



Original Article

Optimization of Laminated Composite Plates for Maximum Biaxial Buckling Load

Pham Dinh Nguyen¹, Quang-Viet Vu², George Papazafeiropoulos³,
Hoang Thi Thiem¹, Pham Minh Vuong¹, Nguyen Dinh Duc^{1,*}

¹*Advanced Materials and Structures Laboratory, VNU University of Engineering and Technology,
Vietnam National University, Hanoi, 144 Xuan Thuy, Cau Giay, Hanoi, Vietnam*

²*Faculty of Civil Engineering, Vietnam Maritime University, 484 Lach Tray, Hai Phong, Vietnam*

³*Department of Structural Engineering, National Technical University of Athens,
Zografou, Athens 15780, Greece*

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Abstract: This paper proposes an optimization procedure for maximization of the biaxial buckling load of laminated composite plates using the gradient-based interior-point optimization algorithm. The fiber orientation angle and the thickness of each lamina are considered as continuous design variables of the problem. The effect of the number of layers, fiber orientation angles, thickness and length to thickness ratios on the buckling load of the laminated composite plates under biaxial compression is investigated. The effectiveness of the optimization procedure in this study is compared with previous works.

Keywords: Optimum design, Fiber angles, Biaxial compression, Laminated composite plates, Abaqus2Matlab.

1. Introduction

Composite materials are widely applied in many heavy duty engineering structures. Composite materials are lightweight and they have low density, high strength and high stiffness. Those properties are a results of the characteristics of the main constituents of composite materials. Therefore, the optimal design of the latter depends on the design of their various components. Optimization problems involving

*Corresponding author.

Email address: ducnd@vnu.edu.vn

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laminated composite plates are often sophisticated because of the numerous design variables and their complex behavior which depends on the properties of the laminae.

In recent years, many studies have been published for the buckling analysis of the laminated composite structures subjected to various loads. The analysis of composite plates using finite element methods (FEMs) has been reported by applying the first-order shear deformation theory (FSDT), Wang et al. [1] presented the results of natural frequencies and buckling load of the laminated composite plates, Ferreira et al. [2] shown the critical buckling load of isotropic and laminated plates, Nguyen-Van et al. [3] presented the free vibration and buckling analysis of composite plates and shells using a smoothed quadrilateral flat element, Thai et al. [4] studied the static, free vibration, and buckling analysis of laminated composite plates with quadratic, cubic, and quartic elements. By using the higher-order shear deformation theories (HSDT), the results of critical buckling load and natural frequencies of cross-ply laminated plates had been reported by Khdeir and Librescu [5] and Faces and Zenkour [6], Chakrabarti and Sheikh [7] investigated the buckling analysis of laminated composite plates using a triangular element. The buckling analysis of composite structures using an analytical method has been reported by Duc et al. [8, 9] using the FSDT for the composite plates resting on elastic foundations, Le et al. [10] presented the nonlinear buckling analysis of functionally graded graphene-reinforced composite laminated cylindrical shells under axial compressive load. The buckling analysis of composite plates and shells using a semi-analytical method has been reported by Kermanidis and Labeas [11] and Mohammad and Arabi [12].

The optimum design is a significant problem in structural engineering which is intended to increase the performance of structures. The optimum values of fiber angles for maximizing the buckling load of the laminated composite plates has been investigated in [13, 14] where the plate had been subjected to uniaxial compression [13], bending load and both [14] under various boundary conditions. Studies for the optimal design of the stacking sequence have been carried out by Riche and Haftka [15] using a genetic algorithm, Jing et al. [16] using a permutation search algorithm and Almeida [17] using a harmony search algorithm, Bargh and Sadr [18] using a the particle swarm optimization algorithm. Both fiber angles and thickness are used as design variables to obtain the maximum buckling load in the studies by Huang and Kroplin [19] using a variable metric algorithm, Akbulut and Sonmez [20] using the simulated annealing algorithm, Ho-Huu et al. [21] using an improved differential evolution algorithm. Chandrasekhar et al. [22] studied the topology optimization of laminated composite plates and shells using optimality criteria.

