



Original Article

A Comparative Study on Two Different Methods
for Calculating Gravity Effect of an Uneven Layer:
Application to Computation of Bouguer Gravity Anomaly
in the East Vietnam Sea and Adjacent Areas

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Abstract: Calculation of gravity anomaly caused by an uneven layer is essential for quantitative interpretation. By comparing calculated anomalies with observed anomalies, we may infer some parameters of subsurface structures. There are many different methods for computing gravity effect of an uneven layer. This paper presents a comparative study of two different forward methods such as the space domain method and the frequency domain method. The performance of each method was evaluated on two synthetic models. Finally, the more effective method was applied to calculate Bouguer gravity anomaly in the East Vietnam Sea and adjacent areas using the latest available dataset from the TOPEX mission.

Keywords: Forward method, space domain, frequency domain, Bouguer gravity anomaly, Vietnam.

1. Introduction

The purpose of gravity methods in natural resources exploration is to determine the geometric parameters of the density structures, including depth, slope, lateral boundaries, etc [1-7]. Knowledge of the parameters of the gravity sources can be important for optimizing drilling operations as well as estimating mineral deposits [8-10]. The gravity anomalies can be calculated from known or assumed information and then compared with measured gravity anomalies to determine some source parameters. A range of different methods have been developed to calculate gravity anomalies caused by density structures. The methods can generally be divided into two main groups, namely the space

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domain methods, and the frequency domain methods. Numerous methods have been developed to calculate gravity anomaly of 2D structures in the space domain [11-16]. Some authors (e.g. [17-23]) derived gravity anomaly expressions of 3D structures in the space domain. Several other authors have presented the different methods for calculating gravity effect of 3D structures in the frequency domain and converted the anomalies into space domain by the inverse Fourier transformation for further analysis [24-27]. The above forward methods are essential in interpreting gravity data, because they provide the basic equations for automatically estimating subsurface structures.

In this paper, I aim to review the performance of the popular forward methods such as the space domain method of Rao et al. (1990) and the frequency domain method of Parker (1973) [20, 24]. These methods were tested on a simple model and then on a complex model. Additionally, Bouguer gravity anomaly in the East Vietnam Sea and adjacent areas were also calculated using the more effective method.

2. Methods

In order to calculate the gravity anomaly caused by an uneven layer, Rao et al. (1990) developed a space domain method that divided the uneven subsurface structure into many rectangular prisms. In a Cartesian coordinate system, let T and W be the half thickness and half width of one such prismatic source with Z_1 and Z_2 are the depths to the top and bottom, respectively. The gravity anomaly of a prism source at any observation (x, y) of a rectangular mesh is given as [20]

$$\Delta g(x, y) = G\Delta\rho \left(z \operatorname{atan} \frac{XY}{zR} + \frac{X}{2} \ln \frac{R-Y}{R+Y} + \frac{Y}{2} \ln \frac{R-X}{R+X} \right) \Big|_{X_1}^{X_2} \Big|_{Y_1}^{Y_2} \Big|_{Z_1}^{Z_2} \quad (1)$$

where

$$X_1 = x + T, X_2 = x - T, Y_1 = y + W, Y_2 = y - W, R = \sqrt{X^2 + Y^2 + Z^2},$$

G is the universal gravitational constant and $\Delta\rho$ is density contrast of the layer. The total gravity anomaly $\Delta g_{total}(x, y)$ is determined by adding the anomaly of all prismatic sources.

Another method was developed by Parker (1973) for rapid calculating gravity and magnetic anomalies in the frequency domain. The method used a sum of the Fourier transforms of the powers of the interface topography h to calculate gravity anomalies. Following Parker (1973), the total gravity anomaly $\Delta g_{total}(x, y)$ caused by an uneven layer is given by [24]

$$\Delta g_{total}(x, y) = 2\pi G\Delta\rho z_0 + F^{-1} \left[2\pi G\Delta\rho e^{(-|k|z_0)} \sum_{n=1}^{\infty} \frac{(-|k|)^{n-1}}{n!} F[h(x, y)^n] \right] \quad (2)$$

where $F[]$ is the Fourier transform operator, $F^{-1}[]$ is the inverse Fourier transform operator, z_0 is the mean depth of the interface and $k = \sqrt{k_x^2 + k_y^2}$ with k_x and k_y are the wavenumbers in the x and y directions, respectively.

