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Original Article

Effective Couplings between Two Photons and Axion in the 3-3-1 Model

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Abstract: The one-loop contribution axion-photon-photon coupling is presented in the framework of the 3-3-1 model, in which the loop diagrams are finite. The decay of axion into two photons is demonstrated. This study shows that it is easy to fulfill dark matter candidate conditions for the axion in the model.

Keywords: extensions of electroweak gauge sector, extensions of electroweak Higgs sector, Electroweak radiative correction

1. Introduction

At present, an axion is a very attractive issue in Particle Physics [1-6]. The axion is a CP-odd scalar field which arises in the solution of the strong-CP problem. It is interesting to note that nowadays the axion is widely considered as a candidate of dark matter (DM) [7]. The dark matter candidate only exists in some beyond the standard model scenarios. Among the SM extensions, the models based on the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetries (called 3-3-1 models, for short) [8-15] have several very interesting features, some of them being the natural explanation of the number of SM fermion families, the electric charge quantization, self-interating dark matter [16] and the solution of the strong CP problem from the PQ symmetry [17], which are automatically fulfilled in the 3-3-1 models. In one of

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the 3-3-1 models, there exist both interesting features, namely the axion dark matter candidate and inflaton for Early Universe [18, 19].

It is well known that the axion plays an important role in explanation of the XENON1T experiment [20]. Moreover, the couplings of the axion with other particles such as gauge bosons, fermions play a pivotal role in colidder search [21].

Our paper is organized as follows: in Section II, we present brief review of the model. Section III is devoted to necessary axion coupling. In Section IV, we present one-loop vertex for axion-photon-photon couplings. The decay width of the axion into two photons is presented in Section V. Finally, we state our conclusions in Section VI.

2. Brief Review of the Model

The model under consideration is based on $SU(3)_C \times SU(3)_L \times U(1)_X$ symmetry and has the following fermion content:

$$\begin{aligned}
&\psi_{aL} = \left(\nu_a,\ l_a,\ \left(\nu_R^c\right)_a\right)_L^T \sim (1,3,-1/3),\ l_{aR} \sim (1,1,-1),\ N_{aR} \sim (1,1,0),\\
&Q_{3L} = \left(u_3,\ d_3,\ U\right)_L^T \sim (3,3,1/3),\ Q_{\alpha L} = \left(d_\alpha,\ -u_\alpha,\ D_\alpha\right)_L^T \sim \left(3,\bar{3},0\right),\\
&u_{aR}, U_R \sim (3,1,2/3),\ d_{aR}, D_{\alpha R} \sim (3,1,-1/3),
\end{aligned} \tag{1}$$

where $\alpha = 1,2$ and $a = 1,2,3$ are family indices. The U and D are exotic quarks with ordinary electric charges, whereas N_{aR} are right-handed neutrinos.

The scalar sector of the model is composed of three $SU(3)_L$ scalar triplets and one $SU(3)_L$ singlet. They have the following transformations under the $SU(3)_C \times SU(3)_L \times U(1)_X$ symmetry:

$$\begin{aligned}
&\chi = \left(\chi_1^0,\ \chi_2^-,\ \chi_3^0\right)^T \sim (1,3,-1/3),\ \eta \sim \left(\eta_1^0,\ \eta_2^-,\ \eta_3^0\right)^T \sim (1,3,-1/3),\\
&\rho = \left(\rho_1^+,\ \rho_2^0,\ \rho_3^+\right)^T \sim (1,3,2/3),\ \phi \sim (1,1,0).
\end{aligned} \tag{2}$$

In order to keep intact the physics results, the Lagrangian of the model must be invariant by the discrete symmetries $Z_{11} \times Z_2$ which are summarised in Table 1. Here, we have used a notation $\omega_k \equiv e^{i2\pi\frac{k}{11}}, k = 0, \pm 1, ..., \pm 5.$

Table 1. $Z_{11} \times Z_2$ charge assignments of the particle content of the model. Here $\alpha = 1,2$ and $a = 1,2,3$.

