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Original Article

# Effect of Confined Optical Phonons on Photo-Stimulated Ettingshausen Effect in Rectangular Quantum Wires with A Perpendicular Magnetic Field

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**Abstract:** We applied the quantum kinetic equation method to investigate the influence of confined optical phonons (confined OP) on the photo-stimulated Ettingshausen effect in rectangular quantum wires (RQW) subjected to a perpendicular magnetic field. We considered the case where the confined electrons-confined OP scattering is the dominant mechanism. Analytical expressions for the kinetic tensors, the EC are obtained. The EC is a function of external fields, the temperature of the system, especially the quantum numbers  $m_1$  and  $m_2$  characterizing confined OP. When the width of the wire increases to infinity, the results of the bulk semiconductors can be gained. The numerical results are numerically evaluated and discussed for the GaAs/AlGaAs RQW. The magnitude of the resonance peaks has been increased for each value of  $m_1$ ,  $m_2$ , found when examining the dependence of the EC on the photon energy. Furthermore, the EC is decreased considerably when the amplitude of EMW increases, which is obtained when investigating the dependence of the EC on the amplitude of EMW (laser). These results are important for further researches and could be helped to complete the theory of the thermo - magnetoelectric effects in the low dimensional system.

*Keywords:* Confined optical phonon, the quantum Ettingshausen effect, rectangular quantum wires, quantum kinetic equation, Photo-stimulated Ettingshausen effect.

# 1. Introduction

The Ettingshausen effect is defined as a thermo-electric (or thermomagnetic) phenomenon that influences the electric current in a conductor when a magnetic field is present, discovered by Albert von

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Ettingshausen and Walther Nernst when examining the Hall effect in Bismuth [1]. This effect is calculated by the Ettigshausen coefficient (EC). In bulk semiconductors, the Ettingshausen has been studied by using the classical Boltzmann equation method [2, 3]. However, this method can only be considered in the case of weak magnetic fields and high temperatures. To overcome this drawback, Malevich and Epstein have applied quantum theory in general, quantum kinetic equation method, in particular, to study the Ettingshausen effect in bulk semiconductors in the presence of laser and indicated that the EC and kinetic tensors are altered on the specific parameters, magnetic field, electric field [4].

In two - dimensional semiconductor system (2DSS), the EC is two order in magnitude larger than the one of the bulk semiconductors when considering the temperature dependence of EC, which is achieved in the parabolic quantum well of GaAs/AlGaAs [5]. In compositional superlattice, the Shubnikovde Haas oscillations haves been appearing when examining the dependence of the quantum EC on the magnetic field (for electrons-acoustic phonons scattering) [6]. In the cylindrical quantum wire of GaAs/AlGaAs, the oscillation of EC is achieved with the transition between low Landau levels [7]. Nevertheless, most of the previous research only examined the case of confined electrons, the case of confined phonons has not been paid much consideration, especially in one-dimensional systems.

The effect of confined phonons on the quantum effects can give rise to new physical properties compared with the case without confined phonons. A value larger than the approximate value of  $\varepsilon_0$  has been produced by confined LO-phonon modes. It was obtained by examining the confined LO-phonon interaction with the electrons and holes [8]. The Hall conductivity increased by the laser radiation and the effect of confined optical phonons [9]. The full width at half-maximum (FWHM) of the optically detected magnetic phonon resonance peaks in the case of bulk phonons has a smaller value in the case of confined phonons, which is obtained in quantum well of GaAs [10]. On the other hand, the photostimulated effects (Ettingshausen effect and Peltier effect) in doped two-dimensional semiconductor superlattice of GaAs:Si/GaAs:Be under the effect of confined acoustic phonons have been examined [11]. The results reported in [11] have indicated that the confinement acoustic phonons can change the peaks in EC oscillations in the magnetic field, the magnitude of EC is significantly changed compared to the case of bulk phonons (unconfined phonons). But in one-dimensional semiconductor system, especially in rectangular quantum wire (RQW), the influence of confined phonons on the Ettingshausen effect until now is an open question.