3. Synthetic models

In this section, I designed two different theoretical models with density contrast of 200 kg/m^3 to test the efficiency of the methods.

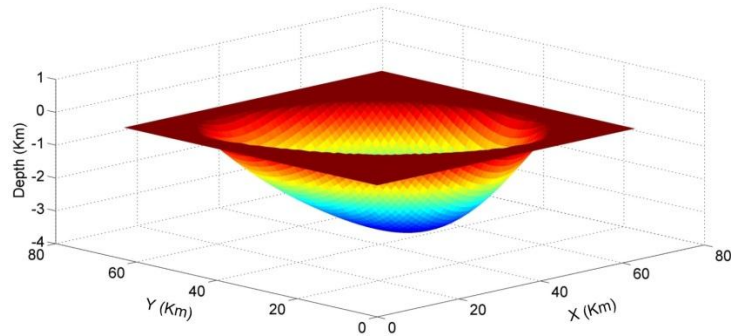


Figure 1. The 3D view of the first model.

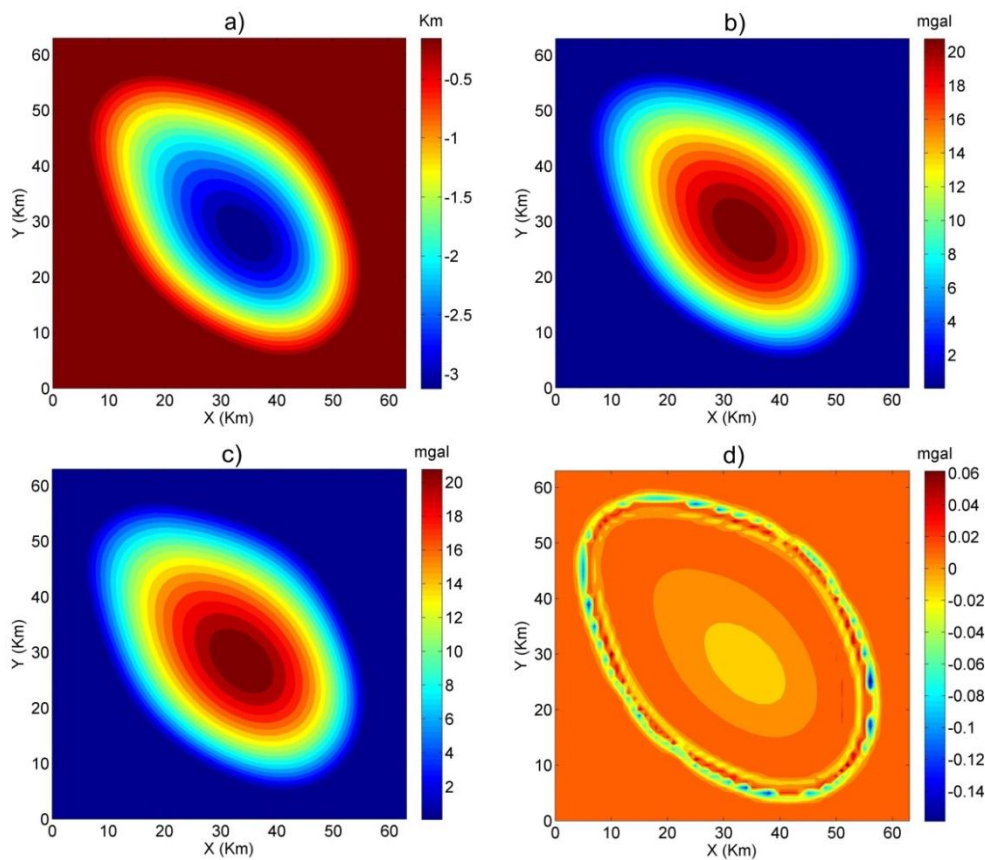


Figure 2. (a) The plan view of the first model, (b) The gravity anomalies calculated by the space domain method, (c) The gravity anomalies calculated by the frequency domain method respectively, (d) The difference between the results in Figure 2b and 2c.