From the above literature review, this paper proposes a new optimization procedure for the laminated composite plates subjected to biaxial compression to obtain maximizing buckling load with design variables are fiber angles and thickness. The optimization procedure is implemented by using Abaqus2Matlab [23] which is designed for transferring model and/or results data from Abaqus to Matlab and vice versa to generate the necessary Abaqus input files, run the analysis and extract the analysis results in Matlab.

2. Methodology

2.1. Buckling Analysis of Laminated Composite Plates

Consider a laminated composite plate that is subjected to biaxial compression, as shown in Figure 1. The composite plate consists of n laminae, each one having its own fiber angle and thickness. The total thickness, length and width of the plate are h, a, b , respectively.

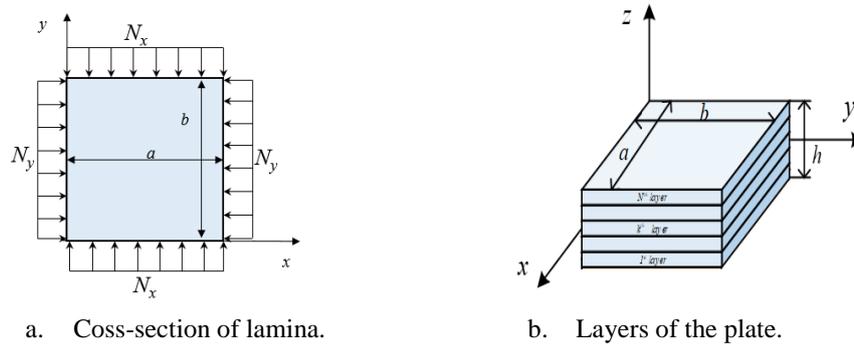


Fig. 1. Model of the composite laminated plate subjected to biaxial load.

In the buckling analysis, the eigenvalues (λ_i) and buckling mode shapes (ϕ_i) are obtained by solving the eigenvalue problem:

$$([K] + \lambda_i [\sigma])\phi_i = 0 \tag{1}$$

in which, $[K], [\sigma]$ are the stiffness and stress matrices, respectively.

The critical eigenvalue buckling analysis (the first eigenvalue λ_{cr}) is used to determine the critical buckling load (F_{cr}) with N is the applied load as follows:

$$F_{cr} = \lambda_{cr} N \tag{2}$$

The buckling coefficient of the laminated composite plates is determined by:

$$k = \frac{\lambda_{cr} a^2}{E_2 h^3} \tag{3}$$

2.2. Optimization Method

2.2.1. Statement of the Problem

The objective of the optimization problem is to maximize the biaxial buckling load factor of the composite plate. The design variables of the optimization problem are the fiber angles and the thicknesses of the laminae of the composite plate, which are continuous variables. The optimization problem contains an equality constraint, stating that the sum of the laminae thicknesses be equal to the total thickness of the plate.

The optimization problem is mathematically described as:

Maximize:

$$\lambda_{cr}(t_i, \theta_i) \tag{4}$$

Subject to $\sum_{i=1}^n t_i = h, \quad t_{lb} \leq t_i \leq t_{ub}, \quad \theta_{lb} \leq \theta_i \leq \theta_{ub}, \quad i = \overline{1 \div n},$

in which, t_i is the i^{th} lamina thickness which varies from lower bound t_{lb} to upper bound t_{ub} , θ_i is the fiber angle of the i^{th} lamina which varies from $\theta_{lb} = -90^\circ$ to $\theta_{ub} = 90^\circ$.

2.2.2. Proposed Optimization Procedure

This section presents an optimization procedure using the gradient-based interior point algorithm (IPA) to calculate the optimum fiber angles and thickness of the laminated composite plates. This optimization procedure integrates Matlab and Abaqus in a loop with the use of Abaqus2Matlab, which is developed by Papazafeiropoulos et al. [23]. All steps of this optimization process are described in Figure 2.

