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Original Article

Fermions, Gauge Bosons and Higgs Masses in the 3-3-1-1 Model with Charged Lepton

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Abstract: In this paper, a new version of 3-3-1-1 model was proposed to solve the Landau pole problem of the previous versions. The masses of fermions where the masses of active neutrinos are generated through the seesaw mechanism, are calculated in detail. All the Higgs bosons and gauge bosons as well as their masses are identified and calculated.

Keywords: 3-3-1-1 model, new charged leptons

1. Introduction

One of the greatest successes of the 20th century physics is the Standard Model (SM) of the electroweak and the strong interactions. The model has been experimentally tested with a very high precision for more than 40 years. However, besides the excellent successes, the SM still has serious problems on both theoretical and experimental sides: (i) Why the mass of top quark is much heavier than the other fermions? (ii) Why there are hierarchies in mass among the generations? (iii) Why the neutrinos have tiny masses? (iv) Why the quarks are small mix while the neutrinos are large mix? (v) The SM cannot explain the asymmetry between matter and antimatter (baryon asymmetry) of the Universe?

Because of the mentioned issues, the SM must be expanded to new models which are called Beyond the SM (BSM). The new BSMs not only have all the SM's triumph but also solve all or part of the above problems. Among the BSMs, the models based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) gauge group [1-7] have some intriguing features: First, they can give partial explanation of the generation number

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problem. Second, the third quark generation is assigned to be different from the first two, so this leads to the possible explanation why top quark is uncharacteristically heavy. The physical phenomena of these series of model were investigated intensively, see, for example, in [8-14] and the references therein. The 3-3-1 model can naturally accommodate an extra $U(1)_N$ symmetry behaving as a gauge symmetry, resulting in some models based on $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N$ (3-3-1-1) gauge symmetry [8-11]. These versions of the 3-3-1-1 model somewhat solve the limited issues of the SM. Notice that, in the 3-3-1 and 3-3-1-1 models, the charged operator and sine of the Weinberg angle θ_W are defined as $Q = T_3 + \beta T_8 + X$ and $\sin\theta_W = g_X / \sqrt{g^2 + (1+\beta^2) g_X^2}$, where T_8 denotes the $SU(3)_L$ generator, X is the $U(1)_X$ gauge charge, g, g_X are respectively the coupling constants of the $SU(3)_L$ and $U(1)_X$ groups. The models face a low Landau pole (Λ) at $\sin^2\theta_W(\Lambda) = 1/(1+\beta^2)$ or $g_X(\Lambda) = \infty$ [11]. In the mentioned models, if the third component of the lepton triplets is new heavy neutral particles then the parameter β has the value of $\pm\sqrt{3}$, resulting that these models' new physics scales are blocked by the Landau pole [12, 13].Threfore , in the present work, we propose a new 3-3-1-1 model where, instead of the heavy neutral particles, the new charged leptons are used, leading to $\beta = 1/\sqrt{3}$ so that the new physics scales are free from the Landau pole. In this study, we mainly focus on the particle content of the model, identify all physical particles of the model as well as their masses. The physical phenomena of the model are reserved for future studies.

2. The 3-3-1-1 model

In this paper, we add a changed lepton to each usual $SU(2)_L$ doublet left-handed lepton to the version considered herein to form a triplet [11]

$$\psi_{aL} \equiv (\nu_{aL} \quad l_{aL} \quad E_{aL}^-)^T \sim (1, 3, -2/3, -2/3), \tag{1}$$

$$\nu_{aR} \sim (1, 1, 0, -1), \quad l_{aR} \sim (1, 1, -1, -1), \quad E_{aR}^- \sim (1, 1, -1, 0), \tag{2}$$

where a = 1, 2, 3 is the generation index. The first two quark generations belong to antitriplets and the third one is in triplet

$$Q_{\alpha L} \equiv (d_{\alpha L} \quad -u_{\alpha L} \quad T_{\alpha L})^T \sim (3, 3^*, 1/3, 0), \quad Q_{3L} \equiv (u_{3L} \quad d_{3L} \quad T_{3L})^T \sim (3, 3, 0, 2/3), \tag{3}$$

$$u_{aR} \sim (3, 1, 2/3, 1/3), \quad d_{aR} \sim (3, 1, -1/3, 1/3), \tag{4}$$

$$T_{3R} \sim (3, 1, -1/3, 4/3), \quad T_{\alpha R} \sim (3, 1, 2/3, -2/3), \quad \alpha = 1, 2. \tag{5}$$

The quantum numbers in the parentheses are defined upon the 3-3-1-1 symmetries, respectively.

The electric charge operator and baryon-minus-lepton charge are defined as

$$Q = T_3 + \frac{1}{\sqrt{3}} T_8 + X I_{3\times 3} = \text{Diag.}\left(\frac{2}{3} + X,\ -\frac{1}{3} + X,\ -\frac{1}{3} + X\right), \tag{6}$$

$$B - L = -\frac{2}{\sqrt{3}} T_8 + N I_{3\times 3} = \text{Diag.}\left(-\frac{1}{3} + N,\ -\frac{1}{3} + N,\ \frac{2}{3} + N\right), \tag{7}$$

where T_8 denotes a diagonal $SU(3)_L$ generator, X is the $U(1)_X$ gauge charge, N is the $U(1)_N$ gauge charge.