Figure 1 displays the 3D view of the first model. Figure 2a displays the plan view of the model. Figure 2b and 2c show the gravity anomalies calculated by the space domain method and the frequency domain method respectively. It can be observed that the results obtained from applying the methods are similar. The difference between these results is shown in Figure 2d. We can see that these differences are insignificantly small and are in the range of -0.25 - $+0.06$ mgal. The root mean square

(RMS) error between them is only 0.0213 mgal. We note here that the frequency domain method took only about 0.1838 s to compute the gravity anomalies at 64×64 grid nodes using a personal computer with Intel (R) Core (TM) i3 at 2.40 GHz CPU, while the space domain method took about 22 s to make the gravity model.

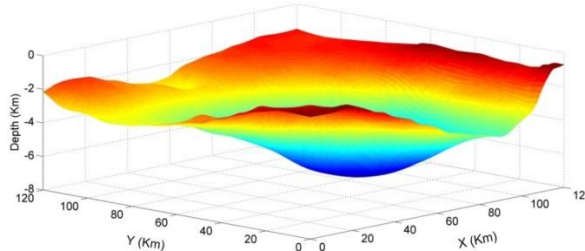


Figure 3. The 3D view of the second model.

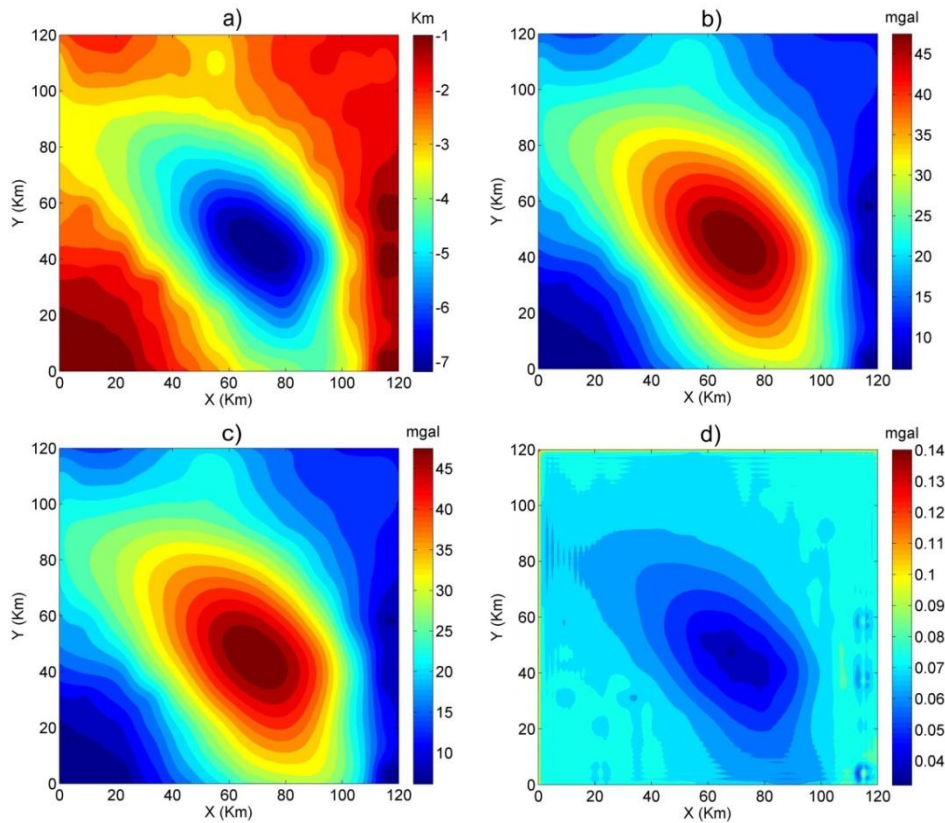


Figure 4. (a) The plan view of the first model, (b) The gravity anomalies calculated by the space domain method, (c) The gravity anomalies calculated by the frequency domain method respectively, (d) The difference between the results in Figure 4b and 4c.