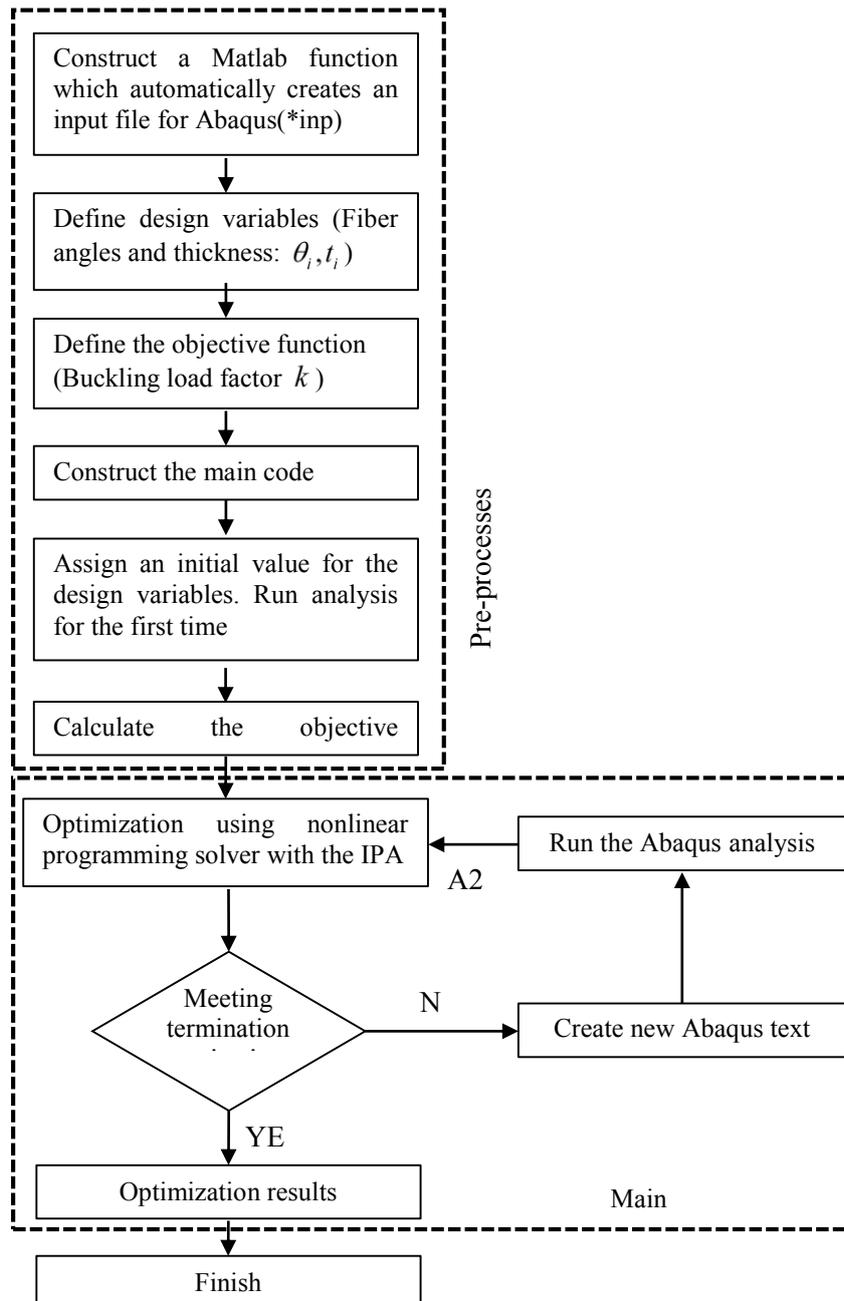


Fig. 2. Flowchart of the optimization procedure.