In order to break the gauge symmetry and generating fermion masses, the 3-3-1-1 model needs the following scalar multiplets [11]

$$\eta = \begin{pmatrix} \eta_1^0 & \eta_2^- & \eta_3^- \end{pmatrix}^T \sim (1, 3, -2/3, 1/3), \quad \rho = \begin{pmatrix} \rho_1^+ & \rho_2^0 & \rho_3^0 \end{pmatrix}^T \sim (1, 3, 1/3, 1/3), \tag{8}$$

$$\chi = \begin{pmatrix} \chi_1^+ & \chi_2^0 & \chi_3^0 \end{pmatrix}^T \sim (1, 3, 1/3, -2/3), \quad \phi \sim (1, 1, 0, 2), \tag{9}$$

with the following VEVs

$$\langle\eta\rangle = \begin{pmatrix} u/\sqrt{2} & 0 & 0 \end{pmatrix}^T, \quad \langle\rho\rangle = \begin{pmatrix} 0 & v/\sqrt{2} & 0 \end{pmatrix}^T, \quad \langle\chi\rangle = \begin{pmatrix} 0 & 0 & w/\sqrt{2} \end{pmatrix}^T, \quad \langle\phi\rangle = w'/\sqrt{2}. \tag{10}$$

To be consistent with low energy phenomenology, we have to impose the following condition $u, v \ll w, w'$.

3. Fermions

The mass of charged leptons (l_a and the new lepton E_a) are obtained from the Yukawa Lagrangian,

$$-\mathcal{L}^l_{\text{Yukawa}} = h^l_{ab}\bar{\psi}_{aL}\rho l_{bR} + h^E_{ab}\bar{\psi}_{aL}\chi E_{bR} + \text{H.c.}, \tag{11}$$

where $\langle\rho\rangle = \begin{pmatrix} 0 & v/\sqrt{2} & 0 \end{pmatrix}^T$, $\langle\chi\rangle = \begin{pmatrix} 0 & 0 & w/\sqrt{2} \end{pmatrix}^T$. The masses of l_a and new lepton E_a are given by

$$(m_l)_{ab} = h^l_{ab}\frac{v}{\sqrt{2}}, \quad (m_E)_{ab} = h^E_{ab}\frac{w}{\sqrt{2}}. \tag{12}$$

For the neutrino sector, the Dirac and Majorana masses are obtained from the following Yukawa Lagrangian,

$$-\mathcal{L}^\nu_{\text{Yukawa}} = h^\nu_{ab}\bar{\psi}_{aL}\eta\nu_{bR} + h'^\nu_{ab}\bar{\nu}^c_{aR}\nu_{bR}\phi + \text{H.c.}, \tag{13}$$

where $\langle\eta\rangle = \begin{pmatrix} u/\sqrt{2} & 0 & 0 \end{pmatrix}^T$, and $\langle\phi\rangle = w'/\sqrt{2}$. From Eq. (13), the Dirac and Majorana mass matrices are derived as

$$(m_\nu)_{ab} = h^\nu_{ab}\frac{u}{\sqrt{2}}, \quad (m^R_\nu)_{ab} = \sqrt{2}h'^\nu_{ab}w'. \tag{14}$$

With the condition $u \ll w'$, the effective neutrino masses are achieved via Type I seesaw mechanism, namely

$$m^L_\nu \simeq m_\nu \left(m^R_\nu\right)^{-1} m^T_\nu \sim \frac{u^2}{w'^2} \ll 1 \tag{15}$$

which can explain the tiny of active neutrino masses.

The quarks getting masses from the Yukawa part,

$$\begin{aligned}-\mathcal{L}^q_{\text{Yukawa}} &= h^T_{33}\bar{Q}_{3L}\chi T_{3R} + h^T_{\alpha\beta}\bar{Q}_{\alpha L}\chi^* T_{\beta R} + h^u_{3a}\bar{Q}_{3L}\eta u_{aR} + h^u_{\alpha a}\bar{Q}_{\alpha L}\rho^* u_{aR} \\ &+ h^d_{3a}\bar{Q}_{3L}\rho d_{aR} + h^d_{\alpha a}\bar{Q}_{\alpha L}\eta^* d_{aR} + \text{H.c.}\end{aligned} \tag{16}$$

When the scalars develop VEVs, the masses of u_a and d_a quarks are given by

$$(m_u)_{\alpha a} = -h^u_{\alpha a}\frac{v}{\sqrt{2}}, \quad (m_u)_{3a} = h^u_{3a}\frac{u}{\sqrt{2}}, \quad (m_d)_{\alpha a} = h^d_{\alpha a}\frac{u}{\sqrt{2}}, \quad (m_d)_{3a} = h^d_{3a}\frac{v}{\sqrt{2}}, \tag{17}$$

whereas the masses of the new quarks, T_a, are derived as $(m_T)_{ab} = h^T_{ab}\frac{w}{\sqrt{2}}$. (18)

4. Gauge Bosons

Gauge bosons' masses arise from the covariant kinetic terms of the Higgs sector,

$$\mathcal{L} = (D_\mu\langle\eta\rangle)^\dagger(D^\mu\langle\eta\rangle) + (D_\mu\langle\rho\rangle)^\dagger(D^\mu\langle\rho\rangle) + (D_\mu\langle\chi\rangle)^\dagger(D^\mu\langle\chi\rangle) + (D_\mu\langle\phi\rangle)^\dagger(D^\mu\langle\phi\rangle). \tag{19}$$

where the covariant derivative is defined as

$$D_\mu = \partial_\mu + igA_{i\mu}T_i + ig_X XB_\mu + ig_N NC_\mu = \partial_\mu + igP^{CC}_\mu + igP^{NC}_\mu, \tag{20}$$

where $T_i, X, N; g, g_X, g_N$ and $A_{i\mu}, B_\mu, C_\mu$ are the generators, the gauge couplings and the fields of the gauge groups $SU(3)_L, U(1)_X$, and $U(1)_N$, respectively; $T_i = \lambda_i/2, i = 1,2,...8, \lambda_i$ are the Gell-Mann matrices.