Since the very complex nature of the geological phenomena, it is necessary to also test the efficiency of the methods in a more complex model. The 3D view of the complex model is shown in Figure 3. Figure 4a displays the plan view of the model. The gravity anomalies calculated by the space domain method and the frequency domain method are shown in Figure 4b and 4c, respectively. It can

be seen from these figures that the result calculated by the space domain method compares reasonably well with those obtained from using the frequency domain method. Figure 4d depicts the differences of the gravity anomalies in Figure 4b and 4c. Clearly, these differences are very small, ranging from +0.04 - +0.14 mgal. The RMS error between them is only 0.0661 mgal. In this case, the frequency domain method took only about 0.2872 s to calculate the anomalies on a 2-D grid of 121×121 data, while the space domain method took about 356 s on the same personal computer to make the similar gravity model.

4. Application

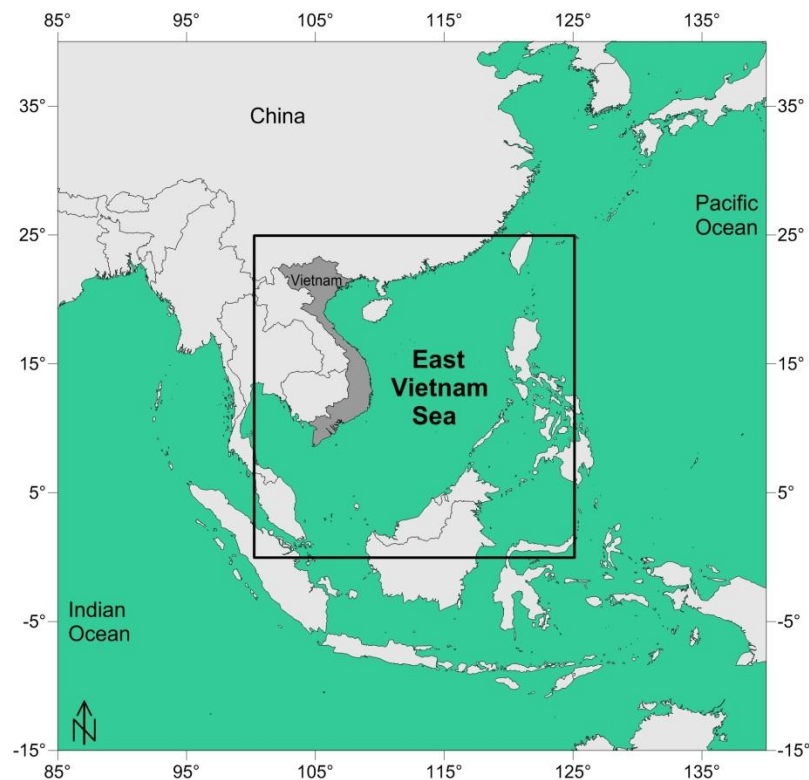


Figure 5. Location of the East Vietnam Sea and adjacent areas.

Since the frequency domain method can perform fast computations with high precision, therefore in this section, I applied it to calculate Bouguer gravity anomaly in the East Vietnam Sea and adjacent areas. Location of the area is shown in Figure 5. The area lies between 100°E and 125°E of the eastern longitudes and 0°N and 25°N of northern latitudes. To compile the Bouguer gravity map of East Vietnam Sea and adjacent areas, we used the latest available data from the TOPEX mission, which includes topographic data (version 18.1) and satellite-derived free-air gravity data (version 28.1) [28-30] (https://topex.ucsd.edu/cgi-bin/get_data.cgi). The data have a grid cell size of 1×1 min. Figure 6 and 7 show the topographic/bathymetric and free-air anomaly maps of the East Vietnam Sea and adjacent areas. The terrain correction is computed using a crustal density of 2670 kg/m^3 and sea-water density of 1030 kg/m^3 , and shown in Figure 8. Figure 9 shows the complete Bouguer anomalies

after applying the terrain correction to the free-air anomalies. It can be seen from Figure 6 and 9 that in general, the Bouguer anomalies are positive over ocean basins and negative over continental areas. The inverse relationship between the Bouguer anomalies and topography is an isostasy manifestation.

