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Original Article

Investigation of Cross-sections for (*α, γ*) Reactions on *p*-nuclei ^{90}Zr, ^{121}Sb, ^{151}Eu, and ^{162}Er using Different Models of Radiative Strength Functions

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Abstract: In this work, we examine cross sections for (*α, γ*) reactions on *p*-nuclei, including ^{90}Zr, ^{121}Sb, ^{151}Eu, and ^{162}Er at astrophysically relevant energies. By using our recently developed *α* optical model potential (*α*-OMP), the (*α, γ*) cross sections were calculated within six models of radiative strength functions (RSF) which consisted of the microscopic HFB-QRPA model based on the BSk-14 Skyrme force, the HF-BCS theory using Skyrme parameters, the EGLO model, SMLO model, the empirical SMLO (SMLOg) and global semi-microscopic (D1M-QRPAg) models. The numerical results is then compared to the measured data of (*α, γ*) cross sections. For the considered (*α, γ*) reactions, the EGLO, SMLO, and HFB results are typically greater than the measured data. In addition, the comparison has indicated that the RSF models of SMLOg and D1M-QRPAg best fit the measured data with *rms* smaller than 0.2 within our proposed *α*-OMP. Therefore, for the (*α, γ*) reactions on the selected targets, RSF models of SMLOg and D1M-QRPAg are strongly recommended. The results are significant for a further systematic examination to evaluate which RSF model is most appropriate for studying (*α, γ*) reactions on *p*-nuclei in general.

Keywords: *p*-nuclei, α optical model potential, radiative strength function.

1. Introduction

It is well known that elements up to iron can be produced in stars through the slow (*s*) and rapid (*r*) neutron capture reactions [1, 2]. These two processes, however, are not thought to directly contribute to the formation of a small number of neutron-deficient isotopes (known as *p*-nuclei) [3]. These nuclei

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could be formed by photodisintegrations on previously generated *s* and *r* nuclei in supernova environments [4], such as the (*γ, p*), (*γ, n*) and (*γ, α*) reactions. Despite being found in small amounts, the abundance of *p*-nuclei predicted by theory is significantly lower than that observed in the solar system in both low and high mass regions [5, 6]. This discrepancy can be attributed to uncertainties in stellar evolution models as well as nuclear physics input, specifically photodisintegrations [7]. As for the latter nuclear parameter, the inverse radiation capture reaction is studied more commonly than the direct photodisintegrations because of the significant influence of the thermally excited state under stellar environment [8]. The photo-decay reaction rate will then be calculated using the inverse kinematics method [9]. As a result, the theoretical and experimental study of radiative light particle capture reactions, particularly the (*α, γ*) cross sections, has received considerable attention recently.

The majority of the (*α, γ*) reaction rates are theoretically calculated using the Hauser-Feshbach (HF) statistical method, with the key inputs being the α-OMP, nuclear level density (NLD), and radiative strength functions (RSF) [10]. Many theoretical and experimental research efforts have been made in recent years to determine the global α-OMPs optical potentials at low energies [11-13], which has significantly improved the ability to theoretically predict the rate of reactions induced by α-particles. For example, α-OMP proposed by Avrigeanu et al., accurately described the experimental data of α-induced reactions on most *p* nuclei [13-15]. However, while the α-OMP model using the double folding method (DFM) used in the recent work [12] achieves high accuracy for the α particle absorption width, the difference between the calculated and experimental cross-sections of the reaction (*α, γ*) are still observable at the energy level below the Coulomb barrier [12]. Therefore, studying the effect of nuclear physics inputs, in particular the RSF on the cross sections of the reactions (*α, γ*) on p-type target nuclei is of great interest and motivates to investigate the cross-sections for (*α, γ*) reactions on *p*-nuclei ^{90}Zr, ^{121}Sb, ^{151}Eu, and ^{162}Er using different RSF models.

In this work, we examined the effect of various RSF models on the cross sections for target nuclei at astrophysically relevant energies, which consisted of ^{90}Zr, ^{121}Sb, ^{151}Eu, and ^{162}Er. The RSF models under evaluation are the microscopic HFB-QRPA model based on the BSk-14 Skyrme force [16], the HF-BCS theory using Skyrme parameters [17], the EGLO model [18], SMLO model [19], the empirical SMLO (SMLOg) and global semi-microscopic (D1M-QRPAg) models. The last two models for electric and magnetic dipole RSF have recently been developed [20, 21], which theoretical calculations were compared to all experiments. They are expected to perform reasonably well when extrapolated to unknown nuclei. To examine the calculated results, the obtained cross sections are compared to the available experimental data. The paper is structured as follows: In Section 2, we present the α-OMPs that was developed in [12]. Section 3 contains the numerical results and discussion for the obtained (*α, γ*) cross sections for four target nuclei. Finally, in Section 4, important conclusions are listed.

