



Original Article

The Photo-stimulated Peltier Effect in Rectangular Quantum Wires under the Influence of Confined Phonons

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Abstract: Based on the kinetic equation method, the quantum Peltier effect has been theoretically studied under the influence of confined optical phonons in a rectangular quantum wire (RQW) of infinite high potential when placed in perpendicular electric and magnetic fields. There were complicated dependences of the analytical expression of the Peltier coefficient (PC) on quantities such as the size of the wires amplitude of the laser radiation, the cyclotron frequency of electrons and temperature of the system. Moreover, the presence of a strong electromagnetic wave (EMW) is also taken into account to determine the influence of confined phonons on the aforementioned effects. We have defined the analytical expressions for the kinetic tensors and PC. In detailed consideration, the quantum numbers m_1 , m_2 were changed in order to characterize the influence of confined optical phonons. The results of setting the m_1 , m_2 to zero showed that this case can be seen as the unconfined optical phonon. This means that the confinement of the phonon affects the Peltier effect quantitatively and qualitatively. The theoretical results have been numerically evaluated and discussed for the GaAs/GaAsAl quantum wires. The change of the confinement and non-confinement phonons upon the change of the magnetic field shows that the height of the resonance peaks in the confinement case is about 4 times larger than the non-confinement case. The position of the resonance peak in the two cases are also shifted. The size of the wire in the case of the confinement phonon is more pronounced and stable than in the non-confinement case. The new results obtained can provide for completing the theory of the Peltier effect in low - dimensional semiconductor systems.

Keywords: Confined optical phonon, the quantum Peltier effect, rectangular quantum wires (RQW), quantum kinetic equation, Photo-stimulated Peltier effect.

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1. Introduction

In 1834, Jean Charles Athanase Peltier, a French physicist, discovered the presence of heat at an electric field junction between two different conductors. This phenomenon is called Peltier effect [1]. The Peltier effect has been used to obtain cooling at low temperatures in semiconductors and semimetals [2]. The amount of heat that is conveyed by an electrical current when it goes through a material is described by the Peltier coefficient [3]. The nonlinear Peltier effect can be utilized to develop the cooling of thin-film micro refrigerator devices [4]. In low dimensional semiconductor systems, Bogachek has discovered the oscillations of the Peltier coefficient in a magnetic field, which is found in quantum point contacts [5] and in nanowires [6]. However, the Peltier effect has not been studied much in one-dimensional systems in general and in rectangular quantum wire (RQW), in particular. Besides, the influence of electromagnetic waves on the effect is still open, especially the effect of confined phonons on the Peltier effect has not been paid much attention.

Therefore, the quantum Peltier effect taking place in a RQW under the influence of confined phonon is still a question to be considered. Hence, to answer this question, we use the quantum kinetic equation method described in [7] to study the effect of confined phonon on the Photo-stimulated Peltier effect in RQW.

2. Analytic Expression for Peltier Coefficient in RQW

We examine a simple model of a RQW, in which the electrons quantized in the x-y plane can move freely in the other direction (z-direction). A magnetic field $\vec{B} = (0, B, 0)$, an electric field with $\vec{E} = (0, 0, E_1)$ and a strong EMW $\vec{E} = (0, 0, E_0 \sin \Omega t)$ were considered. Based on the quantum kinetic equation method, which has been calculated in 2DSS [8-11]. We apply this method to study the Peltier effect in RQW under the influence of the above mentioned conditions. From Hamiltonian of confined electrons-confined optical phonone (OP), we achieved the quantum kinetic equation for electron and the total current density and heat flux density are also found by using the following formulas:

$$\vec{J}_i(m_1, m_2) = \sigma_{ik}(m_1, m_2)E_k + \beta_{ik}(m_1, m_2)\nabla T_k \quad (1)$$

$$\vec{Q}_i(m_1, m_2) = \gamma_{ik}(m_1, m_2)E_k + \xi_{ik}(m_1, m_2)\nabla T_k \quad (2)$$

Here, $\sigma_{ik}(m_1, m_2)$, $\gamma_{ik}(m_1, m_2)$ and $\beta_{ik}(m_1, m_2)$, $\xi_{ik}(m_1, m_2)$ are conductivity tensors and dynamic tensors, the symbols i, j, k correspond for components x, y, z of the Cartesian coordinates. We will use these tensors to calculate the expressions of the tensors in PC such as $\sigma_{xx}(m_1, m_2)$, $\gamma_{xx}(m_1, m_2)$. And then, after some mathematical manipulation, the expression for Ettingshausen coefficient is obtained:

$$PC = \frac{\gamma_{xx}(m_1, m_2)}{\sigma_{xx}(m_1, m_2)} \quad (3)$$

$$\sigma_{xx}(m_1, m_2) = \frac{\tau}{1 + \omega_c^2 \tau^2} \left[ea + \frac{be}{m} (1 - \omega_c^2 \tau^2) \right] \quad (4)$$

$$\gamma_{xx}(m_1, m_2) = -\frac{\tau}{1 + \omega_c^2 \tau^2} cT (1 - \omega_c^2 \tau^2) \quad (5)$$

Here

$$\begin{aligned}
 a &= \sum_{N,n} \frac{e^2 L_z n_0}{\sqrt{2m\pi\beta\hbar}} e^{\beta(\varepsilon_F - \varepsilon_{N,n})} \\
 b &= \sum_{\gamma,\gamma'} \sum_{m_1,m_2} \left| I_{n,l,n',l'}^{m_1,m_2} \right|^2 \cdot \left(\frac{N!}{N!} \right)^2 \cdot \frac{e^{\beta(\varepsilon_F - \varepsilon_\gamma)}}{e^{\frac{\hbar\omega_{m_1,m_2}-1}{KT}}} (b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8) \\
 c &= -\frac{1}{mT} \sum_{\gamma,\gamma'} \sum_{m_1,m_2} \left| I_{n,l,n',l'}^{m_1,m_2} \right|^2 \cdot \left(\frac{N!}{N!} \right)^2 \cdot \frac{e^{\beta(\varepsilon_F - \varepsilon_\gamma)}}{e^{\frac{\hbar\omega_{m_1,m_2}-1}{KT}}} \cdot \\
 &\quad \left\{ (b_1 + b_3)(-\hbar\omega_{m_1,m_2}) + (b_2 + b_4)(\hbar\omega_{m_1,m_2}) + b_5(\hbar\omega_{m_1,m_2} - h\Omega) \right\} \\
 &\quad \left\{ +b_6(-\hbar\omega_{m_1,m_2} - h\Omega) + b_7(\hbar\omega_{m_1,m_2} + h\Omega) + b_8(-\hbar\omega_{m_1,m_2} + h\Omega) \right\} \\
 d &= \frac{1}{mT} \sum_{\gamma,\gamma'} \sum_{m_1,m_2} \left| I_{n,l,n',l'}^{m_1,m_2} \right|^2 \cdot \left(\frac{N!}{N!} \right)^2 \cdot \frac{e^{\beta(\varepsilon_F - \varepsilon_\gamma)}}{e^{\frac{\hbar\omega_{m_1,m_2}-1}{KT}}} \cdot \\
 &\quad \left\{ (b_1 + b_3)(-\hbar\omega_{m_1,m_2})^2 + (b_2 + b_4)(\hbar\omega_{m_1,m_2})^2 + b_5(-\hbar\omega_{m_1,m_2} - h\Omega) \right\} \\
 &\quad \left\{ +b_6(\hbar\omega_{m_1,m_2} - h\Omega) + b_7(-\hbar\omega_{m_1,m_2} + h\Omega) + b_8(\hbar\omega_{m_1,m_2} + h\Omega) \right\} \\
 b_1 &= A_0 e^{-x_1} \left\{ Z_1^{3/4} \left[2K_{\frac{3}{4}}(|x_1|) - F \left(K_{\frac{5}{2}}(|x_1|) + \frac{\hbar^2}{m} K_{\frac{3}{4}}(|x_1|) \right) + 2B_{11}K_0(|x_1|) \right] \right\} \delta(T_1) \\
 b_2 &= A_0 e^{-x_2} \left\{ Z_2^{3/4} \left[2K_{\frac{3}{4}}(|x_2|) - F \left(K_{\frac{5}{2}}(|x_2|) + \frac{\hbar^2}{m} K_{\frac{3}{2}}(|x_2|) \right) + 2B_{22}K_0(|x_2|) \right] \right\} \delta(T_2) \\
 b_3 &= A_1 e^{-x_1} \left\{ Z_1^{3/4} \left[\frac{x_1}{\sqrt{\pi}} K_{\frac{5}{2}}(|x_1|) + 2B_{11}K_{3/2}(|x_1|) - F \left(\frac{K_7(|x_1|)}{\hbar^3\beta} + \frac{K_5(|x_1|)}{\sqrt{\beta}} \right) \right] \right\} \delta(T_1) \\
 b_4 &= A_1 e^{-x_2} \left\{ Z_2^{3/4} \left[\frac{x_2}{\sqrt{\pi}} K_{\frac{5}{2}}(|x_2|) + 2B_{22}K_{3/2}(|x_2|) - F \left(\frac{K_7(|x_2|)}{\hbar^3\beta} + \frac{K_5(|x_2|)}{\sqrt{\beta}} \right) \right] \right\} \delta(T_2) \\
 b_5 &= A_2 e^{-x_3} \left\{ Z_3^{3/4} \left[\frac{x_3}{\sqrt{\pi}} K_{\frac{5}{2}}(|x_3|) + 2B_{33}K_{3/2}(|x_3|) - F \left(\frac{K_7(|x_3|)}{\hbar^3\beta} + \frac{K_5(|x_3|)}{\sqrt{\beta}} \right) \right] \right\} \delta(T_3) \\
 b_6 &= A_2 e^{-x_4} \left\{ Z_4^{3/4} \left[\frac{x_4}{\sqrt{\pi}} K_{\frac{5}{2}}(|x_4|) + 2B_{44}K_{3/2}(|x_4|) - F \left(\frac{K_7(|x_4|)}{\hbar^3\beta} + \frac{K_5(|x_4|)}{\sqrt{\beta}} \right) \right] \right\} \delta(T_4) \\
 b_7 &= A_2 e^{-x_5} \left\{ Z_5^{3/4} \left[\frac{x_5}{\sqrt{\pi}} K_{\frac{5}{2}}(|x_5|) + 2B_{55}K_{3/2}(|x_5|) - F \left(\frac{K_7(|x_5|)}{\hbar^3\beta} + \frac{K_5(|x_5|)}{\sqrt{\beta}} \right) \right] \right\} \delta(T_5) \\
 b_8 &= A_2 e^{-x_6} \left\{ \left(\frac{4m^2 B_{66}^2}{h^4} \right)^{3/4} \left[\frac{x_6}{\sqrt{\pi}} K_{\frac{5}{2}}(|x_6|) + 2B_{66}K_{3/2}(|x_6|) - F \left(\frac{K_7(|x_6|)}{\hbar^3\beta} + \frac{K_5(|x_6|)}{\sqrt{\beta}} \right) \right] \right\} \delta(T_6)
 \end{aligned}$$

$$\begin{aligned}
B_{11} &= \varepsilon_{N,n'} - \varepsilon_{N,n} - \hbar\omega_{m_1,m_2} \\
B_{22} &= \varepsilon_{N,n'} - \varepsilon_{N,n} + \hbar\omega_{m_1,m_2} \\
B_{33} &= \varepsilon_{N,n'} - \varepsilon_{N,n} - \hbar\omega_{m_1,m_2} - \hbar\Omega \\
B_{44} &= \varepsilon_{N,n'} - \varepsilon_{N,n} + \hbar\omega_{m_1,m_2} - \hbar\Omega \\
B_{55} &= \varepsilon_{N,n'} - \varepsilon_{N,n} - \hbar\omega_{m_1,m_2} + \hbar\Omega \\
B_{66} &= \varepsilon_{N,n'} - \varepsilon_{N,n} + \hbar\omega_{m_1,m_2} + \hbar\Omega \\
\varepsilon_{N,n} &= \frac{\pi^2 \hbar^2 n^2}{2mL_x^2} + \omega_c \left(N + \frac{1}{2} \right) - \frac{1}{2m} \left(\frac{eE_1}{\omega_c} \right)^2; \varepsilon_{N',n'} = \frac{\pi^2 \hbar^2 n'^2}{2mL_x^2} + \omega_c \left(N' + \frac{1}{2} \right) - \frac{1}{2m} \left(\frac{eE_1}{\omega_c} \right)^2 \\
Z_i &= \frac{4m^2 B_{ii}^2}{\hbar^4}; F = \frac{l_B^2 (N' + N + 1)}{\hbar^2 (M - 1)(M + 1)}
\end{aligned}$$

with

$$\begin{aligned}
T_1 &= \varepsilon_F + \hbar\sqrt{\omega_0^2 - \gamma^2 q_{m_1,m_2}^2}; \quad T_2 = \varepsilon_F - \hbar\sqrt{\omega_0^2 - \gamma^2 q_{m_1,m_2}^2}; \quad T_3 = T_1 - \hbar\Omega; \quad T_4 = T_2 - \hbar\Omega; \\
T_5 &= T_1 + \hbar\Omega; \quad T_6 = T_2 + \hbar\Omega; \quad \beta = \frac{1}{k_B T}; \quad (M = N' - N); \quad x_i = \frac{mB_{ii}\beta}{\hbar^2}, (i = 1 \div 6)
\end{aligned}$$

ε_F is the Fermi level; τ is the momentum relaxation time. $K_\nu(x)$ is the modified Bessel functions of the second kind.

From the expression of PC in the Eq. (3). It is immediately seen that the PC depends on many quantities such as the external fields, the temperatures, and the length of RQW. Especially the PC depends complicated on the quantum number m_1, m_2 characterizing the effect of confined OP. These results differ from previous studies such as in semiconductor [7] and in doped superlattice [10], the case of unconfined phonon in cylindrical quantum wires [12]. When the length of the wire grows into the bulk size, the results of the bulk phonons case can be achieved. In addition, the results are correct for all temperatures and are also accurate for all numerical methods. In the next section, we present the numerical calculation for the GaAs/AsGaAs RQW to demonstrate above mentioned dependence.

3. Numerical Results and Discussion

To clarify the theoretical results, we present the numerical evaluation of the Ettingshausen coefficient for GaAs/AlGaAs RQW in detail. The characteristic parameters are given by [13]: $m = 0.067m_0 (= 9.1095 \cdot 10^{-31} \text{ kg})$ is the mass of a free electron, $\chi_\infty = 10.9$, $\chi_0 = 12.9$; $\varepsilon_F = 50 \text{ meV}$; $e = 2.07e_0$ (e_0 is a charge of a free electron); $\tau = 10^{-1}$, $n_0 = 10^{23}$, $L_x = 15 \text{ (nm)}$, $L_y = 20 \text{ (nm)}$, $N = 1$, $N' = 3$, n and n' rate from 1 to 3.

Figure 1 gives information about the dependence of the PC on the magnetic field in both cases of phonon: confined OP (the red line) and unconfined OP (the blue line). It can be seen that in the case of the confined OP, the magnitude of PC is not only larger, but also is shifted to a smaller magnetic field domain than in the case of the unconfined OP. Moreover, the PC is increased about 4 times in magnitude by the confined OP. This result is similar to the result of the study on the Peltier effect in parabolic quantum wells of GaAs/AlGaAs reported in [14]. However, the PC value in our case stronger decreased compared to the one of GaAs/AlGaAs quantum well [14]. This can be explained due to the differences in the crystalline structure, the effect of confined phonons, the magnitude of

magnetic field EMW that lead to dissimilarity in the analysis results. Besides, the results are different from the previous results of the published study on the photo-stimulated quantum thermo-magnetoelectric effects in doped two-dimensional semiconductor superlattice (DSS) of GaAs/AlGaAs [10]. It is shown that the PC also was changed with the increased magnetic field, but this change is a form of Shubnikov-de Hass oscillations. These results are also different from the case of bulk semiconductors [7]. This difference can be explained by the influence of the reduction effect in the materials, the electron-phonon scattering between RQW and DSS, and the difference the value of m_1, m_2 .

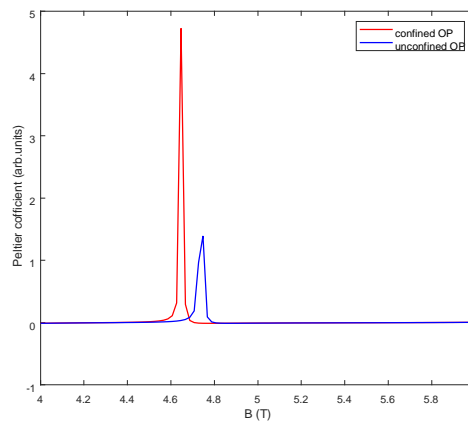


Figure 1. The dependence of the PC on the magnetic-field.

Figure 2 describes the dependence of PC on the size of the wires. It can be seen that the PC is affected by the size of the wires for two phonon models: confined OP and unconfined OP. When the size of the wire is less than 100 nm, due to the effect of confined OP, the value of the PC in this case is larger than in the unconfined OP case. When the size of the wire is larger than 100 nm, the effect of confined OP on the PC is small and can be ignored. In addition, for the wire is larger 120 nm, the result of the bulk case can be achieved [7].

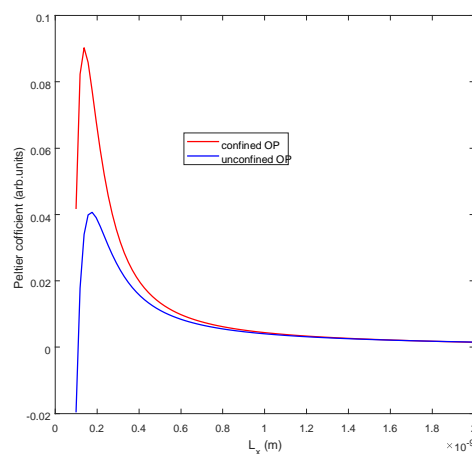


Figure 2. The dependence of the PC on the size of the wire.

4. Conclusion

In this work, we obtained the expression of Peltier coefficient (PC) using the quantum dynamic equation and calculated the number of the theoretical results for GaAs/AlGaAs in a rectangular quantum wire, the magnetic field is perpendicular. The analysis results show that the formula of PC depends on many quantities, especially the quantum number m that characterizes confinement phonon. When the size of the wires increase to infinity, we obtain the result corresponding to the case of bulk semiconductor. All numerical results showed that the influence of magnetic field and the size of the wire on Peltier coefficient in rectangular quantum wire in the presence of phonon confining resonance peaks occurs strongly and resonance position changed when the phonon confinement is absent. Our results are in a good agreement with a number of data including experimental survey results for rectangular quantum wires in two-dimensional systems in particular and low-dimensional semiconductor system of GaAs. Finally, the obtained results can contribute to enriching the theory of thermoelectric cases in general, and the Peltier effect in low-dimensional semiconductor systems, in particular.

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References

- [1] J. C. A. Peltier, *Nouvelles Expériences Sur La Caloricité Des Courans Électriques*, pp. 1834.
- [2] R.T. Delves, *Thermomagnetic Effects in Semiconductors and Semimetals*, Reports on Progress in Physics, Vol. 28, No. 1, 1965, pp. 249.
- [3] D. K. C. MacDonald, *Thermoelectricity: an Introduction to the Principles* (Courier Corporation), 2006.
- [4] M. Zebarjadi, K. Esfarjani, A. Shakouri, *Nonlinear Peltier Effect in Semiconductors*, Applied Physics Letters, Vol. 91, No. 12, 2007, pp. 122104, <https://doi.org/10.1063/1.2785154>.
- [5] E. Bogachek, A. Scherbakov, U. Landman, *Nonlinear Peltier Effect in Quantum Point Contacts*, Solid State Communications, Vol. 108, No. 11, 1998, pp. 851-855, [https://doi.org/10.1016/S0038-1098\(99\)80000-3i](https://doi.org/10.1016/S0038-1098(99)80000-3i).
- [6] E. Bogachek, A. Scherbakov, U. Landman, *Nonlinear Peltier Effect and Thermoconductance in Nanowires*, Physical Review B, Vol. 60, 1999, pp. 11678, <https://doi.org/10.1103/PhysRevB.60.11678>.
- [7] V. Malevich, E. Epshtein E, *Photostimulated Kinetic Effects in Semiconductors*, Soviet Physics Journal, Vol. 19, No. 2, 1976, pp. 230-237, <https://doi.org/10.1007/BF00942874>.
- [8] D. T. Hang, D. T. Huong, D. T. T. Thuy, N. Q. Bau, *The Ettingshausen coefficient in Quantum Wells Under Influence of Laser Radiation in the Case of Electron-Optical Phonon Interaction*, Photonics Letters of Poland, Vol. 8, No. 3, 2016, pp. 79-81.
- [9] N. Q. Bau, D. T. Hang, D. T. Long, *Study of the Quantum Magneto-thermoelectric Effect in the Two-dimensional Compositional Superlattice GaAs/AlGaAs Under the Influence of An Electromagnetic Wave By Using the Quantum Kinetic Equation*, Journal of the Korean Physical Society, Vol. 75, No. 12, 2019, pp. 1004-1016, <https://doi.org/10.3938/jkps.75.1004>.
- [10] N. Q. Bau, N. T. L. Quynh, C. T. V. Ba, L. T. Hung, *Doped Two-dimensional Semiconductor Superlattice: Photo-stimulated Quantum Thermo-magnetoelectric Effects Under the Influence of A Confined Phonon*, Journal of the Korean Physical Society, Vol. 77, No. 12, 2020, pp. 1224-1232, <https://doi.org/10.3938/jkps.77.1224>.
- [11] N. T. L. Quynh, N. T. Huong, N. B. Duc, N. Q. Bau, *The Quantum Ettingshausen Effect in Parabolic Quantum Wells with in-Plane Magnetic Field in the Presence of Laser Radiation Under the Influence of Confined Optical Phonon*, Vol. 1506, No. 1, 2020, pp. 012012.

- [12] H. V. Ngoc, N. Q. Bau, D. M. Quangand, T. H. Hung, One-dimensional Cylindrical Quantum Wire: The Theoretical Study of Photo-stimulated Ettingshausen Effect, *International Journal of Modern Physics B*, Vol. 36, No. 1, 2022, pp. 2250009, <https://doi.org/10.1142/S0217979222500096>.
- [13] N. Q. Bau, N. T. T Nhan, N. V. Nhan, Negative Absorption Coefficient of A Weak Electromagnetic Wave Caused By Electron Confined in Rectangular Quantum Wires in the Presence of Laser Radiation, *Journal of the Korean Physical Society*, Vol. 64, No. 4, 2014, pp. 572-578, <https://doi.org/10.3938/jkps.64.572>.
- [14] C. T. V. Ba, N. T. L. Quynh, N. Q. Bau, N. B. Duc, Quantum Peltier Coefficient Resonance in a GaAs/GaAsAl Parabolic Quantum Well with Confined Optical Phonons, Vol. 2269, No. 1, 2022, pp. 012008.