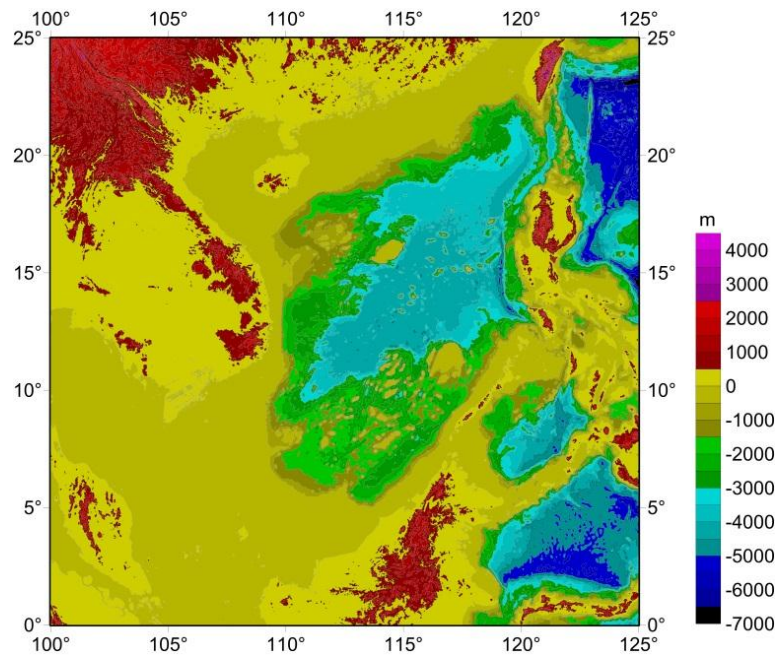


Figure 6. The topographic/bathymetric map of the East Vietnam Sea and adjacent areas.

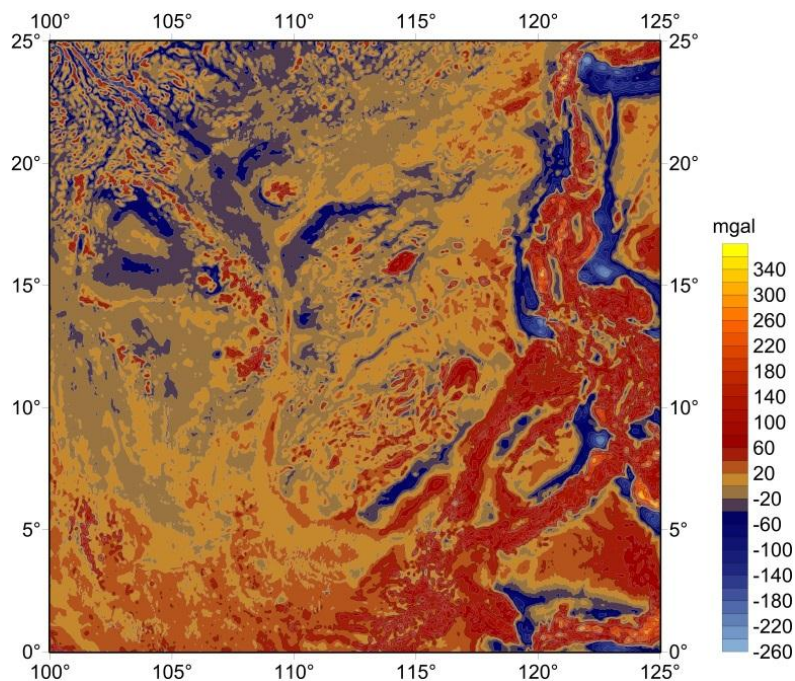


Figure 7. The Free-air anomaly map of the East Vietnam Sea and adjacent areas.