The matrix $A_{i\mu}T_i$ can be written as follows:

$$A_\mu = A_{i\mu}T_i = \frac{1}{2}\times\begin{pmatrix} A_{3\mu} + \frac{A_{8\mu}}{\sqrt{3}} & \sqrt{2}W^+_\mu & \sqrt{2}X^{+Q_X}_\mu \\ \sqrt{2}W^-_\mu & -A_{3\mu} + \frac{A_{8\mu}}{\sqrt{3}} & \sqrt{2}Y^{+Q_Y}_\mu \\ \sqrt{2}X^{-Q_X}_\mu & \sqrt{2}Y^{-Q_Y}_\mu & -\frac{2A_{8\mu}}{\sqrt{3}} \end{pmatrix}, \tag{21}$$

Where

$$W^\mp = \frac{A_1 \pm iA_2}{\sqrt{2}}, \quad X^{\mp Q_X} = \frac{A_4 \pm iA_5}{\sqrt{2}}, \quad Y^{\mp Q_Y} = \frac{A_6 \pm iA_7}{\sqrt{2}}. \tag{22}$$

$$[Q, A_\mu] = Q.A_\mu - A_\mu.Q = Q_A A_\mu, \; Q_A = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}. \tag{23}$$

Therefore, $Q_X = 1, Q_Y = 0$, hence the new gauge bosons X and Y are singly charged and neutral, respectively.

The charged currents are defined as

$$P^{CC}_\mu = \frac{1}{\sqrt{2}}\times\begin{pmatrix} 0 & W^+ & X^+ \\ W^- & 0 & Y^{0*} \\ X^- & Y^0 & 0 \end{pmatrix}, \tag{24}$$

the mass terms of the non-Hermitian gauge bosons are obtained as

$$\mathcal{L}_{\text{mass}}^{\text{charged}} = \frac{1}{4}g^2(u^2+v^2)W^+W^- + \frac{1}{4}g^2(u^2+w^2)X^+X^- + \frac{1}{4}g^2(v^2+w^2)Y^0Y^{0*}, \quad (25)$$

from then we can identify their masses as follows:

$$m_W^2 = \frac{1}{4}g^2(u^2+v^2), \quad m_X^2 = \frac{1}{4}g^2(u^2+w^2), \quad m_Y^2 = \frac{1}{4}g^2(v^2+w^2). \quad (26)$$

We consider W as the SM's W boson (the SM-like gauge boson), so

$$u^2+v^2 = v_w^2 = (246\,\text{GeV})^2. \quad (27)$$

The mass Lagrangian of neutral gauge bosons is given by

$$\mathcal{L}_{\text{mass}}^{\text{neutral}} = \frac{u^2g^2}{24}\left(A_{3\mu} + \frac{A_{8\mu}}{\sqrt{3}} - \frac{2}{3}t_X B_\mu + \frac{2}{3}t_N C_\mu\right)^2 + \frac{v^2g^2}{24}\left(-A_{3\mu} + \frac{A_{8\mu}}{\sqrt{3}} + \frac{2}{3}t_X B_\mu + \frac{2}{3}t_N C_\mu\right)^2$$

$$+\frac{w^2g^2}{6}\left(-\frac{A_{8\mu}}{\sqrt{3}} - \frac{1}{3}t_X B_\mu + \frac{2}{3}t_N C_\mu\right)^2 + 2g^2t_N^2 w' C_\mu = \frac{1}{2}V^T M^2 V, \quad (28)$$

where $V^T = \; A_3 \; A_8 \; B \; C$ and

$$M^2 = \frac{g^2}{2}\times\begin{pmatrix} \frac{1}{2}(u^2+v^2) & \frac{u^2-v^2}{2\sqrt{3}} & -\frac{t_X(2u^2+v^2)}{3} & \frac{t_N(u^2-v^2)}{3} \\ \frac{u^2-v^2}{2\sqrt{3}} & \frac{1}{6}(u^2+v^2+4w^2) & -\frac{t_X(2u^2-v^2+2w^2)}{3\sqrt{3}} & \frac{t_N(u^2+v^2+4w^2)}{3\sqrt{3}} \\ -\frac{t_X(2u^2+v^2)}{3} & -\frac{t_X(2u^2-v^2+2w^2)}{3\sqrt{3}} & \frac{2}{9}t_X^2(4u^2+v^2+w^2) & -\frac{2}{9}t_Xt_N(2u^2-v^2+2w^2) \\ \frac{t_N(u^2-v^2)}{3} & \frac{t_N(u^2+v^2+4w^2)}{3\sqrt{3}} & -\frac{2}{9}t_Xt_N(2u^2-v^2+2w^2) & \frac{2}{9}t_N^2(u^2+v^2+4w^2+36w'^2) \end{pmatrix},$$

where the mass matrix M^2 is symmetric, $t_X = g_X/g = \sqrt{3}\sin\theta_W/\sqrt{3-4\sin^2\theta_W}$, $\sin\theta_W$ is the sine of the Weinberg angle, which can explicitly be identified from the electromagnetic interaction vertices [14] and $t_N = g_N/g$.