2. Theoretical Framework

We calculate (*α, γ*) cross sections using the α-OMP [$U(r,E)$] described in [12]

$$U(r,E) = V_{DF}(r,E) + \Delta V(r,E) + iW(r,E), \quad (1)$$

$$V_{DF}(r,E) = V_N(r,E) + V_C(r) + V_{rep.}(r,E). \quad (2)$$

In Eq. (2), $V_{DF}(r,E)$ is calculated using the double-folding model (DFM), which includes the attractive nuclear potential $V_N(r,E)$, the repulsive Coulomb potential $V_C(r)$, and the additional repulsive potential $V_{rep.}(r,E)$ (DFM+repulsion model). The real term $\Delta V(r,E)$ in Eq. (1) represents the dispersive contribution to the α-OMP, which can be determined through using imaginary term $W(r,E)$ [22].

We first estimated the transmission coefficients using the α-OMP derived from the DFM+repulsion model in the R-matrix computations. The (*α, γ*) cross sections are then calculated using the KEWPIE code [23], which requires transmission coefficients of α particles as inputs. The latter term is computed using the R-matrix method [24] and the α-OMP given from Eq. (1). In the present calculations, KEWPIE code was slightly modified to enable *E1* and *M1* strengths and their corresponding NLDs to be used as inputs simultaneously in order to assure consistency in microscopic RSF model calculations. While Reisdorf's NLDs [25] were used for simulations of the phenomenological RSF models of EGLO, SMLO, D1M-QRPAg, and SMLOg.

3. Numerical Results and Discussion

In this work, we also examined the impact of RSF models on the calculated cross secrions for (*α, γ*) reactions on four target nuclei relevant to *p*-nuclei, such as ^{90}Zr, ^{121}Sb, ^{151}Eu, and ^{162}Er. To begin, we calculated the transmission coefficients using the α-OMP calculated from the DFM+repulsion model in the R-matrix calculations (Fig. 1). These coefficients are integrated in the modified KEWPIE code, which generates (*α, γ*) cross sections using six different RSF models: EGLO, SMLO, SMLOg, D1M-QRPAg, HF-BCS, and HFB-QRPA (Fig. 2). Finally, we assessed the uncertainty in the computed (*α, γ*) cross sections and the reliability of the RSF models using the standard deviation, *rms*, by comparing the predicted results to those obtained from the experimental data, which reads

$$rms = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log\sigma_i - \log\sigma_{exp}\right)^2}.$$

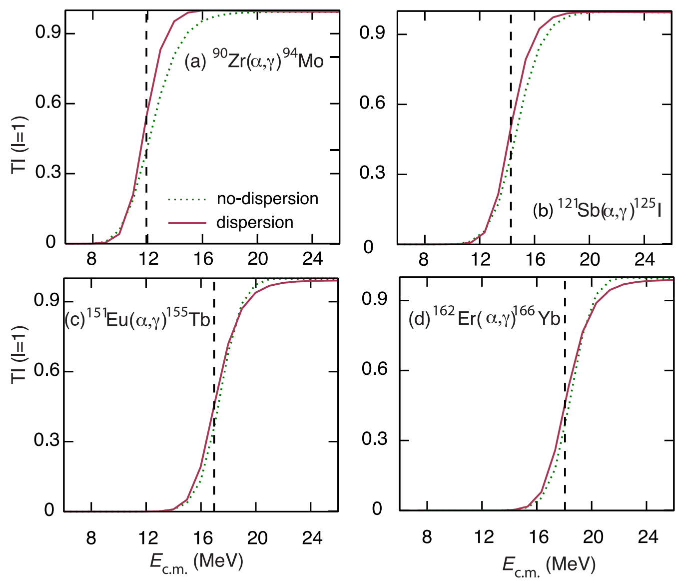

Figure 1. Shows the α transmission coefficients (at l=1) of (*α, γ*) reactions on four target nuclei ^{90}Zr, ^{121}Sb, ^{151}Eu, and ^{162}Er. The solid (dashed) line represents α transmission coefficients with (without) dispersion contribution.