	$Q_{\alpha L}$	Q_{3L}	u_{aR}	d_{aR}	U_R	$D_{\alpha R}$	ψ_{aL}	l_{aR}	N_{aR}	η	χ	ρ	ϕ
Z_{11}	ω_4^{-1}	1	ω_5	ω_2	ω_3	ω_4	ω_1	ω_3	ω_5^{-1}	ω_5^{-1}	ω_3^{-1}	ω_2^{-1}	ω_1^{-1}
Z_2	1	1	-1	-1	-1	-1	1	-1	1	1	-1	-1	1

Assuming fermions of opposite chiralities have opposite PQ charges and $X_d = X_D = 1$, we summarise PQ charges of fermions in Table 2.

Table 2. PQ charges of fermions in the model.

	u_{aL}	d_{aL}	U_L	D_L	ψ_{aL}	l_a	l_{aR}	ν_{aL}	ν_{aR}	N_{aR}
X_{PQ}	-1	1	1	1	1	1	1	1	-1	1

To generate masses for gauge bosons and fermions, the scalar fields should acquire vacuum expectation values (VEVs). These fields can be expanded around the minimum as follows:

$$\chi = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(R_\chi^1 + i I_\chi^1\right) \\ \chi^- \\ \frac{1}{\sqrt{2}}\left(v_\chi + R_\chi^3 + i I_\chi^3\right) \end{pmatrix}, \quad \eta = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_\eta + R_\eta^1 + i I_\eta^1\right) \\ \eta^- \\ \frac{1}{\sqrt{2}}\left(R_\eta^3 + i I_\eta^3\right) \end{pmatrix}, \tag{3}$$

$$\rho = \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}\left(v_\rho + R_\rho + i I_\rho\right) \\ \rho_3^+ \end{pmatrix}, \quad \phi = \frac{1}{\sqrt{2}}\left(v_\phi + R_\phi + i I_\phi\right). \tag{4}$$

Note that since ϕ carries non-zero PQ charge (as shown below), it has to be complex as shown in (4). The VEV v_χ is responsible for the first stage of gauge symmetry breaking, whereas v_η, v_ρ trigger the second stage of electroweak symmetry breaking providing a natural solution to the strong-CP problem.

The VEV v_ϕ is responsible for PQ symmetry breaking resulting in existence of invisible axion due to very high scale around $10^{10} - 10^{11}$ GeV. Then $SU(3)_L \times U(1)_X$ breaks into the SM group by v_χ and two others v_η, v_ρ are needed for the usual $U(1)_Q$ symmetry. Hence,

$$v_\phi \gg v_\chi \gg v_\rho, v_\eta. \tag{5}$$

The constraint conditions of such VEVs were analyzed in Ref. [18].

From an analysis of the scalar potential, we find that the physical CP odd neutral scalar mass eigenstates are:

$$\begin{pmatrix} I_\chi^3 \\ I_\eta^1 \\ I_\rho \\ I_\phi \end{pmatrix} = \begin{pmatrix} \cos\theta_3 & \sin\theta_3\cos\theta_4\cos\theta_\phi & -\sin\theta_3\sin\theta_4 & -\sin\theta_3\sin\theta_\phi\cos\theta_4 \\ \sin\theta_3 & \cos\theta_3\cos\theta_4\cos\theta_\phi & -\sin\theta_4\cos\theta_3 & -\sin\theta_\phi\cos\theta_3\cos\theta_4 \\ 0 & \sin\theta_4\cos\theta_\phi & \cos\theta_4 & \sin\theta_4\sin\theta_\phi \\ 0 & \sin\theta_\phi & 0 & \cos\theta_\phi \end{pmatrix} \begin{pmatrix} G_{Z'} \\ A_5 \\ G_Z \\ a \end{pmatrix}, \tag{6}$$

where the mixing angles in the CP odd scalar sector take the form:

$$\tan\alpha = \frac{v_\eta}{v_\rho}, \qquad \tan\theta_3 = \frac{v_\eta}{v_\chi},$$
$$\tan\alpha = \frac{v_\rho v_\eta}{v_\phi\sqrt{v_\rho^2+v_\eta^2}}, \; \tan\theta_4 \approx \tan\alpha. \tag{7}$$

The mixing angles in the CP odd scalar sector depend on the ratio of v_η to v_ρ, v_χ and v_ϕ.