The mass matrix M^2 has a zero eigenvalue $(m_A = 0)$ which is set as the photon's mass with corresponding eigenstate

$$A = \frac{\sqrt{3}t_X}{\sqrt{3+4t_X^2}}A_3 + \frac{t_X}{\sqrt{3+4t_X^2}}A_8 + \frac{\sqrt{3}}{\sqrt{3+4t_X^2}}B. \quad (29)$$

We can define the SM's Z boson and a new Z' boson as follows:

(30)

$$\begin{aligned} Z &= \frac{\sqrt{3+t_X^2}}{\sqrt{3+4t_X^2}}A_3 - \frac{\sqrt{3}t_X^2}{\sqrt{3+t_X^2}\sqrt{3+4t_X^2}}A_8 - \frac{3t_X}{\sqrt{3+t_X^2}\sqrt{3+4t_X^2}}B, \\ Z' &= \frac{\sqrt{3}}{\sqrt{3+t_X^2}}A_8 - \frac{t_X}{\sqrt{3+t_X^2}}B, \end{aligned} \tag{31}$$

which are orthogonally to A, as usual. At this stage, C is always orthogonal to A, Z, and Z'. Let us change to the new basis $(A_3, A_8, B, C) \to (A, Z, Z', C')$,

$$\begin{pmatrix} A_3 \\ A_8 \\ B \\ C \end{pmatrix} = U_1 \begin{pmatrix} A \\ Z \\ Z' \\ C' \end{pmatrix}, \quad U_1 = \begin{pmatrix} \frac{\sqrt{3}t_X}{\sqrt{3+4t_X^2}} & \frac{\sqrt{3+t_X^2}}{\sqrt{3+4t_X^2}} & 0 & 0 \\ \frac{t_X}{\sqrt{3+4t_X^2}} & -\frac{\sqrt{3}t_X^2}{\sqrt{3+t_X^2}\sqrt{3+4t_X^2}} & \frac{\sqrt{3}}{\sqrt{3+t_X^2}} & 0 \\ \frac{\sqrt{3}}{\sqrt{3+4t_X^2}} & -\frac{3t_X}{\sqrt{3+t_X^2}\sqrt{3+4t_X^2}} & -\frac{t_X}{\sqrt{3+t_X^2}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{32}$$

In the new basis, the mass matrix M^2 becomes

$$M_1^2 = U_1^T M^2 U_1 = \begin{pmatrix} 0 & 0 \\ 0 & M_s^2 \end{pmatrix}, \quad M_s^2 \equiv \begin{pmatrix} m_Z^2 & m_{ZZ'}^2 & m_{ZC'}^2 \\ m_{ZZ'}^2 & m_{Z'}^2 & m_{Z'C'}^2 \\ m_{ZC'}^2 & m_{Z'C'}^2 & m_{C'}^2 \end{pmatrix}. \tag{33}$$

We see that the photon field is physical and decoupled, while Z, Z', C' mix via the 3×3 mass submatrix M_s^2 with the elements given by

$$\begin{aligned} m_Z^2 &= \frac{g^2(3+4t_X^2)(u^2+v^2)}{4(3+t_X^2)}, \quad m_{ZZ'}^2 = \frac{g^2\sqrt{3+4t_X^2}\left[(3+4t_X^2)u^2-(3-2t_X^2)v^2\right]}{12(3+t_X^2)}, \\ m_{ZC'}^2 &= \frac{g^2 t_N\sqrt{3+4t_X^2}(u^2-v^2)}{6\sqrt{3+t_X^2}}, \quad m_{Z'}^2 = \frac{g^2\left[(3+4t_X^2)^2u^2+(3-2t_X^2)^2v^2+4(3+t_X^2)^2w^2\right]}{36(3+t_X^2)}, \\ m_{Z'C'}^2 &= \frac{g^2t_N^2\left[(3+4t_X^2)u^2+(3-2t_X^2)v^2+4(3+t_X^2)w^2\right]}{18\sqrt{3+t_X^2}}, \quad m_{C'}^2 = \frac{g^2t_N^2}{9}(u^2+v^2+4w^2+36w'^2). \end{aligned}$$

Because of the condition $u, v \ll w, w'$, we have $m_Z^2, m_{ZZ'}^2, m_{ZC'}^2 \ll m_{Z'}^2, m_{Z'C'}^2, m_{C'}^2$ and the mixing of Z with the new Z' and C' is negligible. Hence, the Z boson can be considered as a physical particle with mass,

$$m_{Z_1}^2 \simeq \frac{g^2(3+4t_X^2)(u^2+v^2)}{4(3+t_X^2)} \simeq \frac{g^2}{4\cos^2\theta_W}(u^2+v^2). \tag{34}$$

The fields Z' and C' *finitely* mix via a mass matrix obtained by

$$M'^2_s \simeq \begin{pmatrix} m_{Z'}^2 & m_{Z'C'}^2 \\ m_{Z'C'}^2 & m_{C'}^2 \end{pmatrix}. \tag{35}$$

$$\begin{pmatrix} A \\ Z_1 \\ Z' \\ C' \end{pmatrix} = U_2 \begin{pmatrix} A \\ Z_1 \\ Z_2 \\ Z_3 \end{pmatrix}, \quad U_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\varphi & \sin\varphi \\ 0 & 0 & -\sin\varphi & \cos\varphi \end{pmatrix}, \quad M_2^2 = U_2^T M'^2_s U_2 = \text{Diag.}(0, m_{Z_1}^2, m_{Z_2}^2, m_{Z_3}^2). \tag{36}$$

The Z' and C' *mixing* angle and Z_2, Z_3 masses are given by

$$\tan(2\varphi) \simeq \frac{4\sqrt{3+t_X^2}\, t_N w^2}{4t_N^2(9w'^2 + w^2) - (3+t_X^2)w^2}, \tag{37}$$

$$m_{Z_2,Z_3}^2 = \frac{1}{2}\left[m_{Z'}^2 + m_{C'}^2 \mp \sqrt{(m_{Z'}^2 - m_{C'}^2)^2 + 4m_{Z'C'}^4}\right]. \tag{38}$$

We can see that, Z_2, Z_3 getting masses at the *w* scale so that we classify them as the new neutral gauge bosons.

