1\bar{l} \mp \hbar\omega_m + 2\hbar\omega_c$ with $\omega_m = \sqrt{\omega_o^2 - v_s^2 (m\pi / L)^2}$. When m is set to zero, the frequency of optical phonon is defined $\omega_m = \omega_0$. The increase of m leads to the increase of ω_m and make resonance condition change. So, the additional resonance peaks are appeared. Therefore, the confinement of optical phonon not only makes the EC in DSS increase but also causes the appearance of new resonance peaks. It also means that the results for the EC in DSS obtained is different from the case of bulk semiconductor [6].

Figure (2b) shows that the magnetic induction significant impact on locating the resonance peaks. It is so easy to explain. $\omega_c = \dfrac{eB}{m_e}$ absent in the expression to identify resonance frequency of laser radiation mentioned above. Therefore, the change of magnetic field leads to change of resonance condition. In the case of phonon confinement, the EC has negative values when the magnetic field is strong.

Figure 3 indicates the influence of laser amplitude on the EC. The bigger laser amplitude increases, the bigger EC gets. The EC increases fast when laser amplitude is greater than 4.10^5 Vm^{-1}. Similar to the dependence of the EC on temperature, due to the confinement of optical phonon, the value of EC decreases in comparison with un-COP case. In the low laser amplitude condition (less than 2.10^5 Vm^{-1}) the EC is almost unchanged and approaches to zero when the laser amplitude increases. Besides, the EC has positive values with COP and even un-COP. Furthermore, when the magnetic field increases, the EC decreases. That is displayed explicitly by Figure (3b).



(3a) (3b)

Figure 3. The dependence of the EC on the laser amplitude.

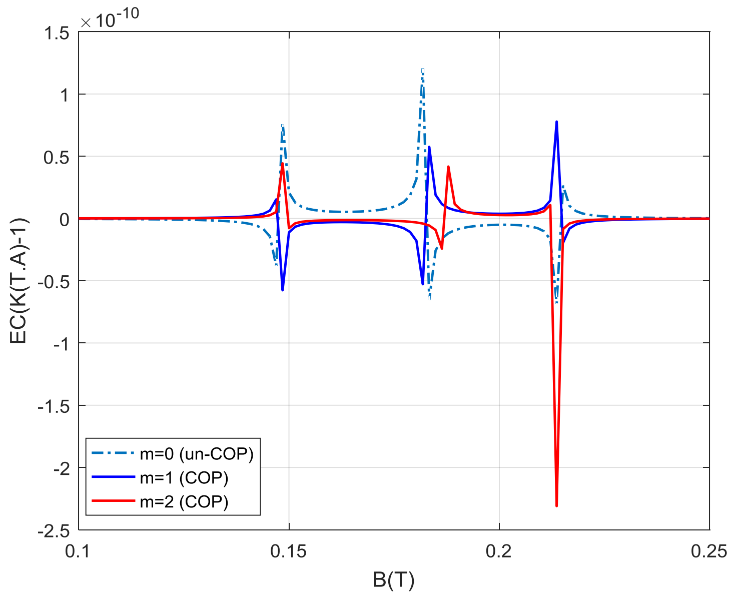


Figure 4. The dependence of the EC on the magnetic field.

Figure 4 shows the EC plotted as a function of magnetic induction at different values of quantum number m. Due to the phonon confinement, the EC changes resonance peaks position. The resonance peaks position is determined by condition: $B = m_e \dfrac{(n-n')\hbar\omega_p + eE_1\bar{l} \pm \hbar\omega_m \pm \hbar\Omega}{2e\hbar}$. For simply, $eE_1\bar{l}$ can be neglected ($eE_1\bar{l} \ll \hbar\omega_0$). When optical phonons are confined, their energy and wave vector are quantized. The COP frequency is defined: $\omega_m = \sqrt{\omega_o^2 - v_s^2 (m\pi / L)^2}$. Each value of m determines a value of COP frequency. So, the increase of quantum number *m* affects resonant condition.

4. Conclusions

Based on the kinetic equation method, we have studied the influence of COP on the Ettingshausen effect in GaAs:Si/GaAs:Be DSS. Due to the remarkable contribution of COP and size effect, the theoretical results for Ettingshausen effect in DSS are different from the previous one [6]. The results are derived from the change of wave vector and energy spectrum of electrons in DSS in comparison with bulk semiconductor. Under the influence of phonon confinement effect, the Ettingshausen coefficient changes in magnitude, the number and position of resonance peaks in comparison with un-COP case. Because of the increase of phonon confinement effect, the resonance condition in DSS changes and the EC decreases. The increase of magnetic field leads to the increase of the EC when temperature increases. It is contrary to the result obtained when investigating the dependence of the EC on the laser amplitude. We also get the results fitting to the un-COP case when the quantum number is set to zero. So far, the results obtained are new for Ettingshausen effects in DSS and contribute to the research and making new semiconductor materials.

Acknowledgments

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