It should be noted that, in contrast to previous DFM models, our proposed α-OMP model includes a dispersive contribution, and the effect of this quantity on α transmission coefficients is depicted in

Fig. 1. As can be seen, this contribution is prioritized near the Coulomb barrier and contributes significantly to the characterization of α-width [12]. After calculating the α transmission coefficients, the modidfied KEWPIE program code was used to calculate the cross section for (*α, γ*) reactions on four target nuclei: ^{90}Zr, ^{121}Sb, ^{151}Eu, and ^{162}Er. As illustrated in Fig. 2, the SMLOg, D1M-QRPAg model accurately describes the experimental data for all four reactions. The reactions on targets ^{90}Zr, ^{121}Sb and ^{151}Eu are poorly described by the HFB-QRPA and HFBCS models, and the calculation results of these HFB-QRPA and HFBCS models differ greatly from the rest of the RSF models, particularly in the region above the Coulomb barrier. The *rms* is then calculated and displayed in Fig. 3 to determine which RSF model best describes four (*α, γ*) reactions considered.

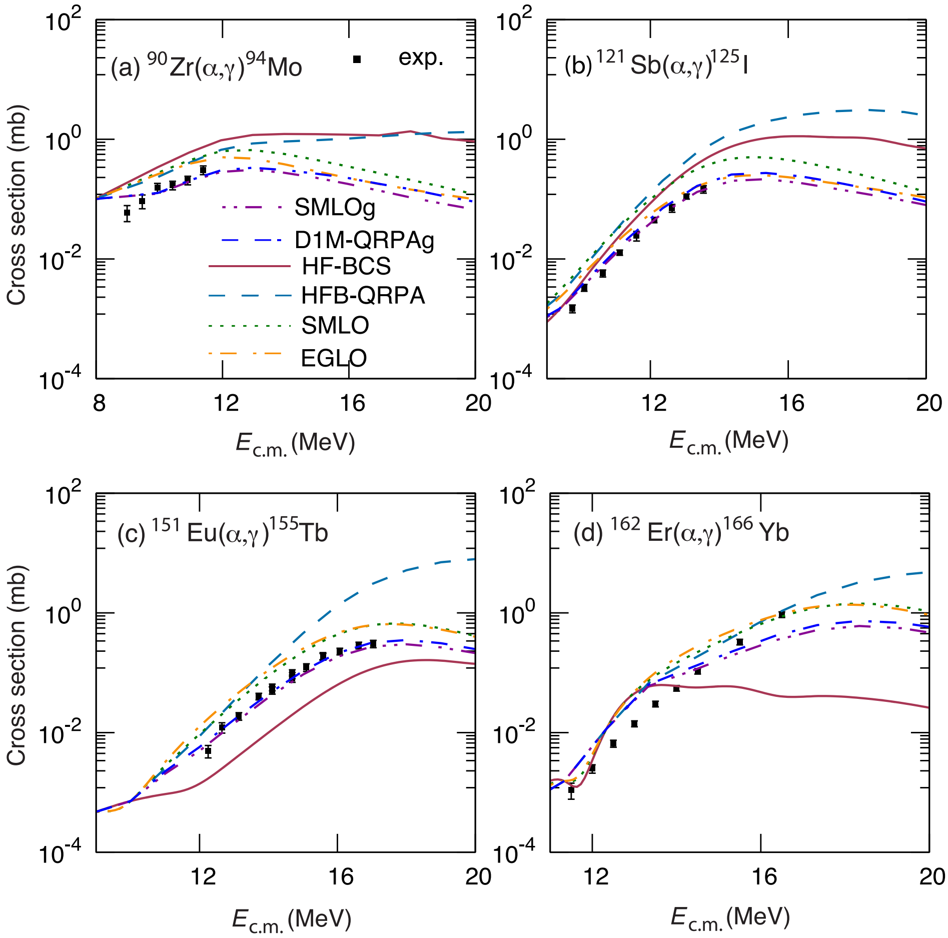

Figure 2. (Color online) Cross section for (*α, γ*) reactions predicted by different RSF models (EGLO, SMLO, SMLOg, D1M-QRPAg, HF-BCS, and HFB-QRPA) as functions of center of mass (c.m.) energy. The calculated results are compared to experimental data for (a) ^{90}Zr [26], (b) ^{121}Sb [27], (c) ^{151}Eu [28], and (d) ^{162}Er [29].