The following Yukawa couplings are given by [22]

$$\begin{aligned} -L^Y &= y_1\bar{Q}_{3L}U_{3R}\chi + y_2^{\alpha\beta}\bar{Q}_{\alpha L}D_{\beta R}\chi^* + y_{3a}\bar{Q}_{3L}u_{aR}\eta + y_4^{\alpha a}\bar{Q}_{\alpha L}d_{aR}\eta^* \\ &+ y_{5a}\bar{Q}_{3L}d_{aR}\rho + y_6^{\alpha a}\bar{Q}_{\alpha L}u_{aR}\rho^* + g_{ab}\bar{\psi}_{aL}l_{bR}\rho \\ &+ \left(y_\nu^D\right)_{ab}\bar{\psi}_{aL}N_{bR}\eta + \left(y_N\right)_{ab}\bar{N}_{aR}^C N_{bR}\phi + H.c., \end{aligned} \tag{8}$$

where $\alpha,\beta = 1,2$ and $a,b = 1,2,3$ are family indices and, for simplicity, we have used Einstein notation for repeated indices.

3. Axion Couplings

The Lagrangian describes the interactions of axions or ALPs to SM particles as follows:

$$L = \frac{1}{2}\partial_\mu a\partial^\mu a + \frac{1}{2}m_a^2 a^2 + \frac{\alpha_s}{8\pi}\frac{a}{f_a}G_{\mu\nu}^a\tilde{G}^{a\mu\nu} + \frac{g_{a\gamma\gamma}}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} + g_{aff}\,a\,\bar{f}\gamma_5 f, \tag{9}$$

where f_a and g_{aff} are the effective axion decay constant and coupling constatnt of axion with fermions, respectively. g_{aff} for QCD axion can be written as

$$g_{aff} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - 1.92\right), \tag{10}$$

where E and N are the mixed anomaly coefficients of the PQ symmetry with EM and QCD respectively, the number 1.92 is contribution from QCD.

The couplings g_{aff} are model dependent, and their bounds are determined from other processes in Early Universe. Fortunately, these coulings are explicitly defined in the model under considereation. Indeed, substitution of (6) to (8) yields the necessary couplings

$$\begin{aligned} L_{(aff)} \supset &-\frac{i}{\sqrt{2}}\left(\sin\theta_3\sin\theta_\phi\cos\theta_4\right)\left(h^U\bar{U}\gamma_5 U - h_{\alpha\beta}^D\bar{D}_\alpha\gamma_5 D_\beta\right)\mathrm{a} \\ &- i\frac{\sqrt{2}m_t}{v_\eta}\sin\theta_\phi\cos\theta_3\cos\theta_4\, a\,\bar{t}\gamma_5 t + H.c. \end{aligned} \tag{11}$$

4. One-Loop Contribution Axion-photon-photon Coupling

Using these interactions, we can calculate one-loop diagrams of axion-photon-photon couplings as in Figure 1.

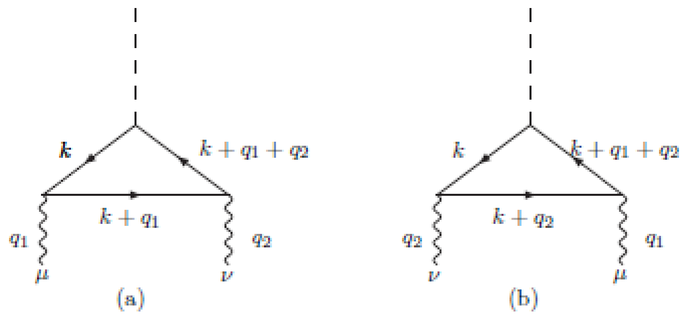

Figure 1. Feynman diagrams for axion-photon-photon couplings. Here, all momenta are incoming.