How the quantum Ettingshausen effect will occur in RQW under the influence of the confined phonon is a question that needs to be answered. Hence, in this work, we focus on the effect of confined OP on the photo-stimulated Ettingshausen effect in RQW under the influence of a magnetic field perpendicular to the plane of motion of the free electrons, based on the quantum kinetic equation method [4]. We achieve the analytical expressions for the kinetic tensors and the EC. The numerical evaluation for the GaAs/AlGaAl RQW is also present to explicate the theoretical results.

# 2. The Photostimulated Ettingshausen Effect in RQW in the Case of Confinedelectrons-Confined Optical Phonons Scattering

The analytical expression for Ettingshausen (EC) is given by [4]:

$$EC = \frac{1}{B} \frac{det \begin{pmatrix} \sigma_{xx}(m_1, m_2) & \sigma_{xy}(m_1, m_2) \\ \gamma_{xx}(m_1, m_2) & \gamma_{xy}(m_1, m_2) \end{pmatrix}}{\sigma_{xx}det \begin{pmatrix} \beta_{xx}(m_1, m_2) & \sigma_{xx}(m_1, m_2) \\ \xi_{xx}(m_1, m_2) & -K_L & \gamma_{xx}(m_1, m_2) \end{pmatrix}},$$
(1)

where  $\sigma_{xx}(m_1,m_2)$ ,  $\sigma_{xy}(m_1,m_2)$ ,  $\gamma_{xx}(m_1,m_2)$ ,  $\gamma_{xy}(m_1,m_2)$  and  $\beta_{xx}(m_1,m_2)$ ,  $\zeta_{xx}(m_1,m_2)$  are conductivity tensors and dynamic tensors.

In this work, we consider in RQW the length  $L_z$  under the influence of an infinite confined potential, a magnetic field,  $\vec{B} = (0, 0, B)$ , a dc electric field  $\vec{E}_1 = (0, E_1, 0)$ , and a strong EMW (laser radiation)  $\vec{E} = (E_0 \sin \Omega t, 0, 0)$ . The wave function and the energy spectrum of an electron are given by:

$$\Psi_{N,n,k_z}\left(x,y,z\right) = \sqrt{\frac{1}{L_z L_y}} e^{ik_z z} \phi\left(x - x_0\right) \sin\left(\frac{\pi n y}{L_y}\right),\tag{2}$$

$$\varepsilon_{N,n}\left(k_{z}\right) = \frac{\hbar^{2}k_{z}^{2}}{2m} + \left(N + \frac{1}{2}\right)\hbar\omega_{c} - \frac{\left(Fl_{B}\right)^{2}}{2\beta_{0}},$$
(3)

where  $\beta_0 = \frac{\hbar^2}{(ml_B^2)}$ ; N = 0, 1, 2,... is the Landau level;  $F = eE_1$ ;  $L_z$  and  $k_z$  respectively being the length

of the quantum wire and wave vector of the electron in the z-direction;  $L_y$  is the normalization length;  $\omega_c = \frac{eB}{m^*}$  is the cyclotron frequency,  $m^*$  being the effective mass of an electron;  $\varphi_N(y - y_0)$  being

harmonic oscillator wave function, centered at  $x_0 = -\frac{\hbar k_z}{m \omega_c}$ , is given by:

$$\phi_N(x-x_0) = \frac{1}{\sqrt{2^N \sqrt{\pi N! a_c}}} e^{-\frac{1}{2a_c^2}(x-x_0)^2} H_N\left(\frac{x-x_0}{a_c}\right)$$
(4)

here  $H_N$  is a Hermite polynomia;  $a_c = \left(\frac{\hbar}{m^* \omega_c}\right)^{\frac{1}{2}}$  is the cyclotron radius.