As can be observed in Fig. 3, the EGLO, SMLO, SMLOg, and D1M-QRPAg models are generally closer to zero, while the HF-BCS calculation has the greatest deviation. In general, all six RSF models produce results in a range of -1 to +1. For considered (*α, γ*) reactions on ^{90}Zr, ^{121}Sb, ^{151}Eu, and ^{162}Er nuclei, the EGLO, SMLO, and HFB predictions are slightly larger than the experimental values. We also calculated *rms* for the above six RSF models with 41 experimental values for all four reactions to assess reliability of the RSF models. The calculated *rms* values for HF-BCS, HFB-QRPA, EGLO, SMLO, SMLOg, and D1M-QRPAg are 0.541; 0.460; 0.308; 0.325; 0.172, and 0.158, respectively. Thus, using our proposed α-OMP, RSF models of SMLOg, and D1M-QRPAg best reproduce the experimental data for considered (α, γ) reactions. This is understandable given that SMLOg and D1M-QRPAg are two new RSF models formulated based on extensive data compilations, thus they certainly perform reasonably well when extended to nuclei with no available RSF data. Meanwhile, the HF-BCS model has the largest discrepancy between calculated and experimantal results. This is due to the fact that the BCS model describes a poorly described pairing effect and the number of nucleons is not conserved. Thus, using our proposed α-OMP, the two RSF models of SMLOg and D1M-QRPAg are highly recommended for the (α, γ) reactions examined. However, systematic investigation is also highly desirable in order to have a general assessment of which RSF model is good for research (α, γ) reactions on *p*-nuclei.

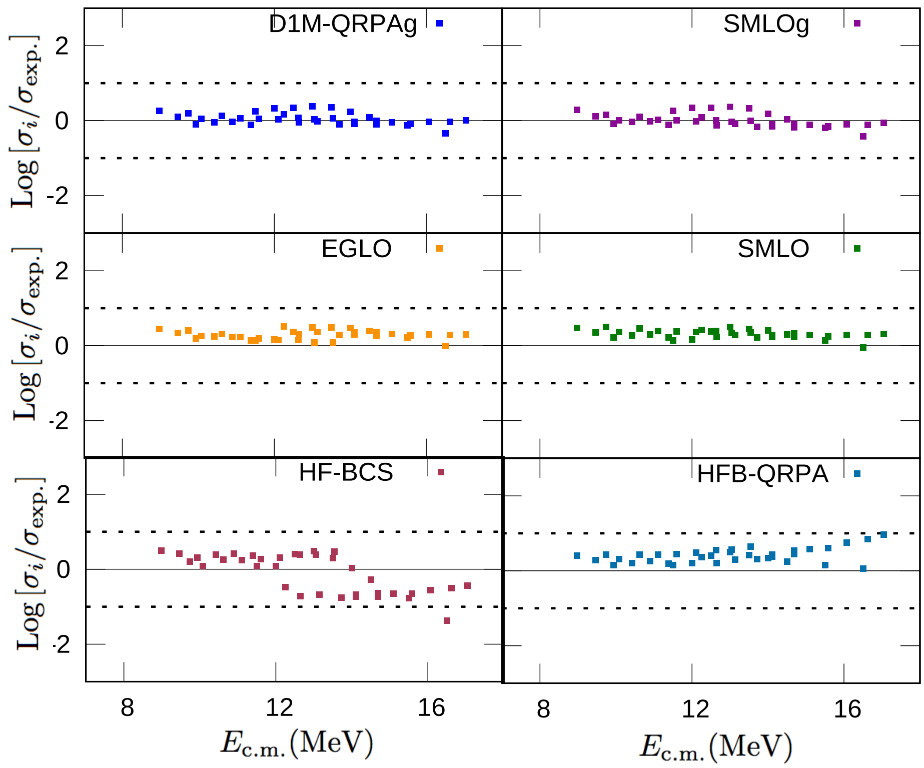

Figure 3. (Online color) Logarithmic ratios of computed cross sections of (α, γ) reactions on discussed nuclei using RSF models of EGLO, SMLO, SMLOg, D1M-QRPAg, HF-BCS, and HFB-QRPA to experimental values.

4. Conclusion

We have theoretically investigated cross sections for (α, γ) reactions on the target nuclei ^{90}Zr, ^{121}Sb, ^{151}Eu, and ^{162}Er. Based on the α-OMP proposed in [12], we have calculated transmission coefficients of α particles using the R matrix method. By using the modified KEWPIE code, we have obtained computationally cross sections for (α, γ) reactions on the target nuclei ^{90}Zr, ^{121}Sb, ^{151}Eu, and ^{162}Er. The comparison between the calculated and measured (α, γ) cross sections has shown that the EGLO, SMLO, and HFB estimations are typically greater than the measured results for considered (*α, γ*) reactions. In addition, RSF models of SMLOg, and D1M-QRPAg best describe the measured data with *rms* less than 0.2 for the considered (*α, γ*) reactions. Hence, RSF models of SMLOg and D1M-QRPAg are highly recommended for the (*α, γ*) reactions on the targets ^{90}Zr, ^{121}Sb, ^{151}Eu, and ^{162}Er. It is also very desirable to conduct a further systematic analysis in order to determine which RSF model is best for studying (*α, γ*) reactions on *p*-nuclei by using our proposed α-OMP.

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