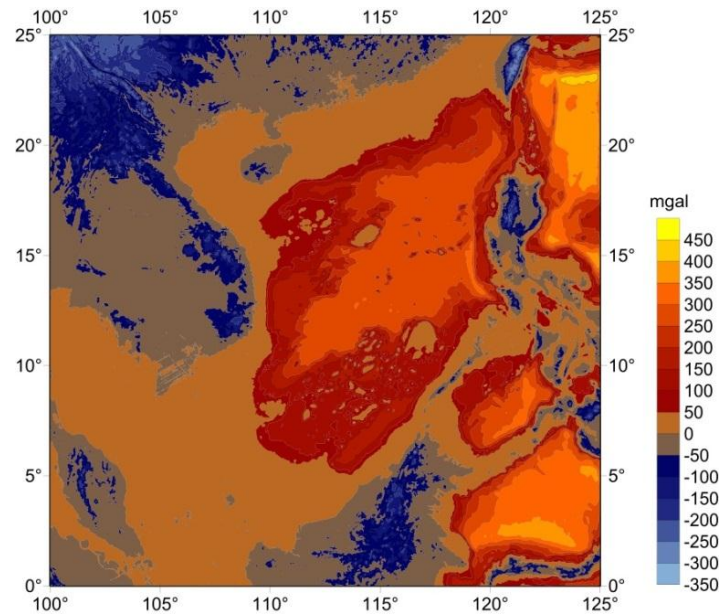


Figure 8. The terrain correction for the East Vietnam Sea and adjacent areas.

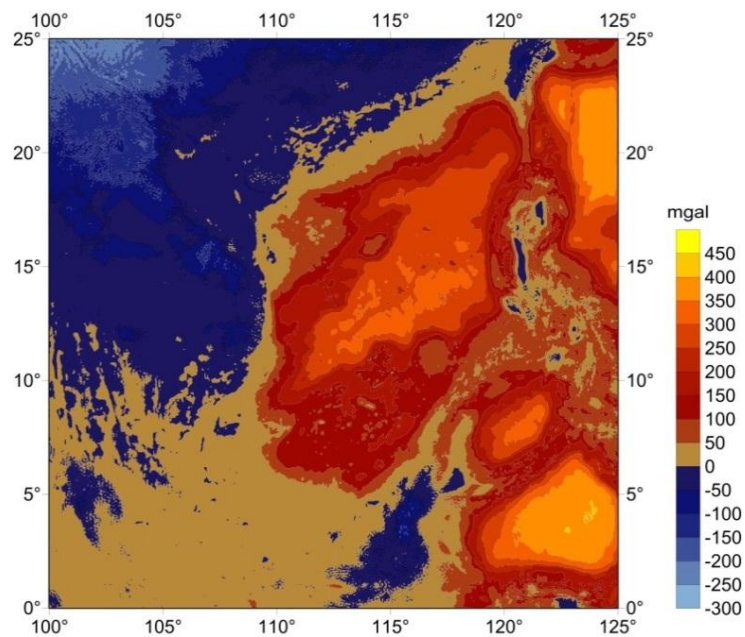


Figure 9. The Bouguer gravity anomaly map for the East Vietnam Sea and adjacent areas.

5. Conclusion

I tried to review the performance of the use of the space domain method of Rao et al. (1990) and the frequency domain method of Parker (1973) for calculating the gravity anomaly caused by an

uneven layer. Test studies were performed on two synthetic models. Although the results obtained from application of the two methods are similar, the frequency domain method can perform very fast computations for the gravity effects. Finally, I applied the frequency domain method for calculating the Bouguer gravity anomaly in the East Vietnam Sea and adjacent areas using the latest available dataset from the TOPEX mission. In this case, the forward of 1498×1498 observation point mesh took only 195 s, which is a very short time for this type of calculation. Thus, it can be concluded that the frequency domain method of Parker (1973) is an efficient tool for calculating the gravity anomaly of the 3D structures, making an improved quantitative interpretation possible.

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