To get the analytical expression for the EC as Eq. (1). We use the quantum kinetic equation method as seen in [4]. From the Hamiltonian of confined electrons-confined optical phonons system in an RQW, we establish the quantum kinetic equation for the electron distribution function. Solving this equation, we obtain the expressions for the total current density and heat flux density. After some mathematical manipulation, the expressions for the conductivity tensors and the kinetic tensors are attained. Finally, the obtained results for the EC in RQW can be written by

$$EC = \frac{1}{B} \frac{ekT}{1 + \omega_c^2 \tau^2} \frac{\left\{ -2\omega_c \tau \left[ \eta + \frac{\rho}{m} \left( 1 - \omega_c^2 \tau^2 \right) \right] - \left( 1 - \omega_c^2 \tau^2 \right)^2 \left[ \frac{\rho}{m} \omega_c \tau + \eta \right] \right\}}{\left\{ \frac{\tau}{1 + \omega_c^2 \tau^2} k^2 T \left( 1 - \omega_c^2 \tau^2 \right) - \left[ \eta + \frac{\rho}{m} \left( 1 - \omega_c^2 \tau^2 \right) \right] \left[ \frac{\tau}{1 + \omega_c^2 \tau^2} \frac{\nu}{e} \left( 1 - \omega_c^2 \tau^2 \right) - K_L \right] \right\}}$$
(5)

with:

$$\begin{split} &\kappa = \sum_{N,n} \frac{e^2 L_N n_0}{\sqrt{2m\pi\beta h}} e^{\beta(x_f - x_{N,n})}, \\ &\rho = \sum_{T,T'} \sum_{m_1,m_2} \left| I_{m,J,n',T'}^{m_1,m_2} \right|^2 \cdot \left( \frac{N'!}{N!} \right)^2 \frac{e^{\beta(x_f - x_f)}}{e^{-k_h T}} \left( P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 \right), \\ &Y = -\frac{1}{m'T} \sum_{T,T'} \sum_{m_1,m_2} \left| I_{m,J,n',T'}^{m_1,m_2} \right|^2 \cdot \left( \frac{N'!}{N!} \right)^2 \frac{e^{\beta(x_f - x_f)}}{e^{-k_h T}} \left( (P_1 + P_3) \left( -h\omega_{m_1,m_2} \right) + (P_2 + P_4) \left( h\omega_{m_1,m_2} \right) + P_3 \left( h\omega_{m_1,m_2} - h\Omega \right) + P_6 \left( -h\omega_{m_1,m_2} - h\Omega \right) + P_7 \left( h\omega_{m_1,m_2} + h\Omega \right) \right), \\ &\Gamma = \frac{1}{mT} \sum_{T,T'} \sum_{m_1,m_1} \left| I_{m,J,n',T'}^{m_1,m_1'} \right|^2 \left( \frac{N'!}{N!} \right)^2 \cdot \frac{e^{\beta(x_f - x_f)}}{e^{-k_h T}} \left( (P_1 + P_3) \left( -h\omega_{m_1,m_2} \right)^2 + P_7 \left( -h\omega_{m_1,m_2} + h\Omega \right) \right), \\ &\Gamma = \frac{1}{mT} \sum_{T,T'} \sum_{m_1,m_1'} \left| I_{m,J,n',T'}^{m_1,m_1''} \left| \frac{N'!}{N!} \right|^2 \cdot \frac{e^{\beta(x_f - x_f)}}{e^{-k_h T}} \left( (P_1 + P_3) \left( -h\omega_{m_1,m_2} \right)^2 + P_7 \left( -h\omega_{m_1,m_2} + h\Omega \right) \right), \\ &\Gamma = \frac{1}{mT} \sum_{T,T''} \sum_{m_1,m_1''} \left| I_{m,J,n',T'}^{m_1,m_1''} \left| \frac{N'!}{N!} \right|^2 \cdot \frac{e^{\beta(x_f - x_f)}}{e^{-k_h T}} \left( (P_1 + P_3) \left( -h\omega_{m_1,m_2} \right)^2 + P_7 \left( -h\omega_{m_1,m_2} + h\Omega \right) \right), \\ &\Gamma = \frac{1}{mT} \sum_{T,T'''} \sum_{m_1,m_1''} \left| \frac{N'!}{2K_{3/4} \left( |x_1| \right) - F \left( \frac{K_{5/2} \left( |x_1| \right) + \frac{h^2}{m} K_{3/4} \left( |x_1| \right) \right) + 2B_{11}K_0 \left( |x_1| \right) \right) \right] \\ &P_1 = A_0 e^{-x_1} \left\{ Z_2^{1/4} \left[ 2K_{3/4} \left( |x_1| \right) - F \left( K_{5/2} \left( |x_1| \right) + \frac{h^2}{m} K_{3/3} \left( |x_2| \right) \right) + 2B_{12}K_0 \left( |x_2| \right) \right) \right] \right\} \delta(T_1), \\ &P_2 = A_0 e^{-x_1} \left\{ Z_1^{3/4} \left[ \frac{X_2}{\sqrt{\pi}} K_{5/2} \left( |x_1| \right) + 2B_{11}K_{3/2} \left( |x_2| \right) - F \left( \frac{K_{7/2} \left( |x_1| \right)}{h^3 \beta} + \frac{K_{5/2} \left( |x_1| \right)}{\sqrt{\beta}} \right) \right) \right] \right\} \delta(T_1), \\ &P_1 = A_1 e^{-x_1} \left\{ Z_2^{3/4} \left[ \frac{X_2}{\sqrt{\pi}} K_{5/2} \left( |x_1| \right) + 2B_{3/3}K_{3/2} \left( |x_2| \right) - F \left( \frac{K_{7/2} \left( |x_1| \right)}{h^3 \beta} + \frac{K_{5/2} \left( |x_1| \right)}{\sqrt{\beta}} \right) \right) \right] \right\} \delta(T_2), \\ &P_3 = A_2 e^{-x_1} \left\{ Z_3^{3/4} \left[ \frac{X_4}{\sqrt{\pi}} K_{5/2} \left( |x_4| \right) + 2B_{3/3}K_{3/2} \left( |x_4| \right) - F \left( \frac{K_{7/2} \left( |x_3| \right)}{h^3 \beta} + \frac{K_{5/2} \left( |x_3| \right)}{\sqrt{\beta}} \right) \right) \right] \right\} \delta(T_3), \\ &P_7 = A_7 e^{-x_7} \left\{ Z_3^{3/4} \left[ \frac{X_4}{\sqrt{\pi}}$$