Contribution from diagram 1(a) is

$$\Gamma^{(a)} = -e^2 g_{a\!f\!f} \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}(S_1)}{D_1}, \tag{12}$$

where

$$S_1 = \gamma_5(\not{k} + \not{q}_1 + \not{q}_2 + m)\gamma_\nu(\not{k} + \not{q}_1 + m)\gamma_\mu(\not{k} + m), \tag{13}$$

$$D_1 = \left[(k+q_1+q_2)^2 - m^2\right]\left[(k+q_1)^2 - m^2\right](k^2 - m^2). \tag{14}$$

Similarly, for diagram 1(b)

$$\Gamma^{(b)} = -e^2 g_{a\!f\!f} \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}(S_2)}{D_2}, \tag{15}$$

where

$$S_2 = \gamma_5(\not{k} + \not{q}_1 + \not{q}_2 + m)\gamma_\mu(\not{k} + \not{q}_2 + m)\gamma_\nu(\not{k} + m),$$

$$D_2 = \left[(k+q_1+q_2)^2 - m^2\right]\left[(k+q_2)^2 - m^2\right](k^2 - m^2). \tag{16}$$

The integrals in (12) and (15) are logarithmically divergent. The presence of γ_5, which is determined only in four dimension, prevents from using dimensional regularization. To solve this difficulty, we should use the following trick: Any combination S of Dirac matrices can be expanded as the following combinations [23]:

$$S = \frac{1}{4}\left[\mathrm{Tr}(S)I + \mathrm{Tr}(\gamma_5 S)\gamma_5 + \mathrm{Tr}(\gamma_\alpha S)\gamma_\alpha + \mathrm{Tr}\gamma_5(\gamma_\alpha S)\gamma^\alpha\gamma_5 + 2\mathrm{Tr}(\sigma_{\alpha\beta}S)\sigma^{\alpha\beta}\right]. \tag{17}$$

Keeping in mind that the expression like S is under trace, so only the first term in (17) is survived. Therefore,

$$S_1 = \frac{1}{4}\mathrm{Tr}(S_1)I. \tag{18}$$

Let us deal with the first term

$$\begin{aligned}
Tr\left(S_1\right) &= Tr[\gamma_5(\not{k}+\not{q}_1+\not{q}_2+m)\gamma_\nu(\not{k}+\not{q}_1+m)\gamma_\mu(\not{k}+m)] \\
&= mTr[\gamma_5(\not{k}+\not{q}_1+\not{q}_2)\gamma_\nu(\not{k}+\not{q}_1)\gamma_\mu+(\not{k}+\not{q}_1+\not{q}_2)\gamma_\nu\gamma_\mu\not{k}+\gamma_\nu(\not{k}+\not{q}_1)\gamma_\mu\not{k}] \\
&= -4im[\varepsilon_{\rho\nu\eta\mu}(k+q_1+q_2)^\rho(k+q_1)^\eta+\varepsilon_{\rho\nu\mu\eta}(k+q_1+q_2)^\rho k^\eta+\varepsilon_{\nu\rho\mu\eta}(k+q_1)^\rho k^\eta] \\
&= -4im[\varepsilon_{\rho\nu\eta\mu}q_2^\rho(k+q_1)^\eta+\varepsilon_{\rho\nu\mu\eta}(q_1+q_2)^\rho k^\eta+\varepsilon_{\nu\rho\mu\eta}q_1^\rho k^\eta] \\
&= 4im\varepsilon_{\rho\eta\nu\mu}q_2^\rho q_1^\eta .
\end{aligned} \tag{19}$$

So in n dimension, we obtain

$$\mathrm{Tr}\left(S_1\right)=im\varepsilon_{\rho\eta\nu\mu}q_2^\rho q_1^\eta \mathrm{Tr}\left(I\right)=imn\varepsilon_{\rho\eta\nu\mu}q_2^\rho q_1^\eta . \tag{20}$$

For S_2, we have to make the following replacements: $\mu \leftrightarrow \nu$ and $q_1 \leftrightarrow q_2$. Hence,

$$\mathrm{Tr}\left(S_2\right)=im\varepsilon_{\rho\eta\nu\mu}q_1^\rho q_2^\eta \mathrm{Tr}\left(I\right)=-imn\varepsilon_{\rho\eta\nu\mu}q_1^\rho q_2^\eta . \tag{21}$$

Finally, we get one loop correction in \$n\$ dimension

$$\Gamma^{total}=\Gamma^{(a)}+\Gamma^{(b)}=-ie^2 g_{aff}mn\varepsilon_{\rho\eta\nu\mu}\left[q_2^\rho q_1^\eta\int\frac{d^nk}{\left(2\pi\right)^n}\frac{1}{D_1}-q_1^\rho q_2^\eta\int\frac{d^nk}{\left(2\pi\right)^n}\frac{1}{D_2}\right]. \tag{22}$$

It is emphasized that the intergals in (22) are finite. For further details on these integrals, the reader is referred to Ref. [24].

4. Decay of Axion into Two Photons

Firstly, assuming that the one-loop correction provides a leading contribution, we consider the axion decay into two photons.

$$a\left(p\right)\to\gamma\left(q_1,\lambda_1\right)+\gamma\left(q_2,\lambda_2\right), \tag{23}$$

where p is momentum of the axion and q_i,λ_i $\left(i=1,2\right)$ are momenta and polarization vectors of outgoing photons, respectively.

Using (22), we have amplitude, in which the loop in the inside contains all fermions of the model

$$M\left(a\to\gamma\gamma\right)=-i4e^2\sum_f m_f g_{aff}\varepsilon_{\rho\eta\nu\mu}\left[q_2^\rho q_1^\eta G_1-q_1^\rho q_2^\eta G_2\right]\varepsilon^\mu\left(q_1,la_1\right)\varepsilon^\nu\left(q_2,la_2\right), \tag{24}$$

where we have replaced n by 4 and denoted $G_i=\int\frac{d^4k}{\left(2\pi\right)^4}\frac{1}{D_i},\ i=1,2.$ Making summation over polarizations of the photons, we get

$$M^2\left(a\to\gamma\gamma\right)=32e^4\sum_f m_f^2 g_{aff}^2\left(q_1.q_2\right)^2\left(G_1G_1^*+G_2G_2^*+G_1G_2^*+G_2G_1^*\right). \tag{25}$$

Then decay width is given by

$$\Gamma(a\to\gamma\gamma)=\frac{M^2(a\to\gamma\gamma)}{32\pi m_a}=4\pi\alpha_{em}^2\sum_f m_f^2 g_{aff}^2 m_a^3\left(G_1G_1^*+G_2G_2^*+G_1G_2^*+G_2G_1^*\right), \tag{26}$$

where m_a is the axion mass.

To evaluate the above decay width, let us take the case of top quark. It is worth mentioning that coupling of axion with fermion is proportional to mass of the latter. Replacing coupling constant g_{aff} following from (10), we get the branching ration in the model under consideration

$$\begin{aligned}\Gamma(a\to\gamma\gamma)&=4\pi\alpha_{em}^2\frac{m_t^4}{v_\eta^2}\cos^2\theta_3\cos^2\theta_4\sin^2\theta_\phi\, m_a^3\left(G_1G_1^*+G_2G_2^*+G_1G_2^*+G_2G_1^*\right)\\ &\simeq 8\pi\alpha_{em}^2 m_t^4\,\frac{m_a^3}{v_\eta^2}\,\frac{v_\rho^4}{v_\eta^2\left(v_\rho^2+v_\eta^2\right)}\left(G_1G_1^*+G_2G_2^*+G_1G_2^*+G_2G_1^*\right).\end{aligned} \tag{27}$$

It is worth mentioning that VEVs v_η, v_ρ are in the ectroweak scale, v_χ in TeV scale, while v_ϕ is in 10^{10} GeV scale. So, decay width in (27) is very small, and this helps it to be DM candidate [25].

The contributions from exotic quarks are quite similar, and it will be published elsewhere.

5. Conclusion

In this paper, we have considered the decay of the axion to two photons in the framework of the special 3-3-1 model. The one-loop contribution axion-photon-photon couplings are presented and showed to be finite. The decay of axion into two photons is demonstrated. Since the mass of axion is evaluated to be around 3 keV, the above considered process is unique. Associated with factor $1/v_\phi^2$, the decay rate is very small, leading to the fact that the axion is a candidate for dark matter.

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