3. Numerical Results and Discussions

3.1. Validation

In this section, the buckling coefficient $k = \frac{\lambda_{cr} a^2}{E_2 h^3}$ of the laminated composite plates under uniaxial and biaxial compression are compared with previous works. The material properties of the laminated composite plates is given as follows: $E_1 / E_2 = 40, G_{12} = G_{13} = 0.6E_2, G_{23} = 0.5E_2, \nu_{12} = 0.25, a = b = 1, a / h = 10$. Tables 1 and 2 compare the buckling load factors of the laminated composite plates (16x16 elements) to verify the Abaqus model developed in this study. Tables 3 and 4 present the comparison results of the optimization of fiber angles and thickness of the laminated composite plates (12x12 elements) when the number of layers is 3, 4, and 10.

From the comparison, it can be shown that the developed model in this paper is reliable to use for the analysis of the laminated composite plates. The last column of Tables 3 and 4 contains the number of structural analyses (NSA) required to reach the optimum solution

Table 1. Comparison of the buckling load factors of [0/90]₅ laminated plates under uniaxial compression

	SSSS	SSCC	SSSC	SSFC	SSFS	SSFF
Wang et al. [1]	25.703	35.162	32.95	14.495	12.658	12.224
Nguyen et al. [3]	25.534	34.531	32.874	14.356	12.543	12.131
Thai et al. [4]	25.5269	35.1784	32.6882	14.4828	12.6174	12.2338
Ho-Huu et al. [21]	25.2562	35.0937	32.7586	14.3433	12.4929	-
Present study	25.2411	34.0573	31.955	14.2248	12.4172	11.992

Table 2. Comparison of the buckling load factors of SSSS [0/90/0] square plates under biaxial compression

E_1 / E_2	10	20	30	40
Nguyen et al. [3]	4.939	7.488	9.016	10.252
Thai et al. [4]	4.9958	7.5155	8.8712	10.0525
Khdeir and Librescu [5]	4.963	7.516	9.056	10.259
Fares and Zenkour [6]	4.963	7.588	8.575	10.202
Present study	4.9461	7.458	8.6595	9.6655

Table 3. Comparison of the optimum ply-angle of SSSS square composite plates under biaxial compression

	No of layers	$[\theta_i^o]$	k	NSA
Ho-Huu et al. [21]	3 $[\theta_1 / \theta_2 / \theta_3]$	[0/90/0]	10.23938	1
		[45/-45/-45]	12.37798	960
Present study		[0/90/0]	9.7377	1
		[45/-45/45]	11.232	99
Ho-Huu et al. [21]	4 $[\theta_1 / \theta_2]_s$	[0 / 90] _s	11.68617	1
		[- 45 / 45] _s	15.66063	520
Present study		[0 / 90] _s	11.634	1
		[- 45 / 45] _s	14.871	68

Ho-Huu et al. [21]	10	$[0/90]_5$	12.71699	1
		$[45/-45/-45/-45/45]_5$	19.50038	1040
Present study	$[\theta_1/\theta_2/\theta_3/\theta_4/\theta_5]_5$	$[0/90]_5$	12.671	1
		$[45/-45/-45/-45/45]_5$	19.524	242