It is worth to note that, the ρ-parameter (or $\Delta\rho = \rho - 1$) is receiving the contributions from two distinct sources, denoted as $\Delta\rho = \Delta\rho_{\text{tree}} + \Delta\rho_{\text{rad}}$, where the first term resulted from the contributions of the tree-level mixing of Z with Z' and C'. The second term originated from the dominant, radiative corrections of a heavy non-Hermitian gauge doublet X and Y, similarly to the 3-3-1 model case [12, 15-17]

$$\Delta\rho_{tree} = \frac{m_W^2}{\cos^2\theta_W m_{Z_1}^2} - 1 = \frac{m_Z^2}{m_{ZZ'}^2 \quad m_{ZC'}^2 \begin{pmatrix} m_{Z'}^2 & m_{Z'C'}^2 \\ m_{Z'C'}^2 & m_{C'}^2 \end{pmatrix}^{-1} \begin{pmatrix} m_{ZZ'}^2 \\ m_{ZC'}^2 \end{pmatrix}} - 1, \tag{39}$$

$$\text{where} \qquad m_{Z_1}^2 \simeq m_Z^2 - \; m_{ZZ'}^2 \quad m_{ZC'}^2 \begin{pmatrix} m_{Z'}^2 & m_{Z'C'}^2 \\ m_{Z'C'}^2 & m_{C'}^2 \end{pmatrix}^{-1} \begin{pmatrix} m_{ZZ'}^2 \\ m_{ZC'}^2 \end{pmatrix}. \tag{40}$$

The explicit results of $\Delta\rho_{\text{tree}}$ and $\Delta\rho_{\text{rad}}$ are obtained as

$$\begin{aligned} \Delta\rho_{\text{tree}} &\simeq \frac{1}{m_Z^2} \; m_{ZZ'}^2 \quad m_{ZC'}^2 \begin{pmatrix} m_{Z'}^2 & m_{Z'C'}^2 \\ m_{Z'C'}^2 & m_{C'}^2 \end{pmatrix}^{-1} \begin{pmatrix} m_{ZZ'}^2 \\ m_{ZC'}^2 \end{pmatrix} \\ &\simeq \frac{\left[u^2 - (1-2\sin^2\theta_W)v^2\right]^2}{4(1-\sin^2\theta_W)(u^2+v^2)w^2} + \frac{1}{36}\left(\frac{\sin^2\theta_W}{1-\sin^2\theta_W}\right)^2 \frac{(u^2+v^2)}{w'^2}, \end{aligned} \tag{41}$$

$$\begin{aligned} \Delta\rho_{\text{rad}} &= \frac{3\sqrt{2}G_F}{16\pi^2}\left(m_X^2 + m_Y^2 - \frac{2m_X^2 m_Y^2}{m_Y^2 - m_X^2}\ln\frac{m_Y^2}{m_X^2}\right) \\ &+ \frac{\alpha}{4\pi^2\sin^2\theta_W}\left(\frac{m_X^2 + m_Y^2}{m_Y^2 - m_X^2}\ln\frac{m_Y^2}{m_X^2} - 2 - \frac{\sin^2\theta_W}{1-\sin^2\theta_W}\ln\frac{m_Y^2}{m_X^2}\right), \end{aligned} \tag{42}$$

where $\alpha \simeq \frac{1}{128}$, $\sin^2\theta_W = 0.231$, $\rho = 1.00039 \pm 0.00019$ [18], and $G_F = \frac{1}{\sqrt{2}(u^2+v^2)}$, $g^2 = \frac{2\pi\alpha}{\sin^2\theta_W}$,

$u^2 + v^2 = (246\text{ GeV})^2$, $m_W^2 = \frac{1}{4}g^2(u^2+v^2)$, $m_X^2 = \frac{1}{4}g^2(u^2+w^2)$, $m_Y^2 = \frac{1}{4}g^2(v^2+w^2)$.

We can see, from Eq. (41), if $w' \gg w$ then $\Delta\rho$ contains only (u, v, w) leading that $\Delta\rho$ is analogous with that of 3-3-1 model with $\beta = 1/\sqrt{3}$. If $w' \sim w$ then $\Delta\rho$ depends on all energy scales (u, v, w, w'), in this case, for simplicity, we set $w' = w$ for numerical investigation. Using the condition $v^2 = (246\ \text{GeV})^2 - u^2$, then $\Delta\rho$ becomes a function of two parameters (u, w). Let $0 \le u \le 246$ GeV, we make the contour plot of $\Delta\rho$ constrained by the experimental data $(0.0002 \le \Delta\rho \le 0.00058)$ [18] in order to find the allowed values of the new physics scale w. The results are plotted in Figure 1 (left panel) for the case of $w' \gg w$ and for the case of $w' = w$ in the right panel. We can see that, the scale of new physics w in both cases are almost similar, that is about several TeV hence the new physics of the model, if it exists, could be detected by the LHC.