Here,

$$Z_{i} = \frac{4m^{2}B_{ii}^{2}}{h^{4}}; F = \frac{l_{B}^{2}(N'+N+1)}{\hbar^{2}(M-1)(M+1)} \text{ with } (M = N'-N); x_{i} = \frac{mB_{ii}\beta}{\hbar^{2}}, (i = 1 \div 6)$$

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$$\begin{split} B_{11} &= \varepsilon_{N,n'} - \varepsilon_{N,n} - h\omega_{m_1,m_2}, \\ B_{22} &= \varepsilon_{N,n'} - \varepsilon_{N,n} + h\omega_{m_1,m_2}, \\ B_{33} &= \varepsilon_{N,n'} - \varepsilon_{N,n} - h\omega_{m_1,m_2} - h\Omega, \\ B_{44} &= \varepsilon_{N,n'} - \varepsilon_{N,n} + h\omega_{m_1,m_2} - h\Omega, \\ B_{55} &= \varepsilon_{N,n'} - \varepsilon_{N,n} - h\omega_{m_1,m_2} + h\Omega, \\ B_{66} &= \varepsilon_{N,n'} - \varepsilon_{N,n} + h\omega_{m_1,m_2} + h\Omega. \end{split}$$

with  $\varepsilon_{\gamma} = \frac{\pi^2 n^2 \hbar^2}{2m^* L_{\gamma}^2} + \hbar \omega_c \left( N + \frac{1}{2} \right); \ \beta = \frac{1}{k_B T}; \ \omega_0 \text{ is the frequency of phonon; } \varepsilon_F \text{ is the Fermilevel; } \tau$ 

is the momentum relaxation time.  $K_{\nu}(x)$  is the modified Bessel functions of the second kind.