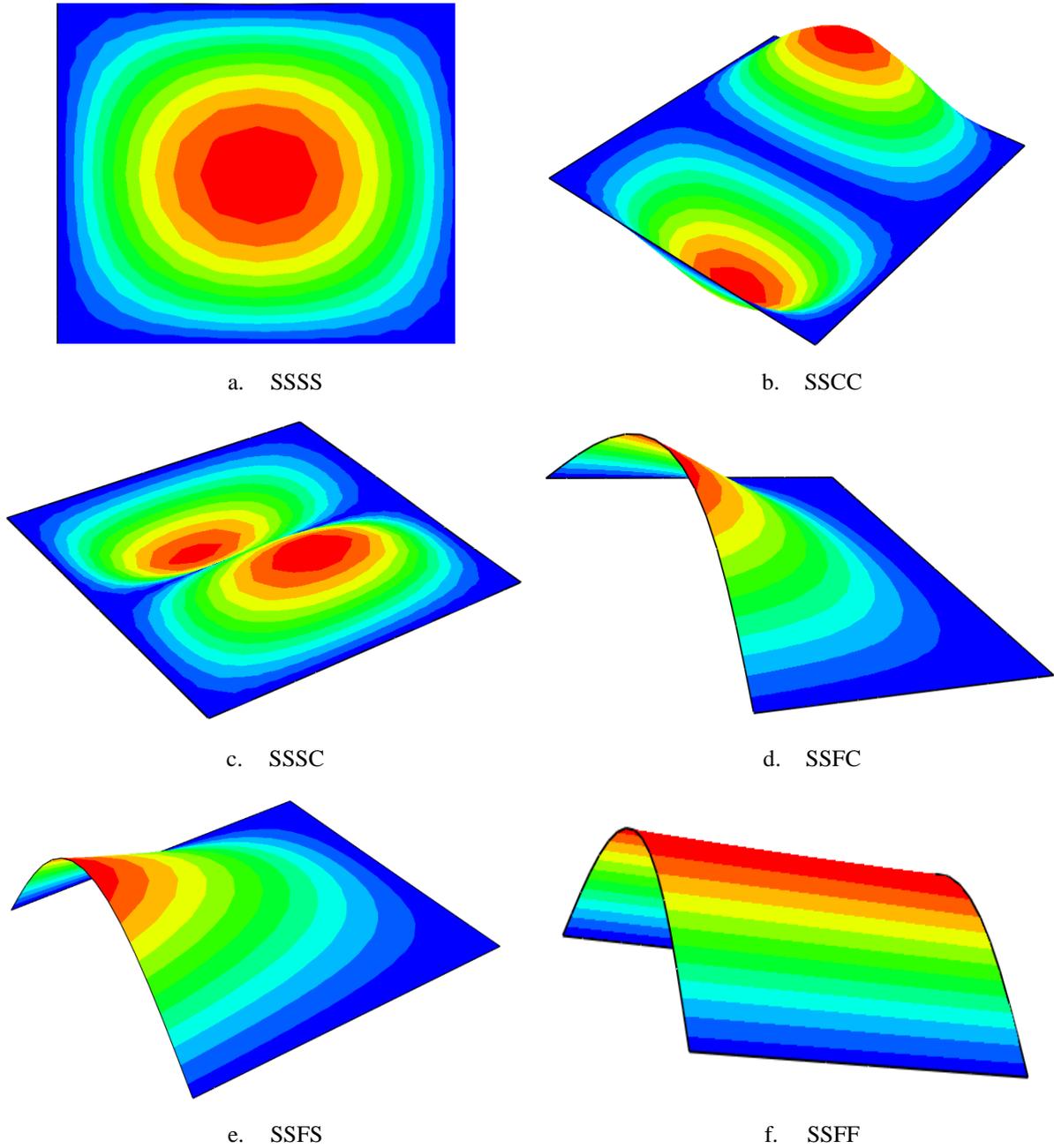


Fig. 3. Buckling modes of laminated plates with various boundary conditions.

Table 4. Comparison the optimum ply-angle and thickness of SSSS square composite plates under bixial compression

	No of layers	$[\theta_i^0; t_i (10^{-3})]$	k	NSA
Ho-Huu et al. [21]	3 $[\theta_1 / \theta_2 / \theta_3]$	[45/-45/45]	19.49077	2180
		[10.213/79.539/10.247]		
Present study	$[t_1 / t_2 / t_3]$	[45/-45/45]	19.5043	208
		[9.5996/80.8009/9.5996]		
Ho-Huu et al. [21]	4 $[\theta_1 / \theta_2]_s$	[45 / -45] _s	19.49077	1260
		[10.242/39.757] _s		
Present study	$[t_1 / t_2]_s$	[45 / -45] _s	19.5043	84
		[9.5996/40.4004] _s		

3.2. Optimum Fiber Angles for Maximizing the Buckling Load

This section presents the optimum fiber orientation angles of the laminated composite plates (12x12 elements) subjected to biaxial compression with simply supported boundary conditions. The objective is to maximize the biaxial buckling load considering only the fiber angles as design variables.