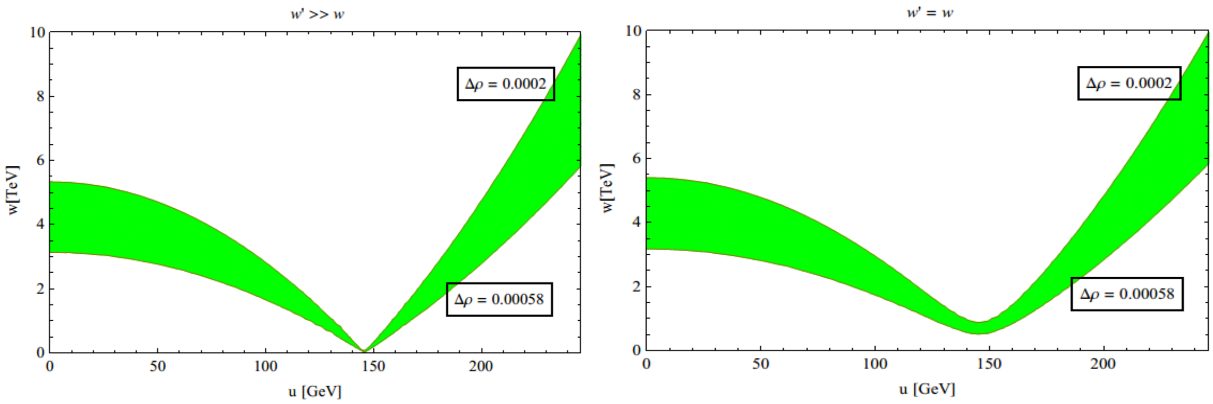

Figure 1. The (u, w) regime that is bounded by the $\Delta\rho$ parameter $(0.0002 \le \Delta\rho \le 0.00058)$ for $w' >> w$ (left panel), for $w' = w$ (right panel).

5. Higgs Sector

The most general form of the Higgs potential can then be written as

$$\begin{aligned} V(\eta,\rho,\chi,\phi) &= \mu_1^2\eta^\dagger\eta + \mu_2^2\rho^\dagger\rho + \mu_3^2\chi^\dagger\chi + \mu_4^2\phi^\dagger\phi + \lambda_1(\eta^\dagger\eta)^2 + \lambda_2(\rho^\dagger\rho)^2 + \lambda_3(\chi^\dagger\chi)^2 \\ &+ \lambda_4(\phi^\dagger\phi)^2 + \lambda_5(\eta^\dagger\eta)(\rho^\dagger\rho) + \lambda_6(\eta^\dagger\eta)(\chi^\dagger\chi) + \lambda_7(\rho^\dagger\rho)(\chi^\dagger\chi) \\ &+ \lambda_8(\phi^\dagger\phi)(\eta^\dagger\eta) + \lambda_9(\phi^\dagger\phi)(\rho^\dagger\rho) + \lambda_{10}(\phi^\dagger\phi)(\chi^\dagger\chi) + \lambda_{11}(\eta^\dagger\rho)(\rho^\dagger\eta) \\ &+ \lambda_{12}(\eta^\dagger\chi)(\chi^\dagger\eta) + \lambda_{13}(\rho^\dagger\chi)(\chi^\dagger\rho) + (\mu\varepsilon^{ijk}\eta_i\rho_j\chi_k + H.c.). \end{aligned} \tag{43}$$

We expand the fields around Higgs' VEVs such as

$$\eta = \begin{pmatrix} \frac{u}{\sqrt{2}} & 0 & 0 \end{pmatrix}^T + \begin{pmatrix} \frac{S_1 + iA_1}{\sqrt{2}} & \eta_2^- & \eta_3^- \end{pmatrix}^T, \ \rho = \begin{pmatrix} 0 & \frac{v}{\sqrt{2}} & 0 \end{pmatrix}^T + \begin{pmatrix} \rho_1^+ & \frac{S_2 + iA_2}{\sqrt{2}} & \frac{S_3 + iA_3}{\sqrt{2}} \end{pmatrix}^T, \tag{44}$$

$$\chi = \begin{pmatrix} 0 & 0 & \frac{w}{\sqrt{2}} \end{pmatrix}^T + \begin{pmatrix} \chi_1^+ & \frac{S_4 + iA_4}{\sqrt{2}} & \frac{S_5 + iA_5}{\sqrt{2}} \end{pmatrix}^T, \quad \phi = \frac{w'}{\sqrt{2}} + \frac{S_6 + iA_6}{\sqrt{2}}. \tag{45}$$

The constraint equations derived from the stationary condition of the scalar potential are given as

$$\mu_1^2 = -\frac{1}{2}\left(\lambda_8 w'^2 + 2\lambda_1 u^2 + \lambda_5 v^2 + \lambda_6 w^2 + \frac{\sqrt{2}\mu v w}{u}\right), \tag{44}$$

$$\mu_2^2 = -\frac{1}{2}\left(\lambda_9 w'^2 + 2\lambda_2 v^2 + \lambda_5 u^2 + \lambda_7 w^2 + \frac{\sqrt{2}\mu u w}{v}\right), \tag{45}$$

$$\mu_3^2 = -\frac{1}{2}\left(\lambda_{10} w'^2 + 2\lambda_3 w^2 + \lambda_6 u^2 + \lambda_7 v^2 + \frac{\sqrt{2}\mu u v}{w}\right), \tag{46}$$

$$\mu_4^2 = -\frac{1}{2}\ 2\lambda_4 w'^2 + \lambda_8 u^2 + \lambda_9 v^2 + \lambda_{10} w^2\ . \tag{47}$$

For the neutral scalar fields A_1, A_2, A_5, A_6 we find out as

$$\mathcal{L}_{\text{mass}}^{A_{1256}} = -\frac{1}{2}\frac{\mu[v^2w^2 + u^2(v^2+w^2)]}{\sqrt{2}uvw}\left(\frac{vwA_1 + uwA_2 + vuA_5}{\sqrt{u^2w^2 + v^2w^2 + u^2v^2}}\right)^2. \tag{48}$$

From this we identify a physical state (physical pseudoscalar) and its's mass as

$$A_P \equiv \frac{vwA_1 + uwA_2 + vuA_5}{\sqrt{u^2w^2 + v^2w^2 + u^2v^2}},\ m_{A_P}^2 = -\frac{\mu[v^2w^2 + u^2(v^2+w^2)]}{\sqrt{2}uvw}. \tag{49}$$

Two other fields are massless that are identified as the Goldstone bosons of Z and Z_1 :

$$G_Z \equiv \frac{-uA_1 + vA_2}{\sqrt{u^2+v^2}}, \quad G_{Z_1} \equiv \frac{-uv(vA_1 + uA_2) + w(u^2+v^2)A_5}{\sqrt{(u^2v^2 + u^2w^2 + w^2v^2)(u^2+v^2)}}. \tag{50}$$

The pseudoscalar A_6 is massless and is identified to the Goldstone boson of Z_2.