From the expression of EC in the Eq. (1), it is immediately seen that the EC depends on many quantities such as the external fields, the temperatures, and the length of RQW. Especially it complicatedly depends on the quantum number  $m_1$ ,  $m_2$  characterizing the effect of confined OP. These results differ from the one of previous works, the reults on the semiconductors [4] and on the doped superlattices [11], or from the case of unconfined phonon in cylindrical quantum wires [7]. When the width of the wire increases to the bulk size, the results of the bulk case can be achieved. In addition, the results are correct for all temperatures and are also accurate for all numerical methods.

#### 3. Numerical Results and Discussion

In detail, we present the numerical evaluation of the Ettingshausen coefficient for GaAs/AlGaAs RQW to clarify the theoretical results. The characteristic parameters are given by [12]:  $m = 0.067m_0$  (= 9.1095.10<sup>-31</sup> kg) is the mass of a free electron,  $\chi_{\infty} = 10.9$ ,  $\chi_0 = 12.9$ ;  $\varepsilon_F = 50meV$ ;  $e = 2.07e_0$  ( $e_0$  is a charge of a free electron);  $\tau = 10^{-12}$ ,  $n_0 = 10^{23}$ ,  $L_x = 15(nm)$ ,  $L_y = 20(nm)$ , N = 1, N = 3, n and n rate from 1 to 3.

Figure 1 gives information about the dependence of the EC on the photon energy.



Figure 1. The dependence of the EC on the photon energy.

When  $E_0 = 4.10^4$  (V/m), B = 3.5T,  $\Omega = 2.10^{13}$  (Hz) in both cases of phonon: confined OP (the red line) and bulk phonon (the dashed - dot black line). It can be seen that there have been many resonance peaks appearing in the graph. These resonance peaks satisfy the condition  $\varepsilon_{N,n} - \varepsilon_{N,n} \pm s\hbar\Omega \pm \hbar\omega_0 = 0$ , which defined the magneto-phonon resonance (MPR) condition [13, 14]. When phonon is confined, the  $\hbar\omega_0$  changed to  $\hbar\omega_{m_1,m_2}$ . It makes the resonance peaks in this case is higher and clearer than the case of unconfined OP. In addition, the results are nearly consistent with the results in parabolic quantum wells with in-plane magnetic field [15]. However, the influence of confined OP in RQW is stronger compared to the quantum well.



Figure 2. The dependence of the EC on the amplitude of laser.

Figure 2 shows the amplitude dependence of the EC of the laser in two cases: i) Confined OP; and ii) The unconfined OP. There was a considerable change in the EC when the amplitude of the laser increases in two phonon models. The EC decreased in a non-linear way, when the amplitude of the laser is smaller than  $3.10^{5}$ (V/m), due to the effect of the confined OP, which makes the EC decline faster. This contrasts with the EC results obtained from previously published work in a parabolic quantum well with an in-plane magnetic field [15].

# 4. Conclusion

The effect of confined optical phonon on the photo-stimulated Ettingshausen effect in RQW of GaAs/AlGaAs subjected to a perpendicular magnetic field has been studied by using the quantum kinetic equation method. The case of confined electrons - confined optical phonons scattering has been considered. The analytic results indicate that the expression of EC depends on many quantities, especially the quantum numbers  $m_1$ ,  $m_2$ . When the width of the wire increases to infinity, we obtained the results of the bulk cases. Besides, the effect of photon energy on the EC is observed. These results are nearly consistent with the one of the previous studies in quantum well of GaAs/AlGaAs [15]. In addition, the EC decreased significantly when the amplitude is smaller than  $3.10^5$ (V/m). The effect of magnetic field on the temperature difference was also observed. The phonon confinement plays an important role in producing the resonance peak of the EC. Therefore, the obtained results in this work can contributed to the completion of the theory of the themor-magnetoelectric effects in one-dimensional systems.

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