Table 5 and Figure 4 present the effects of the optimum fiber orientation angles of laminated composite plates on the critical biaxial buckling load factor (λ_{cr}). In this case, three layers are considered while varying the a/h ratio. As can be seen, the optimum fiber angles do not change when the a/h ratio is increased. The buckling load of the laminated plates is decreased when the a/h ratio increases. The change of a/h ratio doesn't have any effect on the optimum fiber angles of the laminated composite plates.

3.3. Optimum Fiber Angles and Thicknesses for Maximum Buckling Load

The objective in this section is to maximize the biaxial buckling load factor in the case of mixed design variables (i.e. both fiber angles and thicknesses) of the laminated composite plates (12x12 elements) subjected to biaxial compression with simply supported boundary conditions.

Table 5. Effect of a/h ratio on the optimum fiber angle of the laminated composite plates.

a/h	4 layers			6 layers			10 layers		
	$[\theta_1 / \theta_2]_s$	λ_{cr}	NSA	$[\theta_1 / \theta_2 / \theta_3]_s$	λ_{cr}	NSA	$[\theta_1 / \theta_2 / \theta_3 / \theta_4 / \theta_5]_s$	λ_{cr}	NSA
10	[45/-45] _s	14.871	68	[45/-45/-45] _s	18.2	141	[45/-45/-45/-45/45] _s	19.5238	242
15	[45/-45] _s	5.6066	39	[45/-45/-45] _s	7.0367	186	[45/-45/-45/-45/45] _s	7.5743	222
20	[45/-45] _s	2.6246	64	[45/-45/-45] _s	3.328	124	[45/-45/-45/-45/45] _s	3.5858	204
30	[45/-45] _s	0.8458	38	[45/-45/-45] _s	1.0804	86	[45/-45/-45/-45/45] _s	1.1641	110
50	[45/-45] _s	0.4446	43	[45/-45/-45] _s	0.2456	103	[45/-45/-45/-45/45] _s	0.2644	191

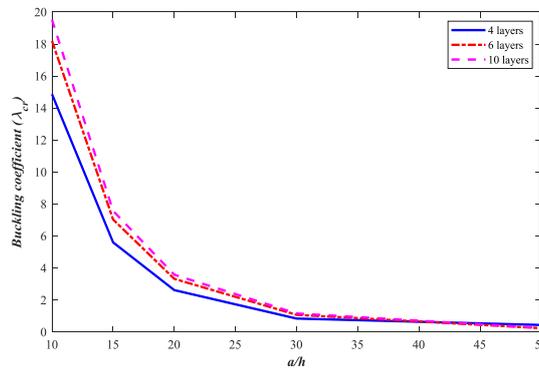


Fig. 4. Effect of the length to thickness ratio on the biaxial buckling load factor.

Table 6 illustrates the optimum results of the fiber angles and thicknesses (θ_i^0, t_i) of the laminated composite plates. As can be seen, the optimum fiber angle of each lamina is $\pm 45^\circ$ for all cases and the optimum thickness of the [45/-45/-45/45] plate is similar to that of the [45/-45/45] plate with the same maximum biaxial buckling load factor $\lambda_{cr}=19.5043$.

Figure 5 presents the comparison of the number of structural analyses (NSA) for various number of layers of the composite plate. From the results of Table 6 and Figure 4, it is clear that the buckling load factor of the composite plate with 6 layers is slightly higher than that of the plates with 3 and 4 layers. In this case, the convergence history of the plate with 4 layers is the fastest among the three cases with only 84 iterations while for the plate with 3 layers it is 208 iterations and for the plate with 6 layers it is 137 iterations.

Table 6. The optimal results of laminated composite plates for biaxial buckling load factor

No of layers	$[\theta_i^0; t_i]$	λ_{cr}	NSA
3 $[\theta_1 / \theta_2 / \theta_3], [t_1 / t_2 / t_3]$	[45 / -45 / 45]	19.5043	208
	[0.0096/0.0808/0.0096]		
4 $[\theta_1 / \theta_2]_s, [t_1 / t_2]_s$	[45 / -45] _s	19.5043	84
	[0.0096/0.0404] _s		
6 $[\theta_1 / \theta_2 / \theta_3]_s, [t_1 / t_2 / t_3]_s$	[- 45 / 45 / 45] _s	19.590	137
	[0.0094/0.0054/0.0352] _s		

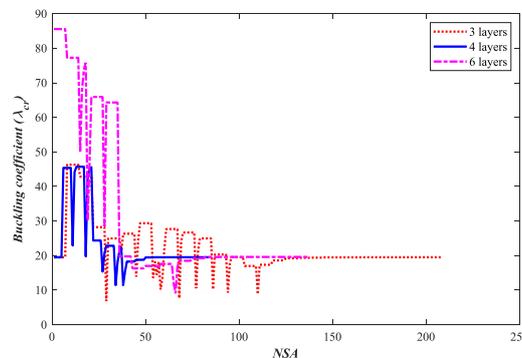


Fig. 5. Convergence histories of the buckling coefficient for composite plates with various numbers of layers.

The effect of a/h ratio on the optimum fiber angles and thicknesses of the laminated composite plates subjected to biaxial load with 3, 4 and 6 layers is presented in Tables 7-9.

From these tables, it is obvious that the biaxial buckling load of the 6-layer plate is the highest among the three cases. Moreover, the buckling load is decreased when a/h ratio increases. The response of the plates with large number of layers is slightly affected in terms of the biaxial buckling load. The optimization of the 4-layer plate is the fastest in terms of convergence rate. This is illustrated in Figure 6.

Examining the data in Tables 7-9, it is found that when the a/h ratio increases the optimum fiber angles do not change and the optimum thickness ratio (thickness of inner layers/thickness of outer layers) shows a minor change. The optimum thickness ratio of the inner layers/outer layers ($t_i^{inner} / t_i^{outer}$) is approximately 4.

Table 7. Effect of a/h ratio on the optimum fiber angle and thickness of the 3-layer plate.

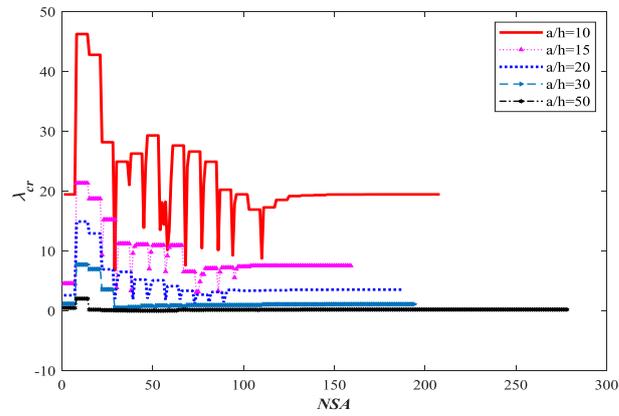
a/h	Optimum fiber angles $[\theta_1 / \theta_2 / \theta_3]$	Optimum thickness $[t_1 / t_2 / t_3]$	λ_{cr}	NSA
$a/h = 10$	[45/-45/45]	[0.0096/0.0808/0.0096]	19.5043	208
$a/h = 15$	[45/-45/45]	[0.0066/0.0534/0.0066]	7.5743	159
$a/h = 20$	[45/-45/45]	[0.005/0.04/0.005]	3.58593	188
$a/h = 30$	[45/-45/45]	[0.0034/0.0265/0.0034]	1.16427	194
$a/h = 50$	[45/-45/45]	[0.0021/0.016/0.0021]	0.26448	278

Table 8. Effect of a/h ratio on the optimum fiber angle and thickness of the 4-layer plate.

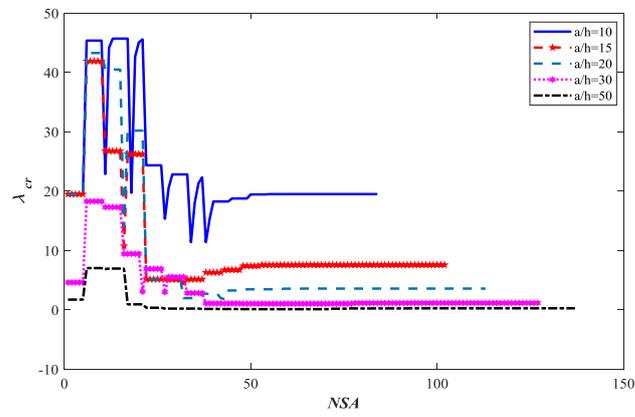
a/h	Optimum fiber angles $[\theta_1 / \theta_2]_s$	Optimum thickness $[t_1 / t_2]_s$	λ_{cr}	NSA
$a/h = 10$	[45/-45] _s	[0.0096/0.0404] _s	19.5043	84
$a/h = 15$	[45/-45] _s	[0.0066/0.0268] _s	7.5743	102
$a/h = 20$	[45/-45] _s	[0.005/0.02] _s	3.58583	113
$a/h = 30$	[45/-45] _s	[0.0034/0.0132] _s	1.16427	127
$a/h = 50$	[45/-45] _s	[0.0021/0.0079] _s	0.26447	137

Table 9. Effect of a/h ratio on the optimum fiber angle and thickness of the 6-layer plate.

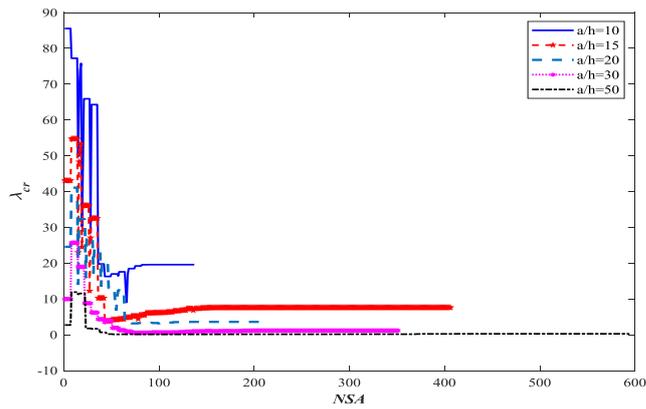
a/h	Optimum fiber angles $[\theta_1 / \theta_2 / \theta_3]_s$	Optimum thickness $[t_1 / t_2 / t_3]_s$	λ_{cr}	NSA
$a/h = 10$	[-45/45/45] _s	[0.0094/0.0054/0.0352] _s	19.590	137
$a/h = 15$	[-45/-45/45] _s	[0.0033/0.0033/0.0268] _s	7.607	406
$a/h = 20$	[-45/-45/45] _s	[0.0025/0.0025/0.02] _s	3.6012	212
$a/h = 30$	[-45/-45/45] _s	[0.00169/0.00169/0.01328] _s	1.1692	351
$a/h = 50$	[45/-45/-45] _s	[0.00207/0.00397/0.003957] _s	0.2645	594



a. 3 layers



b. 4 layers



c. 6 layers

Fig. 6. Convergence histories of the buckling load for composite plates with different a/h ratios.

4. Conclusions

In this paper, a new optimization procedure for the laminated composite plates is proposed which uses the gradient-based interior point algorithm to obtain maximum biaxial buckling load considering the fiber angles and thickness as design variables. Some conclusions are drawn from this study as follows:

- The optimum fiber angle of each lamina of the composite plates subjected to biaxial load is $\pm 45^\circ$ when considering only fiber angle variables as design variables and when considering fiber angle and thickness as design variables.
- The plates with large number of layers are slightly affected in terms of the maximum biaxial buckling load. The 4-layer plate has the fastest convergence rate.
- The variations of a/h ratio have practically no effect on the optimum fiber angles and slightly affect the optimum thickness ratio of the laminated composite plates subjected to biaxial load.

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