For the neutral scalar fields A_3, A_4 , we find

$$\mathcal{L}_{\text{mass}}^{A_{34}} = \frac{1}{2}\left(\frac{\lambda_{13}}{2} - \frac{\sqrt{2}}{2}\frac{\mu u}{vw}\right)(v^2+w^2)\left(\frac{-wA_3 + vA_4}{\sqrt{v^2+w^2}}\right)^2, \tag{51}$$

where we can define the physical state and its' corresponding mass as

$$A_{34} \equiv \frac{-wA_3 + vA_4}{\sqrt{v^2+w^2}},\ m_{A_{34}}^2 = \left(\frac{\lambda_{13}}{2} - \frac{\sqrt{2}}{2}\frac{\mu u}{vw}\right)(v^2+w^2). \tag{52}$$

For the neutral scalar fields S_1, S_2, S_5, S_6 , we define $\mathcal{L}_{\text{mass}}^{S_{1256}} = \frac{1}{2}S^T M_{S_{1256}}^2 S,$ (53)

where $S^T = \ S_1\ S_2\ S_5\ S_6$ and

$$M_{S_{1256}}^2 \equiv \begin{pmatrix} 2\lambda_1 u^2 - \frac{\mu v w}{\sqrt{2}u} & \lambda_5 uv + \frac{\mu w}{\sqrt{2}} & \lambda_6 uw + \frac{\mu v}{\sqrt{2}} & \lambda_8 uw' \\ \lambda_5 uv + \frac{\mu w}{\sqrt{2}} & 2\lambda_2 v^2 - \frac{\mu u w}{\sqrt{2}v} & \lambda_7 vw + \frac{\mu u}{\sqrt{2}} & \lambda_9 vw' \\ \lambda_6 uw + \frac{\mu v}{\sqrt{2}} & \lambda_7 vw + \frac{\mu u}{\sqrt{2}} & 2\lambda_3 w^2 - \frac{\mu v u}{\sqrt{2}w} & \lambda_{10} ww' \\ \lambda_8 uw' & \lambda_9 vw' & \lambda_{10} ww' & 2\lambda_4 w'^2 \end{pmatrix}. \tag{54}$$

Using conditions $u, v \ll w, w'$, we have

$$M_{1S}^2 \equiv \begin{pmatrix} -\frac{\mu v w}{\sqrt{2}u} & \frac{\mu w}{\sqrt{2}} & 0 & 0 \\ \frac{\mu w}{\sqrt{2}} & -\frac{\mu u w}{\sqrt{2}v} & 0 & 0 \\ 0 & 0 & 2\lambda_3 w^2 & \lambda_{10} w w' \\ 0 & 0 & \lambda_{10} w w' & 2\lambda_4 w'^2 \end{pmatrix}, \tag{55}$$

$$m_{S_{H_1}}^2 = 0, \quad H_1 \equiv \frac{uS_1 + vS_2}{\sqrt{u^2+v^2}}, \tag{56}$$

$$m_{S_{H_2}}^2 = -\frac{\mu w(u^2+v^2)}{\sqrt{2}uv}, \quad H_2 \equiv \frac{-vS_1 + uS_2}{\sqrt{u^2+v^2}}, \tag{57}$$

$$m_{S_{H_5}}^2 = \lambda_4 w'^2 + \lambda_3 w^2 + \sqrt{\lambda_3^2 w^4 + (\lambda_{10}^2 - 2\lambda_3\lambda_4) w^2 w'^2 + \lambda_4^2 w'^4}, \tag{58}$$

$$m_{S_{H_6}}^2 = \lambda_4 w'^2 + \lambda_3 w^2 - \sqrt{\lambda_3^2 w^4 + (\lambda_{10}^2 - 2\lambda_3\lambda_4) w^2 w'^2 + \lambda_4^2 w'^4}, \tag{59}$$

where

$$H_5 \equiv \cos\phi\, S_5 + \sin\phi\, S_6, \quad H_6 \equiv -\sin\phi\, S_5 + \cos\phi\, S_6, \tag{60}$$

$$\tan(2\phi) = \frac{\lambda_{10} w w'}{\lambda_3 w^2 - \lambda_4 w'^2}. \tag{61}$$

To diagonalize $M_{S_{1256}}^2$, we transform to a new basis as:

$$S = U\begin{pmatrix} H_1 \\ H_2 \\ H_5 \\ H_6 \end{pmatrix}, \quad U \equiv \begin{pmatrix} \frac{u}{\sqrt{u^2+v^2}} & -\frac{v}{\sqrt{u^2+v^2}} & 0 & 0 \\ \frac{v}{\sqrt{u^2+v^2}} & \frac{u}{\sqrt{u^2+v^2}} & 0 & 0 \\ 0 & 0 & \cos\phi & -\sin\phi \\ 0 & 0 & \sin\phi & \cos\phi \end{pmatrix}, \quad M^2 = U^T M_{S_{1256}}^2 U. \tag{62}$$

At this stage, M^2 has the seesaw form matrix. Diagonalizing this matrix due to the seesaw mechanism [19-22], we obtain the Higgs boson with the mass as follows:

$$m_h^2 = 2\left(\frac{u^4\lambda_1 + v^4\lambda_2 + u^2v^2\lambda_5}{u^2+v^2} + m_0^2 + m_1^2\frac{\mu}{w} + m_2^2\frac{\mu^2}{w^2}\right), \tag{63}$$

where

$$m_0^2 = -\frac{1}{(\lambda_{10}^2 - 4\lambda_3\lambda_4)(u^2+v^2)}\left[-\lambda_4(\lambda_6 u^2 + \lambda_7 v^2)^2 - \lambda_3(\lambda_8 u^2 + \lambda_9 v^2)^2 + \lambda_{10}(\lambda_6 u^2 + \lambda_7 v^2)(\lambda_8 u^2 + \lambda_9 v^2)\right];$$

$$m_1^2 = -\frac{\sqrt{2}uv\left[(\lambda_8\lambda_{10} - 2\lambda_4\lambda_6)u^2 + (\lambda_9\lambda_{10} - 2\lambda_4\lambda_7)v^2\right]}{(\lambda_{10}^2 - 4\lambda_3\lambda_4)(u^2+v^2)}; \quad m_2^2 = \frac{2u^2v^2\lambda_4}{(\lambda_{10}^2 - 4\lambda_3\lambda_4)(u^2+v^2)}.$$

Because w and μ have the same order so m_h has the order of u, hence we can identify h as the

SM's Higgs, namely the SM-like Higgs boson.

Since $u \sim v \ll w \sim -\mu,\ u = v,\ w = -\mu,$ we can simplify the above expressions as

$$\begin{aligned} &m_0^2 \equiv f_0(\lambda)u^2, \quad m_1^2 = f_1(\lambda)u^2, \quad m_2^2 = f_2(\lambda)u^2, \\ &m_h^2 = \ \lambda_1 + \lambda_2 + \lambda_5 \ \ u^2 + 2m_0^2 - 2m_1^2 + 2m_2^2 = f(\lambda)u^2, \end{aligned} \tag{64}$$

where $f_0(\lambda),\ f_1(\lambda),\ f_2(\lambda),\ f(\lambda)$ are functions of only the λ's couplings. Using the Higgs mass $m_h = 125\,\text{GeV}$ [23, 24] and $u = \frac{246}{\sqrt{2}}\,\text{GeV}$, we can estimate that $f_i(\lambda) \simeq 0.52.$

For the neutral scalar fields S_3, S_4, we have

$$\mathcal{L}_{\text{mass}}^{S_{34}} = \frac{1}{4} \ \lambda_{13}vw - \sqrt{2}\mu u \ \ \frac{v^2 + w^2}{vw}\left(\frac{wS_3 + vS_4}{\sqrt{v^2 + w^2}}\right)^2 \tag{65}$$

from this, we define a physical state and its mass as

$$m_{S_{34}}^2 = \frac{1}{2} \ \lambda_{13}vw - \sqrt{2}\mu u \ \ \frac{v^2 + w^2}{vw}, \quad S_{34} \equiv \frac{wS_3 + vS_4}{\sqrt{v^2 + w^2}}. \tag{66}$$

For charged scalars, we derive as

$$\mathcal{L}_{\text{mass}}^{\text{charged}} = \frac{1}{2} \ \lambda_{11}uv - \sqrt{2}\mu w \ \ \frac{u^2 + v^2}{uv} H_1^+ H_1^- + \frac{1}{2} \ \lambda_{12}uw - \sqrt{2}\mu v \ \ \frac{u^2 + w^2}{uw} H_2^+ H_2^-, \tag{67}$$

where the two charged Higgses and their masses are identified as

$$H_1^\pm = \frac{v\eta_2^\pm + u\rho_1^\pm}{\sqrt{u^2 + v^2}}, \quad H_2^\pm = \frac{w\eta_3^\pm + u\rho_1^\pm}{\sqrt{u^2 + w^2}}. \tag{68}$$

$$m_{H_1^\pm}^2 = \frac{1}{2} \ \lambda_{11}uv - \sqrt{2}\mu w \ \left(\frac{u^2 + v^2}{uv}\right), \quad m_{H_2^\pm}^2 = \frac{1}{2} \ \lambda_{12}uw - \sqrt{2}\mu v \ \left(\frac{u^2 + w^2}{uw}\right). \tag{69}$$

Besides, we also find the Goldstone bosons of W and Y bosons as

$$G_{W^\pm} = \frac{u\eta_2^\pm - v\rho_1^\pm}{\sqrt{u^2 + v^2}}, \quad G_{X^\pm} = \frac{u\eta_3^\pm - w\rho_1^\pm}{\sqrt{u^2 + w^2}}. \tag{70}$$

6. Conclusion

In this study, we proposed a new version of 3-3-1-1 model where a new charged lepton for each generation is introduced. The new model can solve the remaining problems of the old versions of 3-3-1-1 model such as the limit of new physics energy scale due to the Landau pole. In this work, fermion, gauge boson and Higgs sectors were studied in detail. We identified all fermions of the SM as well as their masses. The model predicted new charged quarks and charged leptons beyond the SM. These particles received masses on a new physical scale of TeV which was estimated from the ρ parameter. The masses of Dirac and Majorana neutrinos were also determined.

For the gauge boson part, we identified not only the SM's bosons $W^{\pm}$, Z, and photon A but also six new gauge bosons $X^{\pm}, Y^{0,0*}, Z_2$ and , of which, Z_2, Z_3 received masses on a new physical scale. In order to reproduce the SM's $W^{\pm}$ boson mass, we constrained $u^2 + v^2 = 246\,\text{GeV}^2$. For the Higgs region, we identified the Higgs spectrum of the model, in which, h was identical to the Higgs in the SM. Using the SM-like Higgs mass, $m_h = 125\,\text{GeV}$, we could also estimate the values of the parameters in the Higgs' mass at around